KARPAGAM ACADEMY OF HIGHER EDUCATION

COIMBATORE – 641 021

DEPARTMENT OF MATHEMATICS

Curriculum format for under graduate degree program (three year course)

(Proposal for batches to be admitted on 2015 onwards)

SEMESTER – V							
15MMU501	Real Analysis - I	05	40	60	100	3	05
15MMU502	Complex Analysis - I	05	40	60	100	3	05
15MMU503	Numerical Methods	05	40	60	100	3	04
15MMU511	**Numerical Methods-Practical**	05	40	60	100	3	03
15MMU504	MATLAB programming	05	40	60	100	3	05
15MMU505A/ 15MMU505B/ 15MMU505C	Elective – 1	05	40	60	100	3	05
15OEU501	Open elective	-	-	100	100	3	03
	Semester total	**30**	**240**	**460**	**700**	**-**	**30**

Semester – V
L T P C
15MMU502 COMPLEX ANALYSIS –I 5 0 0 5

Scope: This course will enhance the learner to understand the important concepts such as complex number system, complex plane analyticity of a function, function of complex variables etc which plays a crucial role in the application of two dimensional problems in Science.

Objectives: To enable the students to learn various aspects complex number system, complex function and complex integration

UNIT I

Complex number system:Complex number-Field of a complex numbers-Conjugation –Absolute value of a complex number.
Complex plane: Complex number by points-n_{th} root of a complex number-Angle between two rays-Elementary transformation- Stereographic projection.

UNIT II

Analytic functions: Limit of a function –continuity –differentiability – Analytical function defined in a region –necessary conditions for differentiability –sufficient conditions for differentiability – Cauchy-Riemann equation in polar coordinates –Definition of entire function.

UNIT III

Power Series: Absolute convergence –circle of convergence –Analyticity of the sum of a power series-Uniqueness of representation of a function by a power series- Elementary functions : Exponential, Logarithmic, Trigonometric and Hyperbolic functions. Harmonic functions: Definition and determination.

UNIT IV

Bilinear transformation-Circles and Inverse points-Transformation mappings $w=Z^2$,$w=Z^{1/2}$,$w=e^Z$, w = sin Z,and w=cos Z -Conformal mapping-isogonal mapping.

UNIT V

Complex integration: Simple rectifiable oriented curves –Integration of complex functions- Definite integral-Interior and Exterior of a closed curve-Simply connected region-Cauchy"s fundamental theorem-Cauchy"s formula for higher derivatives- Morera"s theorem.

TEXT BOOK

1.Duraipandian.P., Lakshmi Duraipandian.,1997.Complex analysis,Emerald publishers, Chennai-2 .

REFERENCES

1. Lars V.Ahlfors.,1979. Complex Analysis, Third edition, Mc-Graw Hill Book Company,New Delhi
2. Arumugam.S., Thangapandi Isaac., and A.Somasundaram., 2002. Complex Analysis, SCITECH Publications Pvt. Ltd,Chennai.
3. Choudhary.B., 2003. The Elements of Complex Analysis ,New Age International Pvt.Ltd ,New Delhi.

4. Ponnusamy.S., 2004. Foundations of Complex Analysis, Narosa Publishing House, Chennai.
5. Vasishtha A.R ., 2005. Complex Analysis, Krishna Prakashan Media Pvt. Ltd., Meerut.
6. Narayanan .S., T.K Manichavachagam Pillay, 1992. Complex Analysis. S.Viswanathan (printers & publishers) pvt Ltd, Madras.

Unit - I

1. The additive identity of complex number is
   a)(1,1) b)(1,0) c)**(0,0)** d)(0,1)
2. The multiplicative identity of complex number is
   a)(0,1) b)**(1,0)** c)(0,0) d)(0,1)
3. The inverse of (α,β) under addition is
   a)(-α,β) b)**(-α,-β)** c)(α,β) d) (α,-β)
4. | Z1 .Z2|=
   a)‖z1‖ ‖z2‖ b) │z1│ ‖z2‖ c)**│z1│ │z2│** d)│z1│+│z2│
5. The value of i2 is
   a)1 b)**-1** c)0 d) i
6. If Z1 and Z2 are any two complex numbers ,then
   a)arg(Z1Z2) = arg(Z1)+arg(Z2) b)arg(Z1Z2) = arg(Z1)-arg(Z2)
   c)arg(Z1Z2) = arg(Z1)/arg(Z2) d)arg(Z1Z2) = arg(Z1)*arg(Z2)
7. The Equation of the unit sphere is
   a)x2+y2+z2=1 **b)** x2+y2+z2=2 c) x2-y2+z2=1 d)x2-y2-z2=1
8. The element (1,0) is the -------
   a)Additive identity b)**Multiplicative identity** c)identity d)unique
9. The element (0,0) is the ----------
   a)Additive identity b)Multiplicative identity c)identity d)unique
10. If |Z1| = |Z2| and arg(Z1)= arg(Z2) then -----
    a)Z1≠ Z2 b) Z1< Z2 c) Z1> Z2 **d)Z1= Z2**
11. The Equation of the unit circle whose centre is the origin is
    a)|Z| =1 b) |Z-a| =1 c)|Z| =0 d)|Z|≠1
12. The complex plane containing all the finite complex numbers and infinity is called the
    a)infinite complex plane b)**extended complex plane** c)complex plane d) finite complex plane
13. The inversion w = 1/z maps the region │z│< 1 into the region
    a)│w│<1 b)**│w│>1** c)│w│=1 d)│w│≤1
14. The square of real number is ---------
    a)Non negative b)Non positivee c)Negative d)**absolute value**
15. The absolute value of z = x+iy is
    a)√x b)√y c)√x2-y2 d) √**x2+y2**
16. If Z1 and Z2 are any two complex numbers ,then
    a)| Z1 +Z2|£| Z1 |+|Z2| b) | Z1 +Z2|=| Z1 |+|Z2| c)| Z1 +Z2|³ | Z1 |+|Z2| d) | Z1 +Z2|≠| Z1 |+|Z2|
17. The mapping W=1/Z is called an
    a)Linear transformation b)Translation c)**Inversion** d)Rotation
18. The polar form of x+iy is

a)r(cos q +isinq) b) r(cos q -isinq) c) cos q +isinq d)r(cos q -sinq)

19. If Z1 and Z2 are any two complex numbers ,then
a)| Z1 -Z2|£| Z1 |+|Z2| b) | Z1 -Z2|=| Z1 |+|Z2| c) **| Z1 -Z2|³| | Z1 |-|Z2| |** **d)** | Z1 -Z2|≠| Z1 |+|Z2|
20. The complex plane containing all the finite complex numbers is called the
a)infinite complex plane b)extended complex plane c)complex plane
d) **finite complex plane**
21. The conjugation of 5+i3 is
a)5 b)3 c)5+i3 d)**5-i3**
22. If Z1 and Z2 are any two complex numbers ,then
a)arg(Z1/Z2) = arg(Z1)+arg(Z2) **b)arg(Z1 /Z2) = arg(Z1)-arg(Z2)**
c)arg(Z1 /Z2) = arg(Z1)/arg(Z2) d)arg(Z1 /Z2) = arg(Z1)*arg(Z2)
23. The mapping W=Z+b ,b is a complex number, is called the
a. Linear transformation b.**Translation** c.Inversion d.Rotation
24. All the complex numbers except infinity are called
a.Complex numbers b.Complex plane c.**finite complex numbers** d.infinite complex numbers
25. If x= rcosθ , y = rsinθ then for z we get
a.z= rcosθ+ r sinθ b.z= rsinθ + ircosθ c.**z= rcosθ+irsinθ** **d.**z= rcosθ-irsinθ
26. The angle made by the vector (x,y) measured in the anticlockwise direction is
a.mod of z b.norm of z c.**argument of z** **d.0**
27. The argument θ is ------------- as it can take infinite values
a.unique b. **not unique** c.finite d.infinite
28. From x= rcosθ and y = rsinθ weget θ =
a.sin-1 y/x b.cos-1y/x c.**tan-1y/x** d.cot-1y/x
29. The argument of the product of two complex numbers is---- of the complex number
a.The sum of the arguments b.**the argument of the sum**
c.the argument of the division d.the product of the arguments
30. The cross ratio of the form…..
a.(z1-z2)(z2-z4)/(z1-z4)(z2-z3) b. **(z1-z3)(z2-z4)/(z1-z4)(z2-z3)**
c.(z1-z2)(z2-z4)/(z1-z4) d. (z1-z2)/(z1-z4)(z2-z3)
31. If z = -1 + i , then z-1 = ……..
a.-1+i **b.-1-i** c.(-1)/2 + i 1/2 d. (-1)/2 - i 1/2
32. The stereographic projection of the complex point z = (√2 ,1) is
a.(1/√2, 1/√2, 0) b. (0, √2, 1) c.**(1/√2, 1/2 , 1/2)** d. (0, 0,1)
33. The inversion w = 1/z maps the region │z│>1 into the region
a.│w│<1 b.│w│>1 c.│w│=1 d.│w│≤1
34. Under the transformation w = az there are ------ fixed points
a. One b.**two** c.zero d.∞
35. According to De Moivre's theorem (cos θ +isin θ)n = ………………
a.cosn θ + isinn θ b.**cosn θ+isinn θ** c.ncos θ+insin θ d.1

36. The transformation $w = az+b$, where a, b are complex constants ,is a composition of …… tranformations

a.Rotation and Homothetic b.Translation and Rotation

c.**Rotation , Homothetic and Translation** d.Homothetic and Translation

37. The fixed points for $w = (2z-1) / (z+3)$ are

a.0, ∞ b.1/3,0 c. -1/2,-1/3 d.**-1/2+i(√3/2) , -1/2-i(√3/2)**

38. The equation $z\overline{z} + \overline{a}z + a\overline{z} + c = 0$, where c is real and a is complex , is a equation of a

a.Line b.Ray c.Ellipse d.**circle**

Unit - II

1. The functions of the form, $P_n(Z)= a_0+a_1z+a_2z^2+\ldots\ldots+a_nz^n$, $a_n\neq0$ is called a

a.polynomial of degree n b. polynomial of degree 5 c.polynomial of degree 2n d.polynomial of degree n-1

2. If f (z) and g(z) are continuous at z_0 then f(z).g(z) is

a.Continuous at z_0 b.differentiable at z_0 c.Continuous at z d. differentiable at z

3. f(z) = z^2 is a ------------------- valued function.

a.single b.multi c.double d.many

4. If f(z) of f has only one value it is called ----------- valued function.

a.single b.multi c.double d.many

5. If $|f(z)| < M$ for all z in s , then f(z) is said to --------------in S

a.multi valued b.continuous c.bounded d.analytic

6. The limit of a function is -----------

a.unique b.does not exist c.different d.multivalued

7. If f(z) = 2iz is defined then

a. 2 b.2i c.-2 d.i

8. If $|f(z) - f(z_0)| < \epsilon$ for all z in S with $|z - z_0| < \delta$ then f(z) is

a. bounded b.continuous c.unique d.does not exist

9. If f (z) and g(z) are continuous at z_0 then f(z) ± g(z) is

a.Continuous at z_0 b.differentiable at z_0 c. Continuous at z d.differentiable at z

10. If f (z) and g(z) are continuous at z_0 then f(z) /g(z) is

a.Continuous at z_0 b.differentiable at z_0 c. Continuous at z d.differentiable at z

11. In a compact set every continuous function is

a.bounded in s b.uniformly continuous in s c.unique d.does not exist

12. If $|f(z1) - f(z2)| < \epsilon$ for all z1 , z2 S with $|z1 - z2| < \delta$ then f(z) is

a.bounded in s b.uniformly continuous in s c.unique d.does not exist

13. If a function is differentiable at all points in some neighbourhood of a point , then the function is said to be ---- at that point

a.bounded b.analytic c.differentiable d.compact

14. A function which is analytic everywhere in the finite plane is called an ------------------ function.

a. single b.multi c.entire d.continuous

15. f(z) is a function differentiable at z0 , then f(z) is

a.Continuous at z0 b.compact at z c.Continuous at z d.differentiable at z

16. A ---- point of a function is a point at which the function ceases to be analytic

a.non singular b.Singular c.entire d.continuous

17. f(z) = |z|2 is ---------------- everywhere

a.analytic b.not analytic c.continuous d.exist

18. The quotient of two polynomials is called a

a.Exponential function b.logarithmic function c.Continuous function d.rational function

19. If f(z) and g(z) are continuous at z0 then f(z)/g(z), $g(z) \neq 0$ is

a.Continuous at z0 b.differentiable at z0 c.Continuous at z d.differentiable at z

20. If f(1/z) is analytic at 0 then f(z) is

a.Analytic at ∞ b.Continuous at ∞ c.Differentiable at ∞ d.Differentiable at 0

21. The cartesian coordinates of C-R equations are

a.ux=vy and uy= -vx b.ux=vy and uy= -vx c. ux=vy and ux= -vx d. ux=1 and uy= -vx

22. A function of complex variable is sometimes called a

a.complex variable b.variable c.complex function d.constant

23. If the product of the slopes is -1, then the curves cut each other ------

a.diagonally b.orthogonally c.at the origin d. at the point 1

24. The function that is multiple valued is

a.f(z) = z2 b. f(z) = ez c.f(z) = 1/z d.f(z) = z1/2

25. logz is a ------------- valued function

a.single b.multi c.double d.three

26. If

a.0 b.A c.1/A d.∞

27. If

a.0 b.A c.1/A d.∞

28. If f(z) = 1/z2 then

a.0 b.2 c.1 d.-1

29. If f(z0) = ∞, the function f(z) is .... at z = z0

a.continuous b.not continuous c.differentiable d.bounded

30. The function f(z) = Re z/|z |, when z ≠0 ; f(z) = 0 when f(z) = 0 is ....

a.continuous b.not continuous c.differentiable d.bounded

31. The function |z |2 is .... at that point.

a.continuous b.analytic c.not analytic d.bounded

32. If f(z) = u +iv is analytic , then u(x,y) and v(x,y) are .... Functions

A.harmonic B.analytic c.continuous d.bounded

33. The function f(z) = log z,then u(r,θ) = …… v(r,θ) = ...

a.log θ , log r b.r, log θ c.log r , θ d.r,θ

34. If f(z) = 1/z then

a.∞ b.-1 c.0 d.1

35. A continuous function f(z) defined on a set D is uniformly continuous when

a.D is bounded b.D is closed c.D is compact d.D is open

Unit iii

1. The power series with ………. Coefficients are called geometric series.

a. two b.unit c.zero d.three

2. The power series of the form a0 + a1(z –a) + a2(z – a)2 +… converges absolutely in the open disc

a. |z-a|= R b.|z-a|> R c.|z -a|< R d.|z -a| = 0

3. The power series of the form a0 + a1(z –a) + a2(z – a)2 +…. Is said to be a series about

a. z = 0 b.z = -a c.z = a d.z = ∞

4. The power series a0 + a1z + a2z 2 +.. converges absolutely in the open disc ……..

a. |z|= R b.|z|> R |c.z|< R d.|z| = 0

5. The circle of the convergence of the series a0 + a1z + a2z 2 +……

a. |z|> R b.|z|< R c.|z| = 0 d.|z|= R

6. The circle of the convergence of the series a0 + a1(z-a) + a2(z-a) 2 +……

a. |z-a|> R b.|z -a|< R c.|z -a| = 0 d.|z-a|= R

7. A power series … in the exterior of its circle of convergence

a. absolutely convergent b.converges c.diverges d.uniformly convergent

8. If R = 0 the series is divergent in the extended plane except at

a. z = 0 b.z =1 c.z = ∞ d.z = -1

9. The sequence {zn} is bounded if there exists a constant M such that ------ for all n.

a. |zn|= M b.|zn|≤ M c.|zn|≥ M d.|zn|> M

10. For all finite z= h + ik, |ez| = ……

a. eh + k b.eh + ik c.eh d.ek

11. Euler's relation ex + iy =

a. ex(cos y + isin y) b.ex(sin y + icosy) c. ey (cosx + isinx) d. ey(sinx + icosx)

12. The polar form r (cos θ + i sinθ) of a complex numbers in exponential form as

a. reθ b.eiθ c.reiθ d.1/reiθ

13. ez is not defined at

a. z = ∞ b.z =0 c. z = 1 d. z= -1

14. The inverse function of the exponential function is the ……

a. Trignometric functions b.hyberbolic functions c.harmonic functions d.Logarithmic functions

15. Logarithamic function log z = ---------------------- n = 0, ±1, ±2

a. log r + iθ + n(2πi) b.log 1/r + ieiθ + n (2πi) c.log r + ieiθ + n(2πi) d.logr + iθ + n2π

16. (logz) =

a. z b. -z c. ez d.1/z

17. siniz

a. sinz b.sinhz c.isinz d.isinhz

18. cosiz

a. cosz b.icosz c.icoshz d.coshz

19. tanz and secz are analytic in a bounded region in which

a. tanz ≠ 0 b.sec z ≠ 0 c.sinz ≠ 0 d.cos z ≠ 0

20. cot z and cosecz are analytic in a bounded region in which

a. cot z ≠ 0 b.cosecz ≠ 0 c.sinz ≠ 0 d.cosz ≠0

21. cosh2z – sinh2z =

a. 0 b.1 c.-1 d.∞

22. singular points of logz are

a. z = 0 and z = ∞ b.z = 1 and z =0 c,.z =0 and z = -1 d.z =1 and z = ∞

23. Principle value of logz is obtained when n =

a. 0 b.-1 c.1 d.2

24. The logarithmic function is a ------ valued function

a. Single b.multiple c.two d.zero

25. In a complex field z = x + iy then θ = ……….

a. sin-1 (y/x) b.cos-1(y/x) c.tan-1(y/x) d.cot-1(y/x)

26. The sum f(z) of a powerseries is analytic in ………..

a. |z|> R b.|z|< R c.|z| ≤ R d.|z|= R

27. A power series …………… is the interior of the circle of convergence

a. converges b.diverges c.uniformly converges d.converges absolutely

28. The radius of convergence of the series ∑ (2+in)/2n .zn …………

a. 2 b.0 c.∞ d.1

29. Sin (

a. sinz b.cosz c.tanz d.cosecz

30. If u+iv is analytic then v+iu is………..

a. analytic b.not analytic c.continuous d.conjugate

31. coshiz

a. cosz b.cosiz c.sinz d. coshiz

32.az is a …………… valued function

a) single b.double c.multiple d.triple

33.The function az =

a.ezloga b.eloga c.ealogz d.e-zloga

34.The radius of convergence of the series ∑ n2 .zn …………

a.1 b.0 c.2 d.n

35.Cos (z1 + z2) =

a.cosz1 cosz2 - sinz1sinz2 b.cosz1 sinz2 - sinz1cosz2 c.cosz1 cosz2 + sinz1sinz2 d.sinz1 cosz2 - cosz1sinz2

36.The radius of convergence of the series ∑ nn .zn …………

a.1 b.0 c.2 d.n

Unit IV

1. The polar coordinates of C-R equations are
   a. **ur=1/r vɵ and uɵ= -r vr** b.ur= vɵ and uɵ= vr c.ur=1/r vɵ and uɵ= r vr d.uɵ= -r vr
2. Two harmonic functions are said to be ……. Functions if they satisfies the C-R equations.
   a. Conjugate harmonic b.harmonic c.functions d.**analytic**
3. The Laplace equation of the form
   a. **Uxx+Uyy=0** b.Uxx-Uyy=0 c.Vxx+Uyy=0 d.Vxx+Vxy=0
4. If U=x2-y2 then Uyy = ?
   a. 3 b.1 c.0 d.**2**
5. If u(x,y)=excosy then find ux= ?
   a. e^xcosx b. **e^xcosy** c. cosy d. ex
6. The second order partial derivatives exist, continuous and satisfies the laplace equation is called functions
   a. Analytic b.Continuous c.differentiable d. **harmonic**
7. If U=x2-y2 then Uxx = ?
   a. 3 b.**2** c.0 d.1
8. The fixed point's transformation is also known as …… points transformation
   a. Mobius b.invariant c.**constant** d.bilinear
9. The bilinear transformation of the form W=
   a. **az+b/cz+d** **b.** az+b/c+d c.az+b d.az+b/c
10. A function which is ………. in region which is not close may or may not be bounded in it.
    a. **Analytic** b.differentiable c.continuous d.bounded
11. The function 1/(1+z) is analytic at infinity because the function 1/(1+1/z) is
    a. **Analytic at 0** b.continuous at 0 c.differentiable at 0 d.analytic at 1
12. If a function is differentiable at a points then the function is said to be ….
    a. analytic at that point b.continuous at that point c.**differentiabe at that point** d.not differentiable at that point
13. The Laplace equation of the format
    a. **Uxx+Uyy=0** b.Uxx-Uyy=0 c.Vxx+Uyy=0 d.Vxx+Vyy=0
14. The bilinear transformation is also known as ……… transformation
    a. non mobius b.linear c.**mobius** d.non linear
15. The equations ux=vy and uy= -vx are
    a. Polar equation b.Euler equation c.**C - R equation** d.coordinates

16. If u or v is not harmonic , then u+iv is ………

a. analytic b.**not analytic** c.conjugate harmonic d.diffrentiable

17. If f(z) =u(x,y) + iv(x,y) is analytic in domain d iff u(x,y) and v(x,y) are

a. harmonic b.**conjugate harmonic** c.differentiable d.continuous

18. In a two dimensional flow the stream function is tan-1y/x then the velocity potential is

a. **1/2log(x2 + y2)** b.Sin-1 y/x c,log(x2 + y2) d.cos-1 y/x

19. By Milne – Thomson method if u (x,y) = x2 – y2 then f(z) =

a. **Z2** **b.**2x+2y c.x+y d.z

20. The function f(z) = z1/2 is …………Valued function

a. single b.multi c.**double** d.triple

21. The transformation w = z2 maps the -------------- onto the straight lines

a. parabola b.hyperbola c.ellipse d.**rectangular hyperbola**

22. If f(z) = u+iv is an analytic function then -if(z) =

a. u-iv b.v+iu c.u+v d.**v+i(-u)**

23. The value of m such that 2x – x2 + my 2 may be harmonic is ----

a. **1** b.2 c.0 d.3

24. If f(z) = u+iv is an analytic function then(1 -i)f(z) =

a. (u+v)+i(v-u) b.**(u+v)-i(v-u)** c.(u-v)+i(v-u) d.(u+v)+i(v+u)

25. If f(z) = u+iv is an analytic function then(1+i)f(z) =

a. (u+v)+i(v-u) b.(u+v)-i(v-u) c.**(u-v)+i(u+v)** d.(u+v)+i(v+u)

26. Harmonic functions in polar coordinates are

a. **Urr + 1/r ur +1/r2 uθθ** b.Urr + r ur +1/r2 c.uθθ Urr +1/r2 uθθ d.Urr + 1/r ur +1/r2 uθθ

27. The function ----------- is called zhukosky's function

a. 1/z b.**z+1/z** c.z d.sinz

28. If w = u+iv under w = z+1/z then u = ….

a. **u = (r + 1/r)cosθ** b.u = (r - 1/r)cosθ c.u = (r + 1/r)sinθ d.u = r cosθ

29. If w = u+iv under w = z+1/z then v = ….

a. v = (r + 1/r)cosθ b.v = rsinθ c.**v= (r - 1/r)sinθ** d.v = r cosθ

30. A circle whose centre is origin goes onto an …… whose centre is the origin under the zhukosky's transformation.

a. parabola b.hyperbola c.**ellipse** d.rectangular hyperbola

31. A ray emanating from the origin goes onto a ….. Whose centre is the origin under the zhukosky's transformation
    a. parabola b.**hyperbola** c.ellipse d.rectangular hyperbola

32. The principle value of log z are ….
    a. logr b.**logr+iθ** **c.**log1/r d.logr-iθ
33. . The partial derivatives are all ----- in domain D
    a. **analytic** b.not analytic c.does not exists d. continuous

34. w = cos z is a ------ function
    a. **analytic** b.continuous c.not analytic d.limit
35. . f(z) = xy + iy is --------
    a. analytic b.**continuous** c.analytic anywhere d.limit
36. . The function f(z) = |z| is differentiable -----
    a. on real part b. on imaginary part c.**at the origin** d.at the point 2

37. If f(z) has the derivative only at the origin, it is ------analytic everywhere
    a. **not analytic** b.nowhere c.analytic d.nowhere continuous

38. f(z) = 1/z is a ------ function
    a. differentiable b.continuous c, **analytic** d. not analytic
39. An analytic function with constant real part is ------
    a. **constant** b.real c.imaginary d.not analytic
40. An analytic function with constant imaginary part is ------
    a. **constant** b.real c.imaginary d.not analytic
41. An analytic function with constant modulus part is ------
    a. **constant** b.real c. imaginary d,not analytic
42. Both real part and imaginary part of any analytic function satisfies ------

    a.wave equation b.polynomial equation c.del operator d.**laplace's equation**

Unit - V

1. The set of complex points is called ….
   a. arc b.**simple arc** c.closed arc d.open arc
2. If a curve intersects itself at a point then the point is said to be a……
   a. single b.**multiple points** c.double valued d.trile
3. The equation z = cost+isint, 0≤t≤π represents a ……………
   a. arc b.**simple arc** c.closed arc d.curve
4. The unit circle z=cost+isint are
   a. **positively oriented circle** b.negatively oriented circle c. circle d.unit circle
5. The unit circle z = cos(-t) +isin(-t), 0≤t≤2π are ……..
   a. positively oriented circle b.**negatively oriented circle** c. circle d.unit circle
6. It the region lies to the left of a person when he travels along C, then the curve C is called a ……….
   a. **postively oriented simple closed curve** b.negatively oriented simple closed curve c.open curve d.simple closed curve
7. The simple closed rectifible curve is abbreviated as……
   a. **scr curve** b.scro curve c.arc d.curve
8. In cauchy's fundamental theorem, ò f(z) dz=…
   a. 1 b.2 c.**0** d.4
9. The simple closed rectifiable positively oriented curve is abbreviated as …..
   a. curve b.scr curve c.**scro curve** d.arc
10. The simple arc is also known as ….
    a. multiple b.**Jordan** c.double d.multiple

11. The derivative of an analytic function is also …
    a. analytic b.**continuous** c.derivative d.bounded
12. The integral ò f(z) dz=F(b)-F(a) is called a
    a. integral b.indefinite c.definite d.**derivative**
13. The poles of an analytic function are ……
    a. essential b.removable c.pole d.**isolated**
14. If C is a positively oriented circle then ò 1/(z-a) dz=
    a. 2P b.**2Pi** c.0 d. P
15. When the order of the pole is 2,the pole is said to be ……pole
    a. double b.simple c.**multiple** d.triple
16. The limit point of zero's of an analytic function is a ……point of the function
    a. **Singular** b.nonsingular c.poles d.zeros
17. A region which has only one hole is an …..region
    a. origin b.set c.**annular** d.moment
18. A region which is not simply connected is called …
    a. Connected b.**compact** c.multiply- connected d.region.
19. The integrals along scr curves are called….
    a. complex integrals b.integrals c.**contour integrals** d.partial integrals

20. If f(z) is a continuous function defined on a simple rectifiable curve then ʃf(z) dz =
    a. **ʃ (u dx- vdy) + iʃ(udy-vdx)** b.ʃ (u dx- vdy) - iʃ(udy-vdx) c.ʃ (u dx- vdy) +ʃ(udy-vdx) d.ʃ (u dx+ vdy) + iʃ(udy+vdx)

21. ∫ [f1(z) +f2(z)]dz on C is
    a. **∫ f1(z)dz + ∫f2(z)dz** **b.** ∫ f1(z)dz - ∫f2(z)dz c. ∫ f1(z)dz . ∫f2(z)dz d. ∫ f1(z)dz / ∫f2(z)dz
22. If f(z) is analytic in a simply connected domain , then the values of the integrals of f(z) along all paths in the region joining ------ fixed points are the same
    a. one b.**two** c.three d.multiple
23. The bounded region of C is called ……………
    a. **Interior** b.exterior c.interior nor exterior interior and exterior

24. A region D is said to be …………… for every closed curve in D , Ci is contained in D
    a. connected b.**simply - connected** c.disconnected d.disjoint

25. When A is fixed and B(z) moves in D, the integral
    a. **single - valued** b.double -valued c.multi – valued d.zero

26. The function (z-i)2 have a zero i of order…..
    a. **2** b.1 c.0 d.3
27. ………… of an analytic function are isolated
    a. **zeros** b.poles c.residues d.points
28. If f(z) = (z – a)m[a0 + a1(z-a)………] ,a0 ≠0 , then z =a is a zero of order …..
    a. 1 b.2 c.0 d.**m**
29. If C is an arc in D , joining a fixed point z0 and the arbitrary point z then d/dz
    a. 0 b.1 c.**f(z)** d.c
30. A function analytic in D has …………. of all orders in D
    a. **derivatives** b.points c.curves d.zeros

31. A curve is said to be piece-wise smooth if C is not smooth at a …….. number of points in it.
    a. **finite** b.infinite c.zero d.one

KARPAGAM ACADEMY OF HIGHER EDUCATION

COIMBATORE-21

DEPARTMENT OF MATHEMATICS

SUBJECT: COMPLEX ANALYSIS I

SUBJECT CODE: 15MMU502

POSSIBLE QUESTIONS

UNIT I

1. If Z1 and Z_2 are any two complex numbers, then prove that $\overline{Z_1 + Z_2} = \overline{Z_1} + \overline{Z_2}$.

2. Show that every complex numbers z whose absolute value is 1, can be expressed in the form z = (1+it)/(1-it),t is a real number.

3. Explain the Stereographic projection of a complex plane.

4. Show that the argument of the product of two complex numbers is the sum of the arguments of the complex numbers.

5. Show that, if | Z| < ½ then | (1+i) z^3+iz| < ¾

6. Show that stereographic projection maps circles on the Riemann sphere onto circles or Straight lines in the complex plane.

7. If Z1 and Z_2 are any two complex numbers, then prove that $(\overline{Z_1 Z_2}) = \overline{Z_1}.\overline{Z_2}$.

8. If Z1 and Z_2 are any two complex numbers, then $arg\left(\frac{Z_1}{Z_2}\right) = argZ_1 - argZ_2$

9. If Z, Z_1 and Z_2 are any three complex numbers, then prove that

i) -| Z|≤Re Z≤ |Z|, -| Z|≤ Im Z≤ |Z|,

ii) $| Z_1 + Z_2| \leq | Z_1| + |Z_2|$

iii) $| Z_1 - Z_2| \geq | \, | Z_1| - |Z_2| \, |$

10. If Z_1 and Z_2 are the images in the complex plane of two diametrically opposite points on the Riemann sphere,show that $Z_1 \overline{Z_2} = -1$

11. Explain the transformation w=1/z.

12. Explain about the transformation w=az.

13. If z, z1, z2 are any three complex number then prove that i) $|z| = |\bar{z}|$ ii) $\bar{z}z = |z|^2$

iii) $|z_1 z_2| = |z_1||z_2|$

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POSSIBLE QUESTIONS

UNIT II

1. Treating f(z) as a function of x &y and x &y is a function of z & $\bar{z}$ show that

   i) $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$

   ii) $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right) log|f'(z)| = 0.$

2. State and prove C-R equations in polar coordinates.

3. Prove that the necessary condition for a function to be differentiable at a point is the Continuity of the function at the point.

4. Show that in a compact set every continuous function is uniformly continuous.

5. Suppose f(z) is a function differentiable in a region D and the mapping w=f(z) is one to one

and the inverse mapping is $z=\varphi(w)$.If z_0 is a point in D such that $f'(z_0)\neq 0$, then

i) The inverse function $\varphi(w)$ is differential at w_0, where $w_0= f(z_0)$ and

ii) $\varphi'(w_0) =1/ f'(z_0)$

6. Prove that an analytic function f (z) and the C-R equations can be put in the condensed form $\frac{\partial f}{\partial \bar{z}} = 0$.

7. Derive the C-R equations in polar coordinates.

8. Prove that if f(z) is continuous at $z=z_0$ and if for any M>0 there exists a d such that $|f(z)|>M$ for all z in the disc $|z-z_0|<d$ then 1/f(z) is continuous at $z=z_0$.

9. Suppose f(z)= u(x,y)+iv(x,y) is a single valued function defined in a neighbourhood of $z_0=x_0+iy_0$.Then the necessary condition for the differentiability of f(z) at z0 is the existence of the partial derivatives u_x, u_y, v_x, v_y at (x_0,y_0),which satisfy the relations $u_x=v_y, u_y=-v_x$.

10. Show that the single valued continuous function $f(z) = z^{1/2} = r^{\frac{1}{2}}(\cos\frac{1}{2}\theta + \sin\frac{1}{2}\theta)$, $r > 0, 0< \theta < 2\pi$ is analytic, Find $f'(z)$.

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POSSIBLE QUESTIONS
UNIT III

1. Find the radius of convergence of the power series f(z) = $\sum_0^\infty \frac{z^n}{2^n(1+in^2)}$
2. State and prove Euler's relation.
3. State and prove Abel's theorem.
4. Prove that the sum of a convergent power series in z is analytic in the interior of its circle of convergence.
5. Find the domain of convergence of

   i) $\sum_1^\infty \left(\frac{iz-1}{2+i}\right)^n$ ii) $\sum_1^\infty \left(\frac{z+i}{2i}\right)^n$ iii) $\sum_1^\infty \left(\frac{1}{1+z2}\right)^n$
6. State and prove Uniqueness theorem
7. Find the radii of convergence of the following power series

   i) $\sum \frac{n^k}{n^n} z^n$ ii) $\sum \frac{n!}{n^n} z^n$
8. Explain about an exponential function.
9. Find the radii of convergence of the following power series

   i) $\sum \frac{(2)^n}{n!} z^n$ ii) $\sum \frac{2+in}{2^n} z^n$
10. If a power series in z is convergent at z =z_1, then it converges absolutely in the circular Open disc | Z |< | Z_1|.
11. Define circle of convergence.
12. Prove that a power series is divergent in the exterior of its circle of convergence.

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POSSIBLE QUESTIONS

UNIT IV

1. Show that a function f(z)=u(x,y)+iv(x.y) defined in a region D is analytic in it iff u(x,y) and v(x,y) are conjugate harmonic functions.

2. Prove that the cross ratio is preserved by a Bilinear transformation.

3. Show that a bilinear transformation maps straight lines and circles into straight lines and Circles.

4. Find the analytic function f(z)= u + iv given that u- n ni .

5. Find the analytic function f(z) if its real part is u(x,y)=

6. Prove that under a bilinear transformation no two points in z plane go to the same Point in w plane.

7. Prove that the cross ratio is preserved by a bilinear transformation.

8. Show that the function u(x,y) =sin x coshy is harmonic .Find its harmonic conjugate v(x,y) and the analytic function f(z)=u + iv.

9. Prove that the bilinear transformation which transforms z_1,z_2,z_3 into w_1,w_2,w_3 is

$(w-w_1)(w_2-w_3) / (w-w_3)(w_2-w_1) = (z-z_1)(z_2-z_3) / (z-z_3)(z_2-z_1)$.

10. If f (z) = u + iv and u-v = e^x (cos y – sin y), find f(z) in terms of z.

11. Show that the transformation $w=z^2$ transform the families of lines x=h and y=k into co focal parabolas having w=0 as the common focus.

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SUBJECT: COMPLEX ANALYSIS I
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POSSIBLE QUESTIONS

UNIT V

1. State and prove Cauchy's Integral formula for nth derivative**.**

2. State and prove Morera's theorem

3. State and prove Cauchy's formula for first derivative**.**

4. State and prove Goursat's Lemma.

5. If C is an simple arc of length L and f(z) is a continuous function defined on C and if on C, max| f(z) | then |

6. Find the integral of f(z) = along the parabolic arc y= from (0,0) to (1,1).

7. Prove that $\int_C 1/(z-a)\, dz = 2\prod i$, where C is a positively oriented circle whose radius is r and centre is z=a.

8.State and prove Cauchy's integral formula

9. Evaluate the integrals along C,the positively oriented circle |z|=2.

10. State and prove Cauchy's fundamental theorem.

11.Using Cauchy's integral formula, evaluate where C is |z+1+i|=2.