KARPAGAM ACADEMY OF HIGHER EDUCATION
(Deemed to be University Established Under Section 3 of UGC Act 1956)
Pollachi Main Road, Eachanari (Po),
Coimbatore –641 021
DEPARTMENT OF MATHEMATICS

Subject: Numerical Methods	**Subject Code: 15MMU503**	**L T P C**
Class:III B.Sc-B	**Semester:V**	**4 1 0 4**

PO : To enable the students to study numerical techniques as powerful tool in scientific computing.
PLO: This course provides a deep knowledge to the learners to understand the basic concepts of Numerical Methods which utilize computers to solve Engineering Problems that are not easily solved or even impossible to solve by analytical means.

UNIT I

Solution of algebraic and transcendental equations: Bisection method – Iterative method Regula Falsi method – Newton Raphson method – Horners method – Graeffe's root squaring method.

UNIT II

Solution of simultaneous linear algebraic equations: Gauss elimination method – Gauss Jordan method – Method of triangularization – Crout's method – Gauss-Jacobi method – Gauss-seidel method.

UNIT III

Finite Difference: First and higher order differences – Forward and Backward differences –Properties of operator – Difference of a polynomial – Factorial polynomial – Error Propagation in difference table – operator E – Relation between Δ, E and D .

UNIT IV

Interpolation: Gregory Newton Forward and Newton Backward interpolation formula – Equidistant terms with one or more missing values – Interpolation with unequal intervals – Divided differences –Newton's divided difference formula – Lagrange's interpolation formula – Inverse interpolation formula.

UNIT V

Numerical Differentiation and Integration: Newton's Forward and backward differences to compute derivatives – Trapezoidal rule, Simpson's 1/3 &3/8 rule. Solution of ordinary differential equations:R-K method (II order , III order and IV order).

TEXT BOOK

T1: Venkataraman .M.K., Fifth Edition, 2001. Numerical Methods in Science and Engineering National publishing Company ,Madras.

REFERENCES

R1: Jain. M.K., Iyengar S.R.K.,and R.K.Jain., 2004. Numerical Methods for Scientific and Engineering Computation, New Age International Publishers, New Delhi .
R2: Vedamurthy V.N., N.Ch.S.N.Iyenger., 1999. Numerical Methods,Vikas Publishing House Pvt Ltd, New Delhi.
R3: Kandaswamy. P., Thilagavathy K., and K.Gunavathy., 2013 .Numerical Methods, S. Chand &Company Ltd., New Delhi.

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Class:III B.Sc-B **Semester:V** **4 1 0 4**

S.NO	DURATION HOURS	TOPICS TO BE COVERED	SUPPORT MATERIAL
		UNIT-I	
1	1	Solution of Algebraic and Transcendental Equation - Introduction, problems on Bisection Method	T1:chapter-3,Pg.No:81-83
2	1	Continuation of problems on Bisection Method	T1:chapter-3,Pg.No:83-85
3	1	Iterative Method	T1:chapter -3,Pg.No:85-90
4	1	Tutorial 1	
5	1	Regula Falsi Method-Procedure and problems	T1:chapter -3,Pg.No:91-94
6	1	Continuation of problems on Regula Falsi Method	T1:chapter -3,Pg.No:94-97
7	1	Newton- Raphson Method-Procedure & Problems	T1:chapter -3,Pg.No:97-99,102-105
8	1	Continuation of problems on Newton- Raphson Method	T1:chapter -3,Pg.No:102-105
9	1	Tutorial 2	
10	1	Horner’s Method-Problems	R2: chapter-3, Pg.No:3.22-3.24
11	1	Continuation of problems on Horner’s Method	R2: chapter-3, Pg.No:3.24-3.26
12	1	Graeffe’s Root Squaring Method -Problems	R2: chapter-3, Pg.No:3.24-3.26
13	1	Continuation of problems on Graeffe’s Root Squaring Method	R2: chapter-3, Pg.No:3.26-3.29
14	1	Tutorial 3	

15	1	Recapitulation and Discussion of possible questions	
Total	**15 Hours**		

TEXT BOOK:

T1: Venkataraman .M.K., Fifth Edition, 2001. Numerical Methods in Science and Engineering National publishing Company ,Madras.

REFERENCES:

R2: Vedamurthy V.N, N.Ch.S.N.Iyenger., 2008.Numerical Methods, Vikas Publishing House Pvt.Ltd.New Delhi.

UNIT-II

1	1	Solution of Simultaneous Linear algebraic Equations – Introduction, Gauss Elimination Method: Procedure & problems	T1: chapter - 4,Pg.No:113-115
2		Continuation of problems Gauss Elimination Method	T1: chapter - 4,Pg.No:116-118
3	1	Gauss Jordan Method- problems	R2**:** chapter - 4,Pg.No:4.8-4.10
4	1	Continuation of problems Gauss Jordan Method	R2**:** chapter - 4,Pg.No:4.10-4.12
5	1	Tutorial 1	
6	1	Method of Triangularisation	T1: chapter - 4,Pg.No:126-128
7	1	Continuation of Problems on Method of Triangularisation	T1: chapter - 4,Pg.No:128-131
8	1	Crout’s Method-Problems	R2: chapter - 4,Pg.No:4.23-4.32
9	1	Tutorial 2	
10	1	Gauss Jacobi Method-procedure and problems	R3: chapter - 4,Pg.No:146-148
11	1	Continuation of problems on Gauss Jacobi Method	R3: chapter - 4,Pg.No:148-150
12	1	Gauss Seidal Method-procedure and problems	R3: chapter -4, Pg.No:150-154
13	1	Continuation of problems on Gauss Seidal Method	R3: chapter -4, Pg.No:154-158
14	1	Tutorial 3	
15	1	Recapitulation and discussion of possible questions	
Total	**15 Hours**		

TEXT BOOK:

T1: Venkataraman .M.K., Fifth Edition, 2001. Numerical Methods in Science and Engineering National publishing Company ,Madras.

REFERENCES:

R2: Vedamurthy V.N, N.Ch.S.N.Iyenger., 2008.Numerical Methods, Vikas Publishing House Pvt.Ltd.New Delhi.

R3: Kandaswamy.P, Thilagavathy K., and K.Gunavathy. 2013. Numerical Methods, S.Chand & Company Ltd.,New Delhi.

UNIT-III

1	1	Finite Difference: First and higher order differences, Operators	R3: chapter - 5,Pg.No:170-174
2	1	Newtons Forward and Backward Differences - Problems	R3: chapter - 5,Pg.No:174-176
3	1	Continuation of problems on Newtons Forward and Backward Differences	R3: chapter - 5,Pg.No:176-178
4	1	Tutorial 1	
5	1	Difference of a polynomial and Factorial polynomial - problems	R3: chapter - 5,Pg.No:179-181
6	1	Continuation of problems on Factorial polynomial	R3: chapter - 5,Pg.No:181-183
7	1	Error propagation in difference table - Problems	R3: chapter - 5,Pg.No:194-196
8	1	Tutorial 2	
9	1	Continuation of problems on error propagation in difference table	R3: chapter - 5,Pg.No:196-198
10	1	Operator E , Relation between Δ,E and D	T1: chapter - 5,Pg.No:177-180
11	1	Problems based on the operator Δ,E and D	T1: chapter - 5,Pg.No:180-183
12	1	Tutorial 3	
13	1	Recapitulation and Discussion of possible questions	
Total	**13 Hours**		

TEXT BOOK:

T1: Venkataraman .M.K., Fifth Edition, 2001. Numerical Methods in Science and Engineering National publishing Company ,Madras.

REFERENCES:

R3: Kandaswamy.P, Thilagavathy K., and K.Gunavathy. 2013. Numerical Methods, S.Chand & Company Ltd.,New Delhi.

UNIT-IV

1	1	Interpolation: Introduction , Gregory Newton's Forward and Backward Interpolation Formulae & Problems	R3: chapter - 6,Pg.No: 209-213
2	1	Continuation of problems on Gregory	R3: chapter -

		Newton Forward & Backward Interpolation	6,Pg.No: 213-217
3	1	Continuation of problems on Gregory Newton's Forward & Backward Interpolation	R3: chapter - 6,Pg.No: 217-221
4	1	Tutorial 1	Tutorial 1
5	1	problems on Gregory Newton's Forward & Backward Interpolation	R3: chapter - 6,Pg.No: 222-226
6	1	Equidistant terms with one or more missing values	R2: chapter - 6,Pg.No:6.17-6.19
7	1	Interpolation with unequal Intervals: Divided difference -Introduction & Formula	T1: chapter - 8,Pg.No: 244-245
8	1	Tutorial 2	
9	1	Problems on Newton's divided difference formula & properties of divided difference	T1: chapter - 8,Pg.No: 245-247
10	1	Problems on Newton's divided difference	T1: chapter - 8,Pg.No: 247-249
11	1	Lagrange's interpolation formula	R1: chapter - 4,Pg.No:215
12	1	Lagrange's interpolation formula-Problems	R1: chapter -4, 224-225
13	1	Inverse Interpolation Formula	R3: chapter - 8,Pg.No:276-278
14	1	Tutorial 3	
15	1	Recapitulation and discussion of possible questions	
Total	**15 Hours**		

TEXT BOOK:

T1: Venkataraman .M.K., Fifth Edition, 2001. Numerical Methods in Science and Engineering National publishing Company ,Madras.

REFERENCES:

R1: Jain.M.K., Iyengar.S.R.K.,and R.K.Jain.,Jain.,2000.Numerical Methods Scientific and Engineering Computation, New Age International Publishers, New Delhi.

R2: Vedamurthy V.N, N.Ch.S.N.Iyenger., 2008.Numerical Methods, Vikas Publishing House Pvt.Ltd.New Delhi.

R3: Kandaswamy.P, Thilagavathy K., and K.Gunavathy. 2013. Numerical Methods, S.Chand & Company Ltd.,New Delhi.

UNIT-V

1	1	Numerical Differentiation: Newton's Forward & Backward differences to compute derivatives	T1: chapter - 9,Pg.No: 265-268
2	1	Newton's Forward & Backward differences to compute derivatives	T1: chapter - 9,Pg.No: 268-270

3	1	Numerical Integration: Trapezoidal rule-Formula & Problems	T1: chapter - 9,Pg.No:281-282, 290-291
4	1	Tutorial 1	
5	1	Continuation of problems on Trapezoidal rule	T1: chapter - 9,Pg.No: 290-291
6	1	Simpson's 1/3 & 3/8 rule: Formulae & Problems	R3: chapter - 9,Pg.No:303-306
7	1	Continuation of problems on Simpson's 1/3 & 3/8 rule	R3: chapter - 9,Pg.No:306-310
8	1	Continuation of problems on Simpson's 1/3 & 3/8 rule	R3: chapter - 9,Pg.No:310-314
9	1	Tutorial 1	
10	1	Solution of ordinary differential equations: Runge Kutta Method(R-K) (II, III , IV order)-Formulae & problems	R3: chapter - 11,Pg.No:379-384
11	1	Continuation of problems on Runge Kutta Method(R-K)(II, III , IV order)	R3: chapter - 11,Pg.No:385-389
12	1	Continuation of problems on Runge Kutta Method(R-K)(II, III , IV order)	R3: chapter - 11,Pg.No:389-393
13	1	Tutorial 2	
14	1	Recapitulation and discussion of possible questions	
15	1	Discussion on Previous ESE Question Papers	
16	1	Discussion on Previous ESE Question Papers	
17	1	Discussion on Previous ESE Question Papers	
Total	**17 Hours**		

TEXT BOOK:

T1: Venkataraman .M.K., Fifth Edition, 2001. Numerical Methods in Science and Engineering National publishing Company ,Madras.

REFERENCES:

R3: Kandaswamy.P, Thilagavathy K., and K.Gunavathy. 2013. Numerical Methods, S.Chand & Company Ltd.,New Delhi.

Total no.of hours for the course:90 Hours

TEXT BOOK:

T1: Venkataraman .M.K., Fifth Edition, 2001. Numerical Methods in Science and Engineering National publishing Company ,Madras.

REFERENCES:

R1: Jain.M.K., Iyengar.S.R.K.,and R.K.Jain.,Jain.,2000.Numerical Methods Scientific and Engineering Computation, New Age International Publishers, New Delhi.

R2: Vedamurthy V.N, N.Ch.S.N.Iyenger., 2008.Numerical Methods, Vikas Publishing House Pvt.Ltd.New Delhi.

R3: Kandaswamy.P, Thilagavathy K., and K.Gunavathy. 2013. Numerical Methods, S.Chand & Company Ltd.,New Delhi.

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UNIT I

Solution of algebraic and transcendental equations: Bisection method – Iterative method- Regula Falsi method – Newton Raphson method – Horner's method – Graeffe's root squaring method.

TEXT BOOK:

T1: Venkataraman .M.K., Fifth Edition, 2001. Numerical Methods in Science and Engineering National publishing Company ,Madras.

REFERENCES:

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UNIT-I

SOLUTION OF ALGEBRAIC AND TRANSCENDENTAL EQUATIONS

Introduction

The solution of the equation of the form *f(x) = 0* occurs in the field of science, engineering and other applications. If *f(x)* is a polynomial of degree two or more ,we have formulae to find solution. But, if *f(x)* is a transcendental function, we do not have formulae to obtain solutions. When such type of equations are there, we have some methods like Bisection method, Newton-Raphson Method and The method of false position. Those methods are solved by using a theorem in theory of equations, *i.e., If f(x)* is continuous in the interval (a,b) and if f(a) and f(b) are of opposite signs, then the equation f(x) = 0 will have atleast one real root between a and b.

Bisection Method

Let us suppose we have an equation of the form *f(x) = 0* in which solution lies between in the range *(a,b)*. Also *f(x)* is continuous and it can be algebraic or transcendental. If *f(a)* and *f(b)* are opposite signs, then there exist atleast one real root between *a* and *b*.Let *f(a)* be positive and *f(b)* negative. Which implies atleast one root exits between *a* and *b*. We assume that root to be $x_o = (a+b)/2$. Check the sign of $f(x_o)$. If $f(x_0)$is negative , the root lies between *a* and x_o. If $f(x_0)$is positive , the root lies between x_o and b. Subsequently any one of this case occur.

$$X_1 = \frac{X_0+a}{2} \quad \text{(or)} \quad \frac{x_0+b}{2}$$

When $f(x_1)$ is negative , the root lies between xo and x1 and let the root be $x_2=(x_0 +x_1) / 2$.

Again $f(x_2)$ negative then the root lies between x_o and x_2, let $x_3 = (x_0+x_2)/2$ and so on. Repeat the process x_o, x_1,x_2, …. Whose limit of convergence is the exact root.

Steps:

1. Find a and b in which f(a) and f(b) are opposite signs for the given equation using trial and error method.
2. Assume initial root as $x_o = (a+b)/2$.
3. If $f(x_0)$ is negative , the root lies between a and x_o and take the root as $x_1 = (x_o+a)/2$.
4. If $f(x_o)$ is positive , then the root lies between x_o and b and take the root as $x_1 = (x_o+b)/2$.
5. If $f(x_1)$ is negative , the root lies between x_o and x_1 and let the root be $x_2 = (x_0+x_1)/2$.
6. If $f(x_2)$ is negative , the root lies between x_o and x_1 and let the root be $x_3 = (x_0+x_2)/2$.
7. Repeat the process until any two consecutive values are equal and hence the root.

Example:

Find the positive root of $x^3 - x = 1$ correct to four decimal places by bisection method.

Solution:

Let $f(x) = x^3 - x - 1$

$f(0) = 0^3 - 0 - 1 = -1 = -ve$

$f(1) = 1^3 - 1 - 1 = -1 = -ve$

$f(2) = 2^3 - 2 - 1 = 5 = +ve$

So root lies between 1 and 2 , we can take (1+2) /2 as initial root and proceed.

i.e., f(1.5) = 0.8750 = +ve

and f(1) = -1 = -ve

So root lies between 1 and 1.5 ,

Let x_o = (1+1.5) /2 as initial root and proceed.

f(1.25) = - 0.2969

So root lies between x_1 between 1.25 and 1.5

Now $x_1 = (1.25 + 1.5)/2 = 1.3750$

$f(1.375) = 0.2246 = +ve$

So root lies between x_2 between 1.25 and 1.375

Now $x_2 = (1.25 + 1.375)/2 = 1.3125$

$f(1.3125) = -0.051514 = -ve$

Therefore, root lies between 1.375and 1.3125

Now $x_3 = (1.375 + 1.3125) / 2 = 1.3438$

$f(1.3438) = 0.082832 = +ve$

So root lies between 1.3125 and 1.3438

Now $x_4 = (1.3125 + 1.3438) / 2 = 1.3282$

$f(1.3282) = 0.014898 = +ve$

So root lies between 1.3125 and 1.3282

Now $x_5 = (1.3125 + 1.3282) / 2 = 1.3204$

$f(1.3204) = -0.018340 = -ve$

So root lies between 1.3204 and 1.3282

Now $x_6 = (1.3204 + 1.3282) / 2 = 1.3243$

$f(1.3243) = -ve$

So root lies between 1.3243 and 1.3282

Now $x_7 = (1.3243 + 1.3282) / 2 = 1.3263$

$f(1.3263) = +ve$

So root lies between 1.3243 and 1.3263

Now $x_8 = (1.3243 + 1.3263) / 2 = 1.3253$

$f(1.3253) = +ve$

So root lies between 1.3243 and 1.3253

Now $x_9 = (1.3243 + 1.3253) / 2 = 1.3248$

$f(1.3248) = +ve$

So root lies between 1.3243 and 1.3248

Now $x_{10} = (1.3243 + 1.3248) / 2 = 1.3246$

$f(1.3246) = -ve$

So root lies between 1.3248 and 1.3246

Now x_{11} = (1.3248 + 1.3246) / 2 = *1.3247*

f(1.3247) = -ve

So root lies between 1.3247 and 1.3248

Now x_{12} = (1.3247 + 1.3247) / 2 = *1.32475*

Therefore , the approximate root is 1.32475

Example

Find the positive root of x – cos x = 0 by bisection method.

Solution :

Let f(x) = x – cos x

f(0) = 0 - cos (0) = 0 -1 = -1 = -ve

f(0.5) = 0.5 – cos (0.5) = -0.37758 = -ve

f(1) = 1 – cos (1) = 0.42970 = +ve

So root lies between 0.5 and 1

Let x_0 = (0.5 +1) /2 as initial root and proceed.

f(0.75) = *0.75 – cos (0.75) = 0.018311 = +ve*

So root lies between 0.5 and 0.75

x_1 = (0.5 +0.75) /2 =0.625

f(0.625) = *0.625 – cos (0.625)* = - *0.18596*

So root lies between 0.625 and 0.750

x_2 = (0.625 +0.750) /2 = 0.6875

f(0.6875) = - *0.085335*

So root lies between 0.6875 and 0.750

$x3$ = (0.6875 +0.750) /2 = 0.71875

f(0.71875) = 0.71875-cos(0.71875) = - *0.033879*

So root lies between 0.71875 and 0.750

$x4$ = (0.71875 +0.750) /2 = 0.73438

f(0.73438) = -0.0078664 = - ve

So root lies between 0.73438 and 0.750

$x5$ = 0.742190

f(0.742190) = 0.0051999 = + ve

$x6$ = (0.73438 +0.742190) /2 = 0.73829

f(0.73829) = -0.0013305

So root lies between 0.73829 and 0.74219

$x7$ = (0.73829+0.74219) = 0.7402

f(0.7402) = 0.7402-cos(0.7402) = 0.0018663

So root lies between 0.73829 and 0.7402

$x8$ = 0.73925

f(0.73925) = 0.00027593

$x9$ = 0.7388

The root is 0.7388.

Newton-Raphson method (or Newton's method)

Let us suppose we have an equation of the form $f(x) = 0$ in which solution is lies between in the range (a,b). Also $f(x)$ is continuous and it can be algebraic or transcendental. If $f(a)$ and $f(b)$ are opposite signs, then there exist atleast one real root between a and b.

Let $f(a)$ be positive and $f(b)$ negative. Which implies atleast one root exits between a and b. We assume that root to be either a or b, in which the value of f(a) or f(b) is very close to zero. That number is assumed to be initial root. Then we iterate the process by using the following formula until the value is converges.

$f(X_n)$

$$X_{n+1} = X_n - \frac{\qquad}{f'(X_n)}$$

Steps:

1. Find *a* and *b* in which *f(a)* and *f(b)* are opposite signs for the given equation using trial and error method.
2. Assume initial root as $X_o = a$ i.e., *if f(a)*is very close to zero or $X_o = b$ *if f(a)*is very close to zero
3. Find X1 by using the formula

$$X_1 = X_o - \frac{f(X_o)}{f'(X_o)}$$

4. *Find* X_2 *by using the following formula*

$$X_2 = X_1 - \frac{f(X_1)}{f'(X_1)}$$

5. *Find* $X_3, X_4, \ldots X_n$ *until any two successive values are equal.*

Example:

Find the positive root of f(x) = 2x3 - 3x-6 =0 by Newton – Raphson method correct to five decimal places.

Solution:

Let $f(x) = 2x^3 - 3x - 6$; $f'(x) = 6x^2 - 3$

$f(1)$ = 2-3-6 = -7 = -ve

$f(2)$ = 16 – 6-6 = 4 = +ve

So, a root between 1 and 2 . In which 4 is closer to 0 Hence we assume initial root as 2.

Consider $x_0 = 2$

So $X_1 = X_0 - f(X_0)/f'(X_0)$

$= X_0 - ((2X_03 - 3X_0 - 6) / 6\alpha_0 - 3) = (4X_03 + 6)/(6X_02 - 3)$

$X_{i+1} = (4X_i3 + 6)/(6X_i2 - 3)$

$X_1 = (4(2)^2+6)/(6(2)^2- 3) = 38/21 = 1.809524$

$X_2= (4(1.809524)^3+6)/(6(1.809524)^2- 3) = 29.700256/16.646263 = 1.784200$

$X_3= (4(1.784200)^3+6)/(6(1.784200)^2- 3) = 28.719072/16.100218 = 1.783769$

$X_4= (4(1.783769)^3+6)/(6(1.783769)^2- 3) = 28.702612/16.090991 = 1.783769$

Example:

Using Newton's method, find the root between 0 and 1 of $x^3 = 6x – 4$ correct to 5 decimal places.

Solution :

Let $f(x) = x^3 -6x+4$; $f(0) = 4 = +ve$; $f(1) = -1 = -ve$

So a root lies between 0 and 1

f(1) is nearer to 0. Therefore we take initial root as $X_0= 1$

$f'(x) = 3x^2- 6$

$$= x - \frac{f(x)}{f'(x)}$$

$= x - (3x^3 -6x+4)/(3x^2-6)$

$= (2x^3 -4)/(3x^2 -6)$

$X_1 =(2X_0 3 – 4)/(3X_0 2-6) = (2-4)/(3-6) = 2/3 = 0.66666$

$X_2 =(2(2/3)^3 – 4)/(3(2/3)^2-6) = 0.73016$

$X_3= (2(0.73015873)^3 – 4)/(3(0.73015873)^2-6)$

$= (3.22145837/ 4.40060469)$

$= 0.73205$

$X_4= (2(0.73204903)^3 – 4)/(3(0.73204903)^2-6)$

$= (3.21539602/ 4.439231265)$

$= 0.73205$

The root is 0.73205 correct to 5 decimal places.

Method of False Position (or Regula Falsi Method)

Consider the equation $f(x) = 0$ and $f(a)$ and $f(b)$ *are* of opposite signs. Also let $a < b$. The graph $y = f(x)$ *will* Meet the x-axis at some point between A(a, f(a)) and B (b,f(b)). The equation of the chord joining the two points A(a, f(a)) and B (b,f(b)) is

$$\frac{y - f(a)}{x - a} = \frac{f(a) - f(b)}{a - b}$$

The x- Coordinate of the point of intersection of this chord with the x-axis gives an approximate value for the of f(x) = 0. Taking y = 0 in the chord equation, we get

$$\frac{- f(a)}{x - a} = \frac{f(a) - f(b)}{a - b}$$

$$x[f(a) - f(b)] - a f(a) + a f(b) = -a f(a) + b f(b)$$

$$x[f(a) - f(b)] = b f(a) - a f(b)$$

This x_1 gives an approximate value of the root f(x) = 0. $(a < x_1 < b)$

Now $f(x_1)$ and f(a)are of opposite signs or $f(x_1)$ and f(b) are opposite signs.

If $f(x_1), f(a) < 0$. then x2 lies between x_1 and a.

Therefore $$x2 = \frac{a f(x_1) - x_1 f(b)}{f(x_1) - f(a)}$$

This process of calculation of ($x_3, x_4, x_5, \ldots$) is continued till any two successive values are equal and subsequently we get the solution of the given equation.

Steps:

1. Find *a* and *b* in which *f(a)* and *f(b)* are opposite signs for the given equation

using trial and error method.

2. Therefore root lies between *a* and *b if f(a)*is very close to zero select and compute x_1 by using the following formula:

$$x1 = \frac{a\,f(b) - b\,f(a)}{f(b) - f(a)}$$

3. If $f(x_1), f(a) < 0$. then root lies between x_1 and *a*. Compute x_2 by using the following formula:

$$x2 = \frac{a\,f(x_1) - x_1\,f(b)}{f(x_1) - f(a)}$$

4. Calculate the values of ($x_3, x_4, x_5, \ldots$) by using the above formula until any two successive values are equal and subsequently we get the solution of the given equation.

. **Example:**

Solve for a positive root of $x^3-4x+1=0$ by and Regula Falsi method

Solution :

Let $f(x) = x^3-4x+1 = 0$

$f(0) = 0^3-4(0)+1 = 1 = +ve$

$f(1) = 1^3-4(1)+1 = -2 = -ve$

So a root lies between 0 and 1

We shall find the root that lies between 0 and 1.

Here a=0, b=1

$$x1 = \frac{a\,f(b) - b\,f(a)}{f(b) - f(a)}$$

(0 x f(1) – 1 x f(0))

$$= \frac{\quad}{(f(1) - f(0))}$$

$$= \frac{-1}{(-2 - 1)}$$

$= 0.333333$

$f(x_1) = f(1/3) = (1/27)-(4/3) +1 = -0.2963$

Now f(0) and f(1/3) are opposite in sign.

Hence the root lies between 0 and 1/3.

$$x_2 = \frac{(0 \text{ x } f(1/3) - 1/3 \text{ x } f(0))}{(f(1/3) - f(0))}$$

$x_2 = (-1/3)/ (-1.2963) = 0.25714$

Now $f(x_2) = f(0.25714) = - 0.011558 = -ve$

So the root lies between 0 and 0.25714

$x_3 = (0 \text{ x } f(0.25714) - 0.25714 \text{ x } f(0)) / (f(0.25714) - f(0))$

$= -0.25714/-1.011558 = 0.25420$

$f(x_3) = f(0.25420) = -0.0003742$

So the root lies between 0 and 0.25420

$x_4 = (0 \text{ x } f(0.25420)- 0.25420 \text{ x } f(0)) / (f(0.25420) - f(0))$

$= -0.25420 / -1.0003742 = 0.25410$

$f(x_4) = f(0.25410) = - 0.000012936$

The root lies between 0 and 0.25410

$x_5 = (0 \text{ x } f(0.25410) - 0.25410 \text{ x } f(0)) / (f(0.25410) - f(0))$

$= -0.25410 / -1.000012936 = 0.25410$

Hence the root is 0.25410.

Example:

Find an approximate root of $x \log_{10}x - 1.2 = 0$ by False position method.

Solution :

Let $f(x) = x \log_{10}x - 1.2$

$f(1) = -1.2 = -ve$; $f(2) = 2 \times 0.30103 - 1.2 = -0.597940$

$f(3) = 3 \times 0.47712 - 1.2 = 0.231364 = +ve$

So, the root lies between 2 and 3.

$$x_1 = \frac{2f(3) - 3f(2)}{f(3) - f(2)} = \frac{2 \times 0.23136 - 3 \times (-0.59794)}{0.23136 + 0.597}$$

$$= 2.721014$$

$f(x_1) = f(2.7210) = -0.017104$

The root lies between x_1 and 3.

$$x_2 = \frac{{}_1 xf(3) - 3 \times f(x_1)}{f(3) - f(x_1)} = \frac{2.721014 \times 0.231364 - 3 \times (-0.017104)}{0.23136 + 0.017104}$$

$$= 2.740211$$

$f(x_2) = f(2.7402) = 2.7402 \times \log(2.7402) – 1.2$

$= - 0.00038905$

So the root lies between 2.740211 and 3

$$x_3 = \frac{2.7402 \times f(3) – 3 \times f(2.7402)}{f(3) – f(2.7402)} = \frac{2.7402 \times 0.231336 + 3 \times (0.00038905)}{0.23136 + 0.00038905}$$

$$= \frac{0.63514}{0.23175} = 2.740627$$

$f(2.7406) = 0.00011998$

So the root lies between 2.740211 and 2.740627

$$x_4 = \frac{2.7402 \times f(2.7406) – 2.7406 \times f(2.7402)}{f(2.7406) – f(2.7402)}$$

$$= \frac{2.7402 \times 0.00011998 + 2.7406 \times 0.00038905}{0.00011998 + 0.00038905}$$

0.0013950

= ________
0.00050903

= 2.7405

Hence the root is 2.7405

Horner's Method

This numerical methods is employed to determine both the commensurable and the incommensurable real roots of a numerical polynomial equation. Firstly, we find the integral part of the root and then by every iteration. We find each decimal place value in succession.

Suppose a positive root of f(x) = 0 lies between a and a+1.
Let that root be a,a1a2a3….

First diminish the root of f(x)-0 by the integral part a and let $\phi 1(x) = 0$ possess the root 0.a1a2a3…

Secondly , multiply the roots of $\phi 1(x) = 0$ by 10 and let $\phi 2(x) = 0$ possess the root a1.a2a3…as a root.

Thirdly, find the value od a1 and then diminish the roots by a1 and let $\phi 3(x) = 0$ possess a root 0. a2a3…

Now repeating the process we find a2,a3,a4…. each time.

Example:

Find the positive root of $x^3 +3x -1 = 0$, correct to two decimal places by Horner's method.

Solution:

Let $f(x) = x^3 +3x -1 = 0$

f(0) = -ve f(1) = * ve.

The positive root lies between 0 and 1.

Let it be 0.a1a2a3….

Since the integral part is zero, diminishing the root by the integral part is not necessary. Therefore multioly the roots by 10.

Therefore $\phi 1(x) = x^3 +300x -1000 =0$ has root a1.a2a3…

$\phi 1(3)$ = -ve, $\phi 1(4)$ = +ve

Therefore a1=3

Now, the root is 3.a2a3…

Therefore, diminish root of $\phi 1(x) = 0$ by 3

By synthetic division method, we get

$\phi 2(x) = x^3 + 9x^2 + 327x - 73 = 0$ has root 0.a2a3…

Multiply the roots of $\phi 2(x) = 0$ by 10.

$\phi 3(x) = x^3 + 90x^2 + 32700x - 73000 = 0$ has root a2.a3a4…
Now, $\phi 3(2)$ = -ve, $\phi 3(3)$ = +ve

Therefore a2=2

Now diminish the roots of $\phi 3(x)$ by 2.

By synthetic division method, we get

$\phi 4(x) = x^3 + 96x^2 + 33072x - 7232 = 0$ has root 0.a3a4…

Multiply the roots of $\phi 4(x) = 0$ by 10.

$\phi 5(x) = x^3 + 960x^2 + 3307200x - 7232000 = 0$ has root a3.a4…

Now, $\phi 5(2)$ = -ve, $\phi 5(3)$ = +ve

Therefore a3=2

Hence the root is 0.322.

Graeffe's Root SquaringMethod

This is a direct method to find the roots of any polynomial equation with real coefficients. The basic idea behind this method is to separate the roots of the equations by squaring the roots. This can be done by separating even and odd powers of **x** in

$$\mathbf{P_n(x) = x^n + a_1 x^{n-1} + a_2 x^{n-2} + . . . + a_{n-1}x + a_n = 0}$$

and squaring on both sides. Thus we get,

$$\mathbf{(x^n + a_2 x^{n-2} + a_4 x^{n-4} + . . .)^2 = (x^n + a_1 x^{n-1} + a_3 x^{n-3} + . . .)^2}$$

$$x^{2n}{}_n - (a_1^2 - 2a_2)x^{2n-2} + (a_2^2 - 2a_1a_3 + 2a_4)x^{2n-4} + . . . +(-1)^n a_n^2 = 0$$

substituting y for $-x^2$ we have

$$y^n + b_1y^{n-1} + . . . + b_{n-1}y + b_n = 0$$

where

$$b_1 = a_1^2 - 2a_2$$

$$b_2 = a_2^2 - 2a_1a_3 + 2a_4$$

.

.

.

$$b_n = a_n^2$$

Thus all the b_i's ($i = 0, 1, 2, . . . n$) are known in terms of a_i's. The roots of this equation are $-s_1^2, -s_2^2, . . ., s_n^2$ where $s_1, s_2, . . ., s_n$ are the roots of $P_n(x) = 0$.

A typical coefficient b_k of b_i, $i = 1, 2, . . .n$ is obtained by following .The terms alternate in sign starting with a +ve sign. The first term is the square of the coefficient a_k. The second term is twice the product of the nearest neighbouring coefficients a_{i-1} and a_{i+1}. The third is twice the product of the next neighbouring coefficients a_{i-2} and a_{i+2}. This procedure is continued until there are no available coefficients to form the cross products.

This procedure can be repeated many times so that the final equation

$x^n + B_1 x^{n-1} + . . . + B_{n-1}x + B_n = 0$ has the roots $R_1, R_2, . . ., R_n$ such that

$$R_i = -s_i^{(2^m)}, \qquad i = 1, 2, . . ., m$$

if we repeat the process for m times.

If we assume $|s_1| > |s_2| > . . . |s_n|$ then $|R_1| >> |R_2| >> . . .>> |R_n|$

that is the roots R_i are very widely separated for large m.

Now we have $-B_1 = \Sigma R_i \Sigma R_1$

$$B_2 = \Sigma R_iR_j \Sigma R_1R_2$$

$$-B_3 = \Sigma R_iR_jR_k \Sigma R_1R_2R_3$$

.

.

.

$$(-1)^n B_n = R_1 R_2 . . . R_n$$

which gives $\mathbf{R_i = -B_i / B_{i-1}}$, $\mathbf{i = 1, 2, . . . n}$

where $\mathbf{B_0 = 1}$.

since $\mathbf{| s_i |^{2\wedge m} = | R_i |}$ $\mathbf{i = 1, 2, . . . n}$

$\mathbf{\Sigma | s_i | = | R_i |^{2\wedge -m}}$ $\mathbf{i = 1, 2, . . . n}$

This determines the absolute values of the roots and substitution in the original equation will give the sign of the roots.

Example :

Find the roots of $\mathbf{x^3 - 7x^2 + 14x - 8 = 0}$

a[]	1	-7	14	-8
b[]	1	21	84	64
roots =	**4.583**	**2**	**0.873**	
b[]	1	273	4368	4096
roots =	**4.065**	**2**	**0.984**	
b[]	1	65793	1.68E7	1.68E7
roots =	**4.002**	**2**	**0.9995**	

Thus the absolute values of the roots are **4, 2, 1**.

Since **f(1) = 0**, **f(2) = 0** and **f(4) = 0**, the signs of the roots **1, 2** and **4** are all positive.

Possible Questions

Part B (5x8=40 Marks)

1. Find the positive root of $x - \cos x = 0$ by using bisection method.
2. Find the positive root of $e^x = 3x$ by using Bisection method.
3. Solve the equation $x^3 + x^2 - 1 = 0$ for the positive root by iteration method.
4. Find the real root of the equation $\cos x = 3x - 1$ correct to 4 decimal places by iteration method.
5. Find an approximate root of $x \log_{10} x = 1.2$ by False position method.
6. Find an approximate root of $x^3 - 4x + 1 = 0$ by False position method**.**
7. Find the real positive root of $3x - \cos x - 1 = 0$ by Newton's method correct to 3 decimal places.
8. Find the positive root of $x^3 + 3x - 1 = 0$,correct to two decimal places, by Horner's method.
9. Find all the roots of the equation $2x^3 + x^2 - 2x - 1 = 0$ by Graeffe's method (four squaring) .
10. Find all the roots of the equation $x^3 - 9x^2 + 18x - 6 = 0$ by Graeffe's method (root squaring, three times).

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DEPARTMENT OF MATHEMATICS

Subject: Numerical Methods	**Subject Code: 15MMU503**	**L T P C**
Class:III B.Sc-B	**Semester:V**	**4 1 0 4**

UNIT II

Solution of simultaneous linear algebraic equations: Gauss elimination method – Gauss Jordan method – Method of triangularization – Crout's method – Gauss-Jacobi method – Gauss-seidel method.

TEXT BOOK:

T1: Venkataraman .M.K., Fifth Edition, 2001. Numerical Methods in Science and Engineering National publishing Company ,Madras.

R2: Vedamurthy V.N., N.Ch.S.N.Iyenger., 1999. Numerical Methods,Vikas Publishing House Pvt Ltd, New Delhi.

R3: Kandaswamy. P., Thilagavathy K., and K.Gunavathy., 2013 .Numerical Methods, S. Chand &Company Ltd., New Delhi.

UNIT-II

SOLUTIONS OF SIMULTANEOUS LINEAR ALGEBRAIC EQUATIONS

INTRODUCTION

We will study here a few methods below deals with the solution of simultaneous Linear Algebraic Equations

GAUSS ELIMINATION METHOD (DIRECT METHOD).

This is a direct method based on the elimination of the unknowns by combining equations such that the n unknowns are reduced to an equation upper triangular system which could be solved by back substitution.

Consider the n linear equations in n unknowns, viz.

$$a_{11}x_1+a_{12}x_2+....+a_{1n}x_n=b_1$$

$$a_{21}x_1+a_{22}x_2+....+a_{2n}x_n=b_2$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$a_{n1}x_1+a_{n2}x_2+.....+a_{nn}x_n=b_n \quad ....(1)$$

Where a_{ij} and b_i are known constants and x_i's are unknowns.

The system (1) is equivalent to $AX=B$ .....(2)

$$\text{Where } A = \begin{pmatrix} a_{11} & a_{12} \dots\dots\dots a_{1n} \\ a_{21} & a_{22} \dots\dots\dots a_{2n} \\ \dots\dots\dots & \dots\dots\dots\dots\dots\dots \\ a_{n1} & a_{n2} \dots\dots a_{nn} \end{pmatrix} \quad X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \text{ and } B = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

Now our aim is to reduce the augmented matrix (A,B) to upper triangular matrix.

$$(A,B) = \left(\begin{array}{cc|c} a_{11} & a_{12} \dots\dots\dots a_{1n} & b_1 \\ a_{21} & a_{22} \dots\dots\dots a_{2n} & b_2 \\ \dots\dots\dots & \dots\dots\dots\dots\dots\dots & \vdots \\ a_{n1} & a_{n2} \dots\dots\dots a_{nn} & b_n \end{array}\right) \quad ..(3)$$

Now, multiply the first row of (3) (if $a_{11} \neq 0$) by $-\frac{a_{i1}}{a_{11}}$ and add to the ith row of (A,B), where i=2,3,…,n. By thia, all elements in the first column of (A,B) except a_{11} are made to zero. Now (3) is of the form

$$\left(\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ 0 & b_{22} & \cdots & b_{2n} & c_2 \\ \cdots & \cdots & \cdots & & \vdots \\ 0 & b_{n2} & \cdots & b_{nn} & c_n \end{array}\right) \quad \ldots\ldots.(4)$$

Now take the pivot b_{22}. Now, considering b_{22} as the pivot, we will make all elements below b_{22} in the second column of (4) as zeros. That is, multiply second row of (4) by $-\frac{b_{i2}}{b_{22}}$ and add to the corresponding elements of the ith row (i=3,4,…,n). Now all elements below b_{22} are reduced to zero. Now (4) reduces to

$$\left(\begin{array}{ccccc|c} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & b_1 \\ 0 & b_{22} & b_{23} & \cdots & b_{2n} & c_2 \\ 0 & 0 & c_{23} & \cdots & c_{3n} & d_3 \\ \cdots & \cdots & \cdots & \cdots & & \vdots \\ 0 & 0 & c_{n3} & \cdots & c_{nn} & d_n \end{array}\right) \quad \ldots\ldots (5)$$

Now taking c_{33} as the pivot, using elementary operations, we make all elements below c_{33} as zeros. Continuing the process, all elements below the leading diagonal elements of A are made to zero.

Hence, we get (A,B) after all these operations as

$$\left(\begin{array}{cccccc|c} a_{11} & a_{12} & a_{13} & \cdots & & a_{1n} & b_1 \\ 0 & b_{22} & b_{23} & \cdots & & b_{2n} & c_2 \\ 0 & 0 & c_{23} & c_{34} & \cdots & c_{3n} & d_3 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & c_{nn} & d_n \end{array}\right) \quad \ldots(6)$$

From, (6) the given system of linear equations is equivalent to

$$a_{11}x_1+a_{12}x_2+a_{13}x_3+....+a_{1n}x_n=b_1$$

$$b_{22}x_2+b_{23}x_3+....+b_{2n}x_n=c_2$$

$$c_{33}x_3+....+c_{3n}x_n=d_3$$

$$\dots\dots\dots\dots\dots\dots\dots..$$

$$\alpha_{nn}x_n=k_n$$

Going from the bottom of these equation, we solve for $x_n=\frac{k_n}{\alpha_{nn}}$. Using this in the penultimate equation, we get x_{n-1} and so. By this back substitution method for we solve x_n , x_{n-1}, x_{n-2}, ……x_2, x_1.

GAUSS – JORDAN ELIMINATION METHOD (DIRECT METHOD)

This method is a modification of the above Gauss elimination method. In this method, the coefficient matrix A of the system $AX=B$ is brought to a diagonal matrix or unit matrix by making the matrix A not only upper triangular but also lower triangular by making the matrix A not above the leading diagonal of A also as zeros. By this way, the system $AX=B$ will reduce to the form.

$$\left(\begin{array}{cccccc|c} a_{11} & 0 & 0 & 0 & \dots\dots\dots.\ \dots. & a_{1n} & b_1 \\ 0 & b_{22} & 0 & 0 & \dots.\dots\dots\ \dots. & b_{2n} & c_2 \\ \dots\dots\dots & \dots\dots\dots & \dots\dots\dots & \dots\dots\dots & \dots\dots\dots\ \dots & \dots\dots\dots & d_3 \\ 0 & 0 & 0 & 0 & \dots\dots\ \dots\dots\dots & \alpha_{nn} & k_n \end{array}\right) \dots(7)$$

From (7)

$$x_n = \frac{k_n}{\alpha_{nn}}, \ \dots\dots, \ x_2 = \frac{c_2}{b_{22}}, \ x_n = \frac{b_1}{a_{11}}$$

Note: By this method, the values of $x_1, x_2, \dots..x_n$ are got immediately without using the process of back substitution.

Example 1. *Solve the system of equations by (i) Gauss elimination method (ii) Gauss – Jordan method.*

$x+2y+z=3,\ 2x+3y+3z=10,\ 3x-y+2z=13.$

Solution. (By Gauss method)

This given system is equivalent to

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 3 \\ 3 & -1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 10 \\ 13 \end{pmatrix}$$

$$A\,X = B$$

$$(A,B) = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 2 & 3 & 3 & 10 \\ 3 & -1 & 2 & 13 \end{array}\right] \text{ .... (1)}$$

Now, we will make the matrix A upper triangluar.

$$(A,B) = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 2 & 3 & 3 & 10 \\ 3 & -1 & 2 & 13 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & -7 & -1 & 4 \end{array}\right] \quad R_2+(-2)R_1,\ R_3+(-3)R_1$$

Now, take $b_{22}=-1$ as the pivot and make b_{32} as zero.

$$(A,B) \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & -8 & -24 \end{array}\right] \quad R_{32}(-7) \text{ ....(2)}$$

From this, we get

$$x+2y+z = 3, \quad -y+z = 4, \quad -8z = -24$$

$\therefore$ $z = 3,\ y = -1,\ x = 2$ by back substitution.

$x = 2,\ y = -1,\ z = 3$

Solution. (Gauss – Jordan method)

In stage 2, make the element, in the position (1,2), also zero.

$$(A,B) \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & -8 & -24 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 3 & 11 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & -8 & -24 \end{array}\right] \quad R_{12}(2)$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 3 & 11 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & -1 & -3 \end{array}\right] \quad R_3\left(\frac{1}{8}\right)$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & -3 \end{array}\right] \quad R_{13}(3),\ R_{23}(1)$$

i.e., $x = 2,\ y = -1,\ z = 3$

METHOD OF TRIANGULARIZATION (OR METHOD OF FACTORIZATION) (*DIRECT METHOD*)

This method is also called as *decomposition* method. In this method, the coefficient matrix A of the system $AX = B$, decomposed or factorized into the product of a lower triangular matrix L and an upper triangular matrix U. we will explain this method in the case of three equations in three unknowns.

Consider the system of equations

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

This system is equivalent to $AX = B$

Where $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$, $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$, $B = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Now we will factorize A as the product of lower triangular matrix

$$L = \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix}$$

And an upper triangular matrix

$$U = \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix} \text{ so that}$$

$LUX = B$ Let $UX = Y$ And hence $LY = B$

That is, $$\begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$\therefore$ $y_1 = b,\ l_{21}y_1+y_2 = b_2,\ l_{31}y_1+l_{32}y_2+y_3 = b_3$

By forward substitution, y_1, y_2, y_3 can be found out if L is known.

From (4), $$\begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$u_{11}x_1 + u_{12}x_2 + u_{13}x_3 = y_1, \quad u_{22}x_2 + u_{23}x_3 = y_2 \text{ and } \quad u_{33}x_3 = y_3$$

From these, x_1, x_2, x_3 can be solved by back substitution, since y_1, y_2, y_3 are known if U is known.Now L and U can be found from $LU = A$

i.e., $$\begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix}\begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

i.e.,

$$\begin{pmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

Equating corresponding coefficients we get nine equations in nine unknowns. From these 9 equations, we can solve for 3 l's and 6 u's.

That is, L and U re known. Hence X is found out. Going into details, we get $u_{11} = a_{11}$, $u_{12} = a_{12}$, $u_{13} = a_{13}$. That is the elements in the first rows of U are same as the elements in the first of A.

Also, $l_{21}u_{11} = a_{21}$ $\quad l_{21}u_{12}+u_{22} = a_{22}$ $\quad l_{21}u_{13}+u_{23} = a_{23}$

$$l_{21} = \frac{a_{21}}{a_{11}},\quad u_{22} = a_{22} - \frac{a_{21}}{a_{11}}.\,a_{12} \quad \text{and} \quad u_{23} = a_{23} - \frac{a_{21}}{a_{11}}.\,a_{13}$$

again, $l_{31}u_{11} = a_{31}$, $l_{31}u_{12}+l_{32}u_{22} = a_{32}$ and $l_{31}u_{13}+l_{32}u_{23}+u_{33} = a_{33}$

solving, $l_{31} = \frac{a_{31}}{a_{11}}$, $l_{32} = \frac{a_{32} - \frac{a_{31}}{a_{11}} . a_{12}}{a_{22} - \frac{a_{21}}{a_{11}} . a_{12}}$

$$u_{33} = \left\{ a_{33} - \frac{a_{31}}{a_{11}} . a_{13} \right\} - \left[\frac{a_{32} - \frac{a_{31}}{a_{11}} . a_{12}}{a_{22} - \frac{a_{21}}{a_{11}} . a_{12}} \right] a_{33} - \frac{a_{31}}{a_{11}} . a_{13}$$

Therefore L and U are known.

Example 2 *By the method of triangularization, solve the following system.*

5x – 2y + z = 4, 7x + y – 5z = 8, 3x + 7y + 4z = 10.

Solution. The system is equivalent to

$$\begin{pmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \\ 10 \end{pmatrix}$$

$$A \quad X = B$$

Now, let $LU = A$

That is, $$\begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix} = \begin{pmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{pmatrix}$$

Multiplying and equating coefficients,

$$u_{11} = 5, \quad u_{12} = -2, \quad u_{13} = 1$$

$$l_{21}u_{11} = 7 \quad l_{21}u_{12}+u_{22} = 1 \quad l_{21}u_{13}+u_{23} = -5$$

$l_{21} = \frac{7}{5}$, $u_{22} = 1 - \frac{7}{5}.(-2) = \frac{19}{5}$ and

$$u_{23} = -5 - \frac{7}{5}.(1) = -\frac{32}{5}$$

Again equating elements in the third row,

$l_{31}u_{11} = 3$, $l_{31}u_{12}+l_{32}u_{22} = 7$ and $l_{31}u_{13}+l_{32}u_{23}+u_{33} = 4$

$$\therefore \quad l_{31} = \frac{3}{5}, \quad l_{32} = \frac{7 - \frac{3}{5}.(-2)}{\frac{19}{5}} = \frac{41}{19}$$

$$u_{33} = 4 - \frac{3}{5}.(1) - \frac{41}{19}\left(-\frac{32}{5}\right) = 4 - \frac{3}{5} + \frac{1312}{95}$$

$$= \frac{1635}{95} = \frac{327}{19}$$

Now L and U are known.Since $LUX = B$, $LY = B$ where $UX = Y$.

From $LY = B$,

$$\begin{pmatrix} 1 & 0 & 0 \\ \frac{7}{5} & 1 & 0 \\ \frac{3}{5} & \frac{41}{19} & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \\ 10 \end{pmatrix}$$

$$y_1 = 4, \quad \frac{7}{5} y_1 + y_2 = 8, \quad \frac{3}{5} y_1 + \frac{41}{19} y_2 + y_3 = 10$$

$$y_2 = 8 - \frac{28}{5} = \frac{12}{5}$$

$$y_3 = 10 - \frac{12}{5} - \frac{41}{19} \times \frac{12}{5} = 10 - \frac{12}{5} - \frac{492}{95} = \frac{46}{19}$$

$UX = Y$ gives

$$\begin{pmatrix} 5 & -2 & 1 \\ 0 & \frac{19}{5} & -\frac{32}{5} \\ 0 & 0 & \frac{327}{19} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ \frac{12}{5} \\ \frac{46}{19} \end{pmatrix}$$

$$5x - 2y + z = 4$$

$$\frac{19}{5} y - \frac{32}{5} z = \frac{12}{5}$$

$$\frac{327}{19} z = \frac{46}{19}$$

$$z = \frac{46}{327}$$

$$\frac{19}{5} y = \frac{12}{5} + \frac{32}{5}\left(\frac{46}{327}\right)$$

$$y = \frac{284}{327}$$

$$5x = 4 + 2y - z = 4 + 2\left(\frac{568}{327}\right) - \frac{46}{327}$$

$$\therefore \quad x = \frac{366}{327}$$

$$\therefore \quad x = \frac{366}{327},\ y = \frac{284}{327},\ z = \frac{46}{327}$$

Crout's Method

Crout's Method is a root-finding algorithm used in LU decomposition (see Foundation). Also known as Crout Matrix Decomposition and Crout Factorization, the method decomposes a matrix into a lower triangular matrix (L), an upper triangular matrix (U), and a permutation matrix (P). The last matrix is optional and not always needed.

Crout's Method solves the N^2 equations

$$i < j \quad l_{i1}u_{1j} + l_{i2}u_{2j} + \dots + l_{ii}u_{ij} = a_{ij}$$

$$i = j \quad l_{i1}u_{1j} + l_{i2}u_{2j} + \dots + l_{ii}u_{jj} = a_{ij}$$

$$i > j \quad l_{i1}u_{1j} + l_{i2}u_{2j} + \dots + l_{ij}u_{jj} = a_{ij}$$

for the $N^2 + N$ unknowns l_{ij} and u_{ij}.

ITERATIVE METHODS

This iterative methods is not always successful to all systems of equations. If this method is to succeed, each equation of the system must possess one large coefficient and the large coefficient must be attached to a different unknown in that equation. This condition will be satisfied if the large coefficients are along the leading diagonal of the coefficient matrix. When this condition is satisfied, the system will be solvable by the iterative method. The system,

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

will be solvable by this method if

$$|a_{11}| > |a_{12}| + |a_{13}|$$

$$|a_{22}| > |a_{21}| + |a_{23}|$$

$$|a_{33}| > |a_{31}| + |a_{32}|$$

In other words, the solution will exist (iteration will converge) if the absolute values of the leading diagonal elements of the coefficient matrix A of the system $AX=B$ are greater than the sum of absolute values of the other coefficients of that row. The condition is *sufficient* but not *necessary*.

JACOBI METHOD OF ITERATION OR GAUSS – JACOBI METHOD

Let us explain this method in the case of three equations in three unknowns.

Consider the system of equations,

$$a_1x+b_1y+c_1z = d_1$$

$$a_2x+b_2y+c_2z = d_2$$

$$a_3x+b_3y+c_3z = d_3 \quad \ldots\ldots\ldots\ldots (1)$$

Let us assume $|a_1| > |b_1| + |c_1|$

$$|b_2| > |a_2| + |c_2|$$

$$|c_3| > |a_3| + |b_3|$$

Then, iterative method can be used for the system (1). Solve for x, y, z (whose coefficients are the larger values) in terms of the other variables. That is,

$$x = \frac{1}{a_1}(d_1 - b_1y - c_1z)$$

$$y = \frac{1}{b_2}(d_2 - a_2x - c_2z)$$

$$z = \frac{1}{c_3}(d_3 - a_3x - b_3y) \quad \ldots\ldots\ldots. (2)$$

If x^0, y^0, z^0 are the initial values of *x, y, z* respectively, then

$$x^{(1)} = \frac{1}{a_1}(d_1 - b_1y^{(0)} - c_1z^{(0)})$$

$$y^{(1)} = \frac{1}{b_2}(d_2 - a_2x^{(0)} - c_2z^{(0)})$$

$$z^{(1)} = \frac{1}{c_3}(d_3 - a_3x^{(0)} - b_3y^{(0)}) \quad \ldots\ldots\ldots. (3)$$

Again using these values $x^{(2)}$, $y^{(2)}$, $z^{(2)}$ in (2), we get

$$x^{(2)} = \frac{1}{a_1}(d_1 - b_1y^{(1)} - c_1z^{(1)})$$

$$y^{(2)} = \frac{1}{b_2}(d_2 - a_2x^{(1)} - c_2z^{(1)})$$

$$z^{(2)} = \frac{1}{c_3}(d_3 - a_3x^{(1)} - b_3y^{(1)}) \quad \ldots.(4)$$

Proceeding in the same way, if the rth iterates are $x^{(r)}$, $y^{(r)}$, $z^{(r)}$, the iteration scheme reduces to

$$x^{(r+1)} = \frac{1}{a_1}(d_1 - b_1y^{(r)} - c_1z^{(r)})$$

$$y^{(r+1)} = \frac{1}{b_2}(d_2 - a_2x^{(r)} - c_2z^{(r)})$$

$$z^{(r+1)} = \frac{1}{c_3}(d_3 - a_3x^{(r)} - b_3y^{(r)}) \quad \ldots..(5)$$

The procedure is continued till the convergence is assured (correct to required decimals).

GAUSS – SEIDEL METHOD OF ITERATION:

This is only a refinement of Guass – Jacobi method. As before,

$$x = \frac{1}{a_1}(d_1 - b_1 y - c_1 z)$$

$$y = \frac{1}{b_2}(d_2 - a_2 x - c_2 z)$$

$$z = \frac{1}{c_3}(d_3 - a_3 x - b_3 y)$$

We start with the initial values y^0, z^0 for y and z and get $x^{(1)}$ from the first equation. That is,

$$x^{(1)} = \frac{1}{a_1}(d_1 - b_1 y^{(0)} - c_1 z^{(0)})$$

While using the second equation, we use $z^{(0)}$ for z and $x^{(1)}$ for x instead of x^0 as in Jacobi's method, we get

$$y^{(1)} = \frac{1}{b_2}(d_2 - a_2 x^{(1)} - c_2 z^{(0)})$$

Now, having known $x^{(1)}$ and $y^{(1)}$, use $x^{(1)}$ for x and $y^{(1)}$ for y in the third equation, we get

$$z^{(1)} = \frac{1}{c_3}(d_3 - a_3 x^{(1)} - b_3 y^{(1)})$$

In finding the values of the unknowns, we use the latest available values on the right hand side. If $x^{(r)}$, $y^{(r)}$, $z^{(r)}$ are the rth iterates, then the iteration scheme will be

$$x^{(r+1)} = \frac{1}{a_1}(d_1 - b_1 y^{(r)} - c_1 z^{(r)})$$

$$y^{(r+1)} = \frac{1}{b_2}(d_2 - a_2 x^{(r+1)} - c_2 z^{(r)})$$

$$z^{(r+1)} = \frac{1}{c_3}(d_3 - a_3 x^{(r+1)} - b_3 y^{(r+1)})$$

This process of iteration is continued until the convergence assured. As the current values of the unknowns at each stage of iteration are used in getting the values of unknowns, the convergence in Gauss – seidel method is very fast when compared to Gauss – Jacobi method. The rate of convergence in Gauss – Seidel method is roughly two times than that of Gauss – Jacobi method. As we saw the sufficient condition already, the sufficient condition for the convergence of this method is also the same as we stated earlier. That is, *the method of iteration will converge if in each equation of the given system, the absolute value of the largest* coefficient is greater than the sum of the absolute values of all the remaining coefficients. (The largest coefficients must be the coefficients for different unknowns).

Example 3 Solve the following system by Gauss – Jacobi and Gauss – Seidel methods:

10x-5y-2z = 3 ; 4x-10y+3z = -3 ; x+6y+10z = -3.

Solution: Here, we see that the diagonal elements are dominant. Hence, the iteration process can be applied.

That is, the coefficient matrix $\begin{bmatrix} 10 & -5 & -2 \\ 4 & -10 & 3 \\ 1 & 6 & 10 \end{bmatrix}$ is diagonally dominant, since $|10| > |-5| + |-2|$,

$|-10| > |4| + |3|$,

$|10| > |1| + |6|$

Gauss – Jacobi method, solving for x, y, z we have

$x = \frac{1}{10}(3+5y+2z)$ ………………. (1)

$y = \frac{1}{10}(3+4x+3z)$ ………………. (2)

$z = \frac{1}{10}(-3-x-6y)$ ………………. (3)

First iteration: Let the initial values be (0, 0, 0).

Using these initial values in (1), (2), (3), we get

$$x^{(1)} = \frac{1}{10}(3 + 5(0) + 2(0)) = 0.3$$

$$y^{(1)} = \frac{1}{10}(3 + 4(0) + 3(0)) = 0.3$$

$$z^{(1)} = \frac{1}{10}(-3-(0) - 6(0)) = -0.3$$

Second iteration: using these values in (1), (2), (3), we get

$x^{(2)} = \frac{1}{10}(3 + 5(0.3) + 2(-0.3)) = 0.39$

$y^{(2)} = \frac{1}{10}(3 + 4(0.3) + 3(-0.3)) = 0.33$ $z^{(2)} = \frac{1}{10}(-3 - (0.3) - 6(0.3)) = -0.51$

Third iteration: using these values of $x^{(2)}$, $y^{(2)}$, $z^{(2)}$ in (1), (2), (3), we get,

$x^{(3)} = \frac{1}{10}(3 + 5(0.33) + 2(-0.51)) = 0.363$

$y^{(3)} = \frac{1}{10}(3 + 4(0.39) + 3(-0.51)) = 0.303$

$z^{(3)} = \frac{1}{10}(-3 - (0.39) - 6(0.33)) = -0.537$

Fourth iteration:

$x^{(4)} = \frac{1}{10}(3 + 5(0.303) + 2(-0.537)) = 0.3441$

$y^{(4)} = \frac{1}{10}(3 + 4(0.363) + 3(-0.537)) = 0.2841$

$z^{(4)} = \frac{1}{10}(-3 - (0.363) - 6(0.303)) = -0.5181$

Fifth iteration:

$x^{(5)} = \frac{1}{10}(3 + 5(0.2841) + 2(-0.5181)) = 0.33843$

$y^{(5)} = \frac{1}{10}(3 + 4(0.3441) + 3(-0.5181)) = 0.2822$

$z^{(5)} = \frac{1}{10}(-3 - (0.3441) - 6(0.2841)) = -0.50487$

Sixth iteration:

$x^{(6)} = \frac{1}{10}(3 + 5(0.2822) + 2(-0.50487)) = 0.340126$

$y^{(6)} = \frac{1}{10}(3 + 4(0.33843) + 3(-0.50487)) = 0.283911$

$z^{(6)} = \frac{1}{10}$ (-3- (0.33843) – 6(0.2822)) = - 0.503163

Seventh iteration:

$x^{(7)} = \frac{1}{10}$ (3 + 5(0.283911) +2(-0.503163)) =0.3413229

$y^{(7)} = \frac{1}{10}$ (3 + 4(0.340126) +3(-0.503163)) = 0.2851015

$z^{(7)} = \frac{1}{10}$ (-3- (0.340126) – 6(0.283911)) = - 0.5043592

Eighth iteration:

$x^{(8)} = \frac{1}{10}$ (3 + 5(0.2851015) +2 (-0.5043592))

=0.34167891

$y^{(8)} = \frac{1}{10}$ (3 + 4(0.3413229) + 3(-0.5043592))

= 0.2852214

$z^{(8)} = \frac{1}{10}$(-3-(0.3413229)– 6(0.2851015))

= - 0.50519319

Ninth iteration:

$x^{(9)} = \frac{1}{10}$ (3 + 5(0.2852214) + 2 (-0.50519319))

= 0.341572062

$y^{(9)} = \frac{1}{10}$ (3 + 4(0.34167891) + 3(-0.50519319))

= 0.285113607

$z^{(9)} = \frac{1}{10}$ (-3- (0.34167891) – 6(0.2852214)) = - 0.505300731

Hence, correct to 3 decimal places, the values are

x = 0.342, y = 0.285, z = -0.505

Gauss – seidel method: Initial values : y = 0, z = 0.

First iteration: $x^{(1)} = \frac{1}{10}(3 + 5(0) + 2(0)) = 0.3$

$y^{(1)} = \frac{1}{10}(3 + 4(0.3) + 3(0)) = 0.42$

$z^{(1)} = \frac{1}{10}(-3 - (0.3) - 6(0.42)) = -0.582$

Second iteration:

$x^{(2)} = \frac{1}{10}(3 + 5(0.42) + 2(-0.582)) = 0.3936$

$y^{(2)} = \frac{1}{10}(3 + 4(0.3936) + 3(-0.582)) = 0.28284$

$z^{(2)} = \frac{1}{10}(-3 - (0.3936) - 6(0.28284)) = -0.509064$

Third iteration:

$x^{(3)} = \frac{1}{10}(3 + 5(0.28284) + 2(-0.509064)) = 0.3396072$ $y^{(3)} = \frac{1}{10}(3 + 4(0.3396072) + 3(-0.509064)) = 0.28312368$

$z^{(3)} = \frac{1}{10}(-3 - (0.3396072) - 6(0.28312368))$

$= -0.503834928$

Fourth iteration:

$x^{(4)} = \frac{1}{10}(3 + 5(0.28312368) + 2(-0.503834928))$

$= 0.34079485$

$y^{(4)} = \frac{1}{10}(3 + 4(0.34079485) + 3(-0.503834928))$

$= 0.285167464$

$z^{(4)} = \frac{1}{10}(-3 - (0.34079485) - 6(0.285167464))$

$= -0.50517996$

Fifth iteration:

$x^{(5)} = \frac{1}{10}(3 + 5(0.285167464) + 2(-0.50517996))$

$= 0.34155477$

$y^{(5)} = \frac{1}{10}(3 + 4(0.34155477) + 3(-0.50517996))$

$= 0.28506792$

$z^{(5)} = \frac{1}{10}(-3 - (0.34155477) - 6(0.28506792))$

$= -0.505196229$

Sixth iteration:

$x^{(6)} = \frac{1}{10}(3 + 5(0.28506792) + 2(-0.505196229))$

$= 0.341494714$

$y^{(6)} = \frac{1}{10}(3 + 4(0.341494714) + 3(-0.505196229))$

$= 0.285039017$

$z^{(6)} = \frac{1}{10}(-3 - (0.341494714) - 6(0.28506792))$

$= -0.5051728$

Seventh iteration:

$x^{(7)} = \frac{1}{10}(3 + 5(0.285039017) + 2(-0.5051728))$

$= 0.3414849$

$y^{(7)} = \frac{1}{10}(3 + 4(0.3414849) + 3(-0.5051728))$

$= 0.28504212$

$z^{(7)} = \frac{1}{10}(-3 - (0.3414849) - 6(0.28504212))$

$= -0.5051737$

The values at each iteration by both methods are tabulated below:

Iteration	Gauss - jacobi method			Gauss – seidel method		
	x	y	z	x	y	z
1	0.3	0.3	-0.3	0.3	0.42	-0.582
2	0.39	0.33	-0.51	0.3936	0.2828	-0.5090
3	0.363	0.303	-0.537	0.3396	0.2831	-0.5038
4	0.3441	0.2841	-0.5181	0.3407	0.2851	-0.5051
5	0.3384	0.2822	-0.5048	0.3415	0.2850	-0.5051
6	0.3401	0.2839	-0.5031	0.3414	0.2850	-0.5051
7	0.3413	0.2851	-0.5043	0.3414	0.2850	-0.5051
8	0.3416	0.2852	-0.5051			
9	0.3411	0.2851	-0.5053			

The values correct to 3 decimal places are

$$x = 0.342, \ y = 0.285, \ z = -0.505$$

Possible Questions

Part -B (5x8=40 Marks)

1. Solve the following system by Gauss elimination method.

$$x + 2y + z = 3$$
$$2x + 3y + 3z = 10$$
$$3x - y + 2z = 13$$

2. Solve the following system by Gauss elimination method

$$x + y + z + w = 2$$
$$2x - y + 2z - w = -5$$
$$3x + 2y + 3z + 4w = 7$$
$$x - 2y - 3z + 2w = 5$$

3. Solve the following system by Gauss Jordan method

$$10x + y + z = 12$$
$$x + 10y + z = 12$$
$$x + y + 10z = 12$$

4. Solve the following system by Gauss Jordan method

$$x + y + 2z = 4$$
$$3x + y - 3z = -4$$
$$2x - 3y - 5z = -5$$

5. Solve the following system by triangularisation method.

$$5x - 2y + z = 4$$
$$7x + y - 5z = 8$$
$$3x + 7y + 4z = 10$$

6. Solve the following system by triangularisation method.

$$5x - 2y + z = 4$$
$$7x + y - 5z = 8$$
$$3x + 7y + 4z = 10$$

7. Solve the following system of equations by Crout's method.

$$2x + 3y + z = -1$$
$$5x + y + z = 9$$
$$3x + 2y + 4z = 11$$

8. Solve the following system of equations by Gauss-Jacobi method

$$10x - 5y - 2z = 3$$
$$4x - 10y + 3z = -3$$
$$x + 6y + 10z = -3$$

9. Solve the following system of equations by Gauss-seidal method

$$28x + 4y - z = 32$$
$$x + 3y + 10z = 24$$
$$2x + 17y + 4z = 35$$

10. Solve the following system of equations by Gauss-Seidel method

$$8x - 3y + 2z = 20$$
$$4x + 11y - z = 33$$
$$6x + 3y + 12z = 35$$

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DEPARTMENT OF MATHEMATICS

Subject: Numerical Methods	**Subject Code: 15MMU503**	**L T P C**
Class:III B.Sc-B	**Semester:V**	**4 1 0 4**

UNIT III

Finite Difference: First and higher order differences – Forward and Backward differences –Properties of operator – Difference of a polynomial – Factorial polynomial – Error Propagation in difference table – operator E – Relation between Δ, E and D .

TEXT BOOK:

T1: Venkataraman .M.K., Fifth Edition, 2001. Numerical Methods in Science and Engineering National publishing Company ,Madras.

REFERENCES:

R3: Kandaswamy.P, Thilagavathy K., and K.Gunavathy. 2013. Numerical Methods, S.Chand & Company Ltd.,New Delhi.

UNIT-III

FINITE DIFFERENCES

First Differences:

Let y=f(x) be a give function of x and let y0,y1,y2……yn be the values of y corresponding to x0,x1,x2…..xn

The values of x. the independent variable x is called the argument and the corresponding dependent value y is called the entyr. In general the difference between any two consecutive values of x need not be same or equal.

Forward, backward, and central differences

Only three forms are commonly considered: forward, backward, and central differences.

A **forward difference** is an expression of the form

$$\Delta_h[f](x) = f(x+h) - f(x).$$

Depending on the application, the spacing *h* may be variable or constant.

A **backward difference** uses the function values at *x* and *x* − *h*, instead of the values at *x* + *h* and *x*:

$$\nabla_h[f](x) = f(x) - f(x-h).$$

Finally, the **central difference** is given by

$$\delta_h[f](x) = f(x+\tfrac{1}{2}h) - f(x-\tfrac{1}{2}h).$$

Relation with derivatives

The derivative of a function *f* at a point *x* is defined by the limit

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}.$$

If *h* has a fixed (non-zero) value, instead of approaching zero, then the right-hand side is

$$\frac{f(x+h) - f(x)}{h} = \frac{\Delta_h[f](x)}{h}.$$

Hence, the forward difference divided by h approximates the derivative when h is small. The error in this approximation can be derived from Taylor's theorem. Assuming that f is continuously differentiable, the error is

$$\frac{\Delta_h[f](x)}{h} - f'(x) = O(h) \quad (h \to 0).$$

The same formula holds for the backward difference:

$$\frac{\nabla_h[f](x)}{h} - f'(x) = O(h).$$

However, the central difference yields a more accurate approximation. Its error is proportional to square of the spacing (if f is twice continuously differentiable):

$$\frac{\delta_h[f](x)}{h} - f'(x) = O(h^2).$$

The main problem with the central difference method, however, is that oscillating functions can yield zero derivative. If f(nh)=1 for n uneven, and f(nh)=2 for n even, then f'(nh)=0 if it is calculated with the central difference scheme. This is particularly troublesome if the domain of f is discrete.

Higher-order differences

In an analogous way one can obtain finite difference approximations to higher order derivatives and differential operators. For example, by using the above central difference formula for $f'(x + h / 2)$ and $f'(x - h / 2)$ and applying a central difference formula for the derivative of f' at x, we obtain the central difference approximation of the second derivative of f:

2nd Order Central

$$f''(x) \approx \frac{\delta_h^2[f](x)}{h^2} = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}.$$

Similarly we can apply other differencing formulas in a recursive manner. **2nd Order Forward**

$$f''(x) \approx \frac{\Delta_h^2[f](x)}{h^2} = \frac{f(x+2h) - 2f(x+h) + f(x)}{h^2}.$$

More generally, the n^{th}-order forward, backward, and central differences are respectively given by:

$$\Delta_h^n[f](x) = \sum_{i=0}^{n} (-1)^i \binom{n}{i} f(x + (n-i)h),$$

$$\nabla_h^n[f](x) = \sum_{i=0}^{n} (-1)^i \binom{n}{i} f(x - ih),$$

$$\delta_h^n[f](x) = \sum_{i=0}^{n} (-1)^i \binom{n}{i} f\left(x + \left(\frac{n}{2} - i\right) h\right).$$

Note that the central difference will, for odd n, have h multiplied by non-integers. This is often a problem because it amounts to changing the interval of discretization. The problem may be remedied taking the average of $\delta^n[f](x - h / 2)$ and $\delta^n[f](x + h / 2)$.

The relationship of these higher-order differences with the respective derivatives is very straightforward:

$$\frac{d^n f}{dx^n}(x) = \frac{\Delta_h^n[f](x)}{h^n} + O(h) = \frac{\nabla_h^n[f](x)}{h^n} + O(h) = \frac{\delta_h^n[f](x)}{h^n} + O(h^2).$$

Higher-order differences can also be used to construct better approximations. As mentioned above, the first-order difference approximates the first-order derivative up to a term of order h. However, the combination

$$\frac{\Delta_h[f](x) - \frac{1}{2}\Delta_h^2[f](x)}{h} = -\frac{f(x+2h) - 4f(x+h) + 3f(x)}{2h}$$

approximates $f'(x)$ up to a term of order h^2. This can be proven by expanding the above expression in Taylor series, or by using the calculus of finite differences, explained below.

If necessary, the finite difference can be centered about any point by mixing forward, backward, and central differences.

Relations between Difference operators

1. We note that

$$Ef(x) = f(x+h) \quad = [f(x+h) - f(x)] + f(x) \quad = \Delta f(x) + f(x) = (\Delta + 1) f(x).$$

Thus,

$$\boxed{E \equiv 1 + \Delta} \quad \text{or} \quad \Delta \equiv E - 1.$$

2. Further, $\nabla(E(f(x)) = \nabla(f(x+h)) = f(x+h) - f(x).$ Thus,

$$(1-\nabla)Ef(x) = E(f(x)) - \nabla(E(f(x)) = f(x+h) - [f(x+h) - f(x)] = f(x).$$

Thus $E \equiv 1 + \Delta,$ gives us

$$(1-\nabla)(1+\Delta)f(x) = f(x) \text{ for all } x.$$

So we write,

$$(1+\Delta)^{-1} = 1 - \nabla \quad \text{or} \quad \boxed{\nabla = 1 - (1+\Delta)^{-1},} \quad \text{and}$$

$$(1-\nabla)^{-1} = 1 + \Delta = E.$$

Similarly,

$$\Delta = (1-\nabla)^{-1} - 1.$$

3. Let us denote by $E^{\frac{1}{2}} f(x) = f(x + \frac{h}{2}).$ Then, we see that

$$\delta f(x) = f(x + \frac{h}{2}) - f(x - \frac{h}{2}) = E^{\frac{1}{2}} f(x) - E^{-\frac{1}{2}} f(x).$$

Thus,

$$\boxed{\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}.}$$

Recall,

$$\delta^2 f(x) = f(x+h) - 2f(x) + f(x-h) = [f(x+h) + 2f(x) + f(x-h)] - 4f(x) = 4(\mu^2 - 1)f(x).$$

So, we have,

$$\boxed{\mu^2 \equiv \frac{\delta^2}{4} + 1} \quad \text{or} \quad \boxed{\mu \equiv \sqrt{1 + \frac{\delta^2}{4}}}.$$

That is, the action of $\sqrt{1 + \frac{\delta^2}{4}}$ is same as that of μ.

4. We further note that,

$$\Delta f(x) = f(x+h) - f(x) = \frac{1}{2}\left[f(x+h) - 2f(x) + f(x-h)\right] + \frac{1}{2}\left[f(x+h) - f(x-h)\right]$$

$$= \frac{1}{2}\delta^2(f(x)) + \frac{1}{2}\left[f(x+h) - f(x-h)\right]$$

5. and

$$\delta\mu f(x) = \delta\left[\frac{1}{2}\left\{f(x+\frac{h}{2}) + f(x-\frac{h}{2})\right\}\right] = \frac{1}{2}\left[\{f(x+h) - f(x)\} + \{f(x) - f(x-h)\}\right]$$

$$= \frac{1}{2}\left[f(x+h) - f(x-h)\right].$$

6.

$$\Delta f(x) = \left[\frac{1}{2}\delta^2 + \delta\mu\right] f(x),$$

7.

8.In view of the above discussion, we have the following table showing the relations between various difference operators:

	E	Δ	∇	δ
E	E	$\Delta+1$	$(1-\nabla)^{-1}$	$\frac{1}{2}\delta^2+\delta\sqrt{1+\frac{\delta^2}{4}}+1$
Δ	$E-1$	Δ	$(1-\nabla)^{-1}-1$	$\frac{1}{2}\delta^2+\delta\sqrt{1+\frac{1}{4}\delta^2}$
∇	$1-E^{-1}$	$1-(1+\nabla)^{-1}$	∇	$-\frac{1}{2}\delta^2+\delta\sqrt{1+\frac{1}{4}\delta^2}$
δ	$E^{1/2}-E^{-1/2}$	$\Delta(1+\Delta)^{-1/2}$	$\nabla(1-\nabla)^{-1/2}$	δ

Difference of a polynomial:

Theorem:

The n^{th} difference of a polynomial of n^{th} degree are constants.

Proof

We have a polynomial f(x), where, in fact, the x's are specific values

$$f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots a_{n-2} x^2 + a_{n-1} x + a_n$$ [2.01]

Suppose the steps along the x axis are h. The next f(x) value at x+h is:

$$f(x+h) = a_0 (x+h)^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots a_{n-2} (x+h)^2 + a_{n-1} (x+h) + a_n$$

[2.02]

We recall, by definition:

$$\Delta f(x) = f(x+h) - f(x)$$ [2.03]

That is, the difference is equation 2.02 minus equation 2.01:

$$\Delta f(x) = \{a_0 (x+h)^n - a_0 x^n\} + \{a_1 (x+h)^{n-1} - a_1 x^{n-1}\} + \{a_2 (x+h)^{n-2} - a_2 x^{n-2}\} + \ldots \{a_{n-2} (x+h)^2 - a_{n-2} x^2\} + \{a_{n-1} (x+h) - a_{n-1} x\}\{a_n - a_n\}$$

[2.03]

If we expand the left-hand parts of each term, we find:

$$= a_0\{x^n + n.x^{n-1}h\ldots - x^n\} + a_1\{x^{n-1} + (n-1)x^{n-2}h - x^{n-1}\} + a_2\{x^{n-2} + (n-2)x^{n-2}h\ldots - x^{n-2}\} + \ldots a_{n-2}\{x^2 + 2xh + h^2 - x^2\} + a_{n-1}\{(x+h) - x\} + \{a_n - a_n\}$$

[2.04]

The last term, a_n cancels, leaving a new constant, $a_{n-1}h$, (which will cancel out in the 2nd difference):

$$= a_0\{n.x^{n-1}h\ldots\} + a_1\{(n-1)x^{n-2}h\} + a_2\{(n-2)x^{n-3}h\ldots\} + \ldots a_{n-2}(2xh + h^2) + a_{n-1}h$$

[2.05]

Therefore for a polynomial of degree n, step h

$$\Delta f(x) = n a_0 x^{n-1}.h + \text{terms of degree n}-2\text{ and lower} \quad [2.06]$$

This is reminiscent of:

$$\frac{d}{dx}(x^n) = n x^{n-1} \quad [2.07]$$

Applying 2.06 again, we get:

$$\Delta^2 f(x) = n(n-1)a_0 x^{n-2}.h^2 + \text{terms of degree n}-3\text{ and lower} \quad [2.07]$$

If we apply the formula 2.06 n times, we have:

$$\Delta^n f(x) = a_0 n(n-1)(n-2)\ldots 1.h^n$$

Or

$$\Delta^n f(x) = a_0 n!h^n$$

Note:

1. Of course, because this is a constant (it is independent of x), the n+1 difference and further differences will be zero, so:

   $$\Delta^{n+1} f(x) = 0$$

2. When h=1, we can write for a polynomial of degree n:

   $$\Delta^n f(x) = a_0 n!$$

Factorial Polynomial:

A factorial polynomial looks like this:

$$f(k)\text{ or }k^{(2)} = k(k-1)$$

$$f(k)\text{ or }k^{(3)} = k(k-1)(k-2)$$

In general a factorial polynomial of degree n, (y_k or k^n) is:

$$k^{(n)} = k\,(k-h)\,(k-2\,h)\ldots(k-n\,h)\,(k-(n-1)\,h) \quad [1.01]$$

We assume that n is an integer greater than zero (A natural number).
We can call this k to the n falling (because there is a rising version!) with step h.
k to the n+1 falling is:

$$(k+1)^{(n)} = (k+1)\,k\,(k-h)\,(k-2\,h)\ldots(k-n\,h)\,(k-(n-1)\,h-h)$$

Which, simplifying the last term:

$$(k+1)^{(n)} = (k+1)\,k\,(k-h)\,(k-2\,h)\ldots(k-n\,h) \quad [1.02]$$

$k^{(0)}$ is defined as 1

Finding the First Difference

By definition, the first difference for the factorial polynomial, $k^{(n)}$, is

$$\Delta k^{(n)} = (k+1)^{(n)} - k^{(n)} \quad [1.03]$$

Substituting our values from 1.01 and 1.02 for k^{n+1} and k^n in 1.03:

$$\Delta k^{(n)} = \left[(k+1)\,k\,(k-h)\,(k-2\,h)\ldots(k-n\,h+h)\,(k-n\,h)\right] - \left[k\,(k-h)\,(k-2\,h)\ldots\right.$$

[1.04]

Factorising gives us:

$$\Delta k^{(n)} = k\,(k-h)\,(k-2\,h)\ldots(k-n\,h)\,(k-n\,h)\left[(k+h)-(k-(n-1)\,h)\right]$$

[1.05]

And further simplifying the final term by cancelling the x's and rounding up the h's:

$$\Delta k^{(n)} = k\,(k-h)\,(k-2\,h)\ldots(k-n\,h+h)\,(k-n\,h)\left[n\,h\right] \quad [1.06]$$

We note that, substituting n-1 for n in 1.06:

$$k^{(n-1)} = k\,(k-h)\,(k-2\,h)\ldots(k-n\,h)\,(k-(n-1+1)\,h)$$

Simplifying the final factor:

$$k^{(n-1)} = k\,(k-h)\,(k-2\,h)\ldots(k-n\,h+h)\,(k-n\,h) \quad [1.07]$$

First Difference and General Formula for n>0

From 1.06 substituting 1.07, we have:

$$\Delta k^{(n)} = n\cdot h\cdot k^{(n-1)} \quad [1.08]$$

So we can determine any of the differences using 1.08, for instance:

$$\Delta^2 k^{(n)} = n\cdot(n-1)\cdot h^2\cdot k^{(n-2)}$$

$$\Delta^3 k^{(n)} = n\cdot(n-1)\cdot(n-2)\cdot h^3\cdot k^{(n-3)}$$

In general, the mth difference is:

$$\Delta^m k^{(n)} = n \cdot (n-1) \cdot (n-2) \ldots (n-m+1) \cdot h^m \cdot k^{(n-m)} \quad [1.09]$$

This is reminiscent of differentiating using the infinitesimal calculus.

$$\frac{d}{dx}\left(x^n\right) = n x^{n-1} \quad [1.10]$$

1.08 also reminds us of similar result for regular polynomials, repeated below:

$$\Delta f(x) = n a_0 x^{n-1}.h + \text{terms of degree n}-2\text{ and lower}$$

With regular polynomials, the difference isn't so neat as that with factorial polynomials. However, we can convert regular polynomials to factorials and obtain clearer results for differences.

Often, the factorial polynomials we use have a step of 1, or h=1, so:
$k^{(n)}$=k(k-1)(k-2)...(k-n)(k-n+1) [1.11]
And the mth difference when h=1 is:

$$\Delta^m k^{(n)} = n \cdot (n-1) \cdot (n-2) \ldots (n-m+1) \cdot k^{(n-m)} \quad ■[1.12]$$

Possible Questions

Part-B (5x8=40 Marks)

1. Find y(-1) if y(0) = 2, y(1) = 9, y(2) = 28, y(3) = 65, y(4) = 126, y(5) = 217.

2. Find the 7^{th} term of the sequence 2,9,28,65,126,217 and also. Find the General term.

3. i) Explain the Relation between Δ,E and D

   ii) Find the first term of the series whose second and subsequent terms are 8, 3, 0, -1, 0, ...

4. Find $\Delta^3 f(x)$ if

   i) $f(x) = (3x+1)(3x+4)\dots\dots\dots(3x+19)$

   ii) $f(x) = x(3x+1)(3x+4)\dots\dots\dots(3x+19)$

5. Evaluate i) $\Delta^n(e^{ax+b})$ ii) $\Delta^n[\sin(ax+b)]$

   iii) $\Delta^n[\cos(ax+b)]$ iv) $\Delta[\log(ax+b)]$

6. Express i) $x^4 + 3x^3 – 5x + 6x – 7$

   ii) $x^3 + x^2 + x + 1$ in factorial polynomials and get their successive differences taking h = 1.

7. Estimate the production for 1964 & 1966 from the following data

Year :	1961	1962	1963	1964	1965	1966	1967
Production :	200	220	260	-	350	-	430

8. Prove that n^{th} difference of a polynomial of the n^{th} degree are constants.

9. The following table gives the values of y which is a polynomial of degree 5. It is known that y = f(3) is in error. Correct the error.

x :	0	1	2	3	4	5	6
y :	1	2	33	254	1025	3126	7777

10. If y = f(x) is a polynomial of degree 3 and the following table gives the values of x & y. Locate and correct the wrong values of y.

x :	0	1	2	3	4	5	6
y :	4	10	30	75	160	294	490

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Subject: Numerical Methods	Subject Code: 15MMU503	L T P C
Class:III B.Sc-B	Semester:V	4 1 0 4

UNIT-IV

Interpolation: Gregory Newton Forward and Newton Backward interpolation formula – Equidistant terms with one or more missing values – Interpolation with unequal intervals – Divided differences –Newton's divided difference formula – Lagrange's interpolation formula – Inverse interpolation formula.

TEXT BOOK:

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UNIT-IV
INTERPOLATION

Introduction

Interpolation means the process of computing intermediate values of a function a given set of tabular values of a function. Suppose the following table represents a set of values of x and y.

x : x_1	x_2	x_3............x_n
y : y_1	y_2	y_3...........y_n

We may require the value of $y = y_i$ for the given $x = x_i$, where x lies between x_0 to x_n

Let $y = f(x)$ be a function taking the values $y_0, y_1, y_2, \ldots\ y_n$ corresponding to the *values* $x_0,\ x_1, x2,\ \ldots\ldots\ldots\ x_n$. Now we are trying to find $y = y_i$ for the given $x = x_i$ under assumption that the function f(x) is not known. In such cases , we replace f(x) by simple fan arbitrary function and let Φ(x) denotes an arbitrary function which satisfies the set of values given in the table above . The function Φ(x) is called interpolating function or smoothing function or interpolation formula.

Newton's forward interpolation formula (or) Gregory-Newton forward interpolation formula (for equal intervals)

Let $y = f(x)$ denote a function which takes the values $y_0, y_1, y_2 \ldots\ldots\ldots, y_n$ corresponding to the values $x_0, x_1, x_2 \ldots\ldots\ldots, x_n$.

Let suppose that the values of x i.e., $x_0, x_1, x_2 \ldots\ldots\ldots, x_n$ are equidistant .

$$x_1 = x_0 + h\ ;\ x_2 = x_1 + h\ ;\ \text{and so on}\quad x_n = x_{n-1} + h\ ;$$

Therefore $xi = x0 + i\,h$, *where* $i = 1,2, \ldots, n$

Let $P_n(x)$ be a polynomial of the n^{th} degree in which x is such that

$$y_I = f(x_i) = P_n(x_i),\ \ I = 0,1,2, \ldots\ n$$

Let us assume Pn(x) in the form given below

$$P_n(x) = a_0 + a_1(x - x_0)^{(1)} + a_2(x - x_0)^{(2)} + \ldots\ldots\ldots + a_r(x - x_0)^{(r)} + \ldots\ldots +$$

$+ \ldots\ldots + a_n (x - x_0)^{(n)} \ldots\ldots (1)$

This polynomial contains the n + 1 constants $a_0, a_1, a_2, \ldots\ldots\ldots a_n$ can be found as follows :

$P_n(x_0) = y_0 = a_0$ (setting x = x0, in (1))

Similarly $y_1 = a_0 + a_1 (x_1 - x_0)$

$y_2 = a_0 + a_1 (x_2 - x_0) + a_2 (x_2 - x_0)$

From these, we get the values of $a_0, a_1, a_2, \ldots\ldots a_n$

i.e.,

Therefore, $a_0 = y_0$

$\Delta y_0 = y_1 - y0 = a_1 (x_1 - x_0)$

$= a_1 h$

$\Rightarrow a_1 = \Delta y_0 / h$

lly $\Rightarrow a_2 = (\Delta y_1 - \Delta y_0) / 2h^2 = \Delta^2 y_0 / 2!\, h^2$

lly $\Rightarrow a_3 = \Delta^3 y_0 / 3!\, h^3$

Putting these values in (1), we get

$$P_n(x) = = y_0 + (x - x_0)^{(1)}\Delta y_0 /h + (x - x_0)^{(2)} \Delta^2 y_0/(2!\, h^2) + \ldots\ldots + (x - x_0)^{(r)} \Delta^r y_0/(r!\, h^r) + \ldots\ldots + (x - x_0)^{(n)} \Delta^r y_0/(n!\, h^n)$$

By substituting $\frac{x - x_0}{h} = u$ *, the above equation becomes*

$$y(x_0 + uh) = y_u = y_0 + u\,\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \ldots\ldots \quad \ldots$$

By substituting $u = u^{(1)}$,

$u(u-1) = u^{(2)}$,

$u(u-1)(u-2) = u^{(3)}$, *… in the above equation, we get*

$$P_n(x) = P_n y(x_0 + uh) = y_0 + u^{(1)}\Delta y_0 + \frac{u^{(2)}}{2!}\Delta^2 y_0 + \frac{u^{(3)}}{3!}\Delta^3 y_0 + \ldots\ldots + \frac{u^{(r)}}{r!}\Delta^r y0 + \ldots. + \frac{u^{(n)}}{n!}\Delta^n y_0$$

The above equation is known as **Gregory-Newton forward formula or Newton's forward interpolation formula.**

Note : 1. This formula is applicable only when the interval of difference is uniform.

2. This formula apply forward differences of y_0, hence this is used to interpolate the values of y nearer to beginning value of the table (i.e., x lies between x0 to ^{x}1 or x1 to x_2)

Example.

Find the values of y at x = 21 from the following data.

x: x : 20

0.3420 23 26

0.3907

0.4384

29

0.4848

Solution.

Step 1. Since x = 21 is nearer to beginning of the table. Hence we apply Newton's forward formula.

Step 2. Construct the difference table

x	y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$
20	0.3420	(0.3420-0.3907)		
		0.0487	(0.0477-0.0487)	
23	0.3907		-0.001	
		0.0477		-0.0003
26	0.4384		-0.0013	
		0.0464		
29	0.4848			

Step 3. Write down the formula and put the various values :

$$P_n(x) = P_n y(x_0 + uh) = y_0 + u^{(1)}\Delta y_0 + \frac{u^{(2)}}{2!}\Delta^2 y_0 + \frac{u^{(3)}}{3!}\Delta^3 y_0 + \ldots\ldots + \frac{u^{(r)}}{r!}\Delta^r y0 + \ldots + \frac{u^{(n)}}{n!}\Delta^n y_0$$

Where $u^{(1)} = (x - x_0)/h = (21 - 20)/3 = 0.3333$

$u(2) = u(u-1) = (0.3333)(0.6666)$

$$P_n(x=21) = y(21) = 0.3420 + (0.3333)(0.0487) + (0.3333)(-0.6666)(-0.001)$$
$$+ (0.3333)(-0.6666)(-1.6666)(-0.0003)$$

$$= \mathbf{0.3583}$$

Example: . From the following table of half yearly premium for policies maturing at different ages, estimate the premium for policies maturing at age 46.

Age	x:	45	50	55	60	65
Premium	y:	114.84	96.16	83.32	74.48	68.48

Solution.

Step 1. Since $x = 46$ is nearer to beginning of the table and the values of x is equidistant i.e., $h = 5$.. Hence we apply Newton's forward formula.

Step 2. Construct the difference table

x	y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$
45	114.84				
		-18.68			
50	96.16		5.84		
		-12.84		-1.84	
55	83.12		4.00		0.68
		-8.84		-1.16	
60	74.48		2.84		
		-6.00			
65	68.48				

Step 3. Write down the formula and put the various values :

$$P_n(x) = P_n y(x_0 + uh) = y_0 + u^{(1)}\Delta y_0 + \frac{u^{(2)}}{2!}\Delta^2 y_0 + \frac{u^{(3)}}{3!}\Delta^3 y_0 + \ldots\ldots + \frac{u^{(r)}}{r!}\Delta^r y0 + \ldots + \frac{u^{(n)}}{n!}\Delta^n y_0$$

Where $u = (x - x_0) / h = (46 - 45) / 5 = 01/5 = 0.2$

$$P_n(x=46) = y(46) = 114.84 + [0.2(-18.68)] + [0.2(-0.8)(5.84)/3]$$

$$+ [0.2(-0.8)(-1.8)(-1.84)/6]$$

$$+ [0.2(-0.8)(-1.8)(-2.8)(0.68)]$$

$$= 114.84 - 3.7360 - 0.4672 - 0.08832 - 0.228$$

$= \mathbf{110.5257}$

Example . From the following table , find the value of tan $45^0\,15'$

x^0:	45	46	47	48	49	50
tan x^0:	1.0	1.03553	1.07237	1.11061	1.15037	1.19175

Solution.

Step 1. Since $x = 45^o\,15'$ is nearer to beginning of the table and the values of x is equidistant i.e., $h = 1$. Hence we apply Newton's forward formula.

Step 2. Construct the difference table to find various Δ's

x	y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$	$\Delta^5 y_0$
45^0	1.0000					
		0.03553				
460	1.03553		0.00131			
		0.03684		0.00009		
47^0	1.07237		0.00140		0.00003	
		0.03824		0.00012		-0.00005
48^0	1.11061		0.00152		-0.00002	
		0.03976		0.00010		
49^0	1.15037		0.00162			
		0.04138				
50^0	1.19175					

Step 3. Write down the formula and substitute the various values :

$$P_n(x) = P_n y(x_0 + uh) = y_0 + u^{(1)}\Delta y_0 + \frac{u^{(2)}}{2!}\Delta^2 y_0 + \frac{u^{(3)}}{3!}\Delta^3 y_0 + \ldots\ldots + \frac{u^{(r)}}{r!}\Delta^r y0 + \ldots. + \frac{u^{(n)}}{n!}\Delta^n y_0$$

Where $u = (45^o\,15' - 45^0) / 1^0$

$= 15' / 1^0$

$= 0.25$ ……………(since $1^0 = 60$ ')

$$y\,(x=45^o\,15') = P_5\,(45^o\,15') = 1.00 + (0.25)(0.03553) + (0.25)(-0.75)(0.00131)/2$$

$$+(0.25)(-0.75)(-1.75)(0.00009)/6$$

$$+(0.25)(-0.75)(-1.75)(-2.75)(0.0003)/24$$

$$+(0.25)(-0.75)(-1.75)(-2.75)(-3.75)(-0.00005)/120$$

$$= 1.000 + 0.0088825 - 0.0001228 + 0.0000049$$

$= \mathbf{\mathit{1.00876}}$

Newton's backward interpolation formula (or) Gregory-Newton backward interpolation formula (for equal intervals)

Let $y = f(x)$ denote a function which takes the values $y_0, y_1, y_2 \ldots\ldots\ldots, y_n$ corresponding to the values $x_0, x_1, x_2 \ldots\ldots\ldots, x_n.$

Let suppose that the values of x i.e., $x_0, x_1, x_2 \ldots\ldots\ldots, x_n$ are equidistant .

$x_1 = x_0 + h$; $x_2 = x_1 + h$; and so on $x_n = x_{n-1} + h$;

Therefore $xi = x0 + i h$, where $i = 1,2, \ldots, n$

Let $P_n(x)$ be a polynomial of the n^{th} degree in which x is such that

$y_I = f(x_i) = P_n(x_i),\ I = 0,1,2, \ldots. n$

$$P_n(x) = a_0 + a_1(x - x_n)^{(1)} + a_2(x - x_n)(x - x_{n-1})^{)} + \ldots\ldots\ldots + a_n(x - x_n)(x - x_{n-1}) \ldots(x - x_1) \ldots\ldots(1)$$

Let us assume Pn(x) in the form given below

$$P_n(x) = a_0 + a_1(x - x_n)^{(1)} + a_2(x - x_n)^{(2)} + \ldots\ldots\ldots + a_r(x - x_n)^{(r)} + \ldots\ldots + \ldots\ldots + a_n(x - x_n)^{(n)} \ldots\ldots(1.1)$$

This polynomial contains the n + 1 constants $a_0, a_1, a_2, \ldots\ldots\ldots a_n$ can be found as follows :

$P_n(x_n) = y_n = a_0$ (setting x = xn, in (1))

Similarly $y_{n-1} = a_0 + a_1(x_{n-1} - x_n)$

$y_{n-2} = a_0 + a_1(x_{n-2} - x_n) + a_2(x_{n-2} - x_n)$

From these, we get the values of $a_0, a_1, a_2, \ldots\ldots a_n$

Therefore, $a_0 = y_n$

$\check{}yn = y_n - yn\text{-}1 = a_1(x_{n-1} - x_n)$

$= a_1 h$

$\Rightarrow a_1 = y_n / h$

lly $\Rightarrow a_2 = (\check{} y_1 - \check{} y_n) / 2h^2 = \check{}^2 y_n / 2!\, h^2$

lly $\Rightarrow a_3 = \check{}^3 / 3!\, h^3$

Putting these values in (1), we get

$$P_n(x) = \; = y_n + (x - x_n)^{(n)} \nabla y_n / h \;+ (x - x_n)^{(2)} \nabla^2 y_n / (2!\, h^2) + (x - x_n)^{(r)} \nabla^r y_n / (r!\, h^r)$$
$$+ \ldots\ldots \;+ (x - x_n)^{(n)} \nabla^n y_n / (n!\, h^n)$$

By substituting $\dfrac{x - x_n}{h} = v$ *, the above equation becomes*

$$y(x_n + vh) = y_n + v \nabla y_n + \frac{v(v+1)}{2!} \nabla^2 y_n + \frac{v(v+1)(v+2)}{3!} \nabla^3 y_n + \ldots\ldots$$

By substituting $v = v^{(1)}$,

$v(v+1) = v^{(2)}$,

$v(v+1)(v+2) = v^{(3)}$, *… in the above equation, we get*

$$P_n(x) = P_n y(x_n + vh) \;= y_n + v^{(1)} \nabla y_n + \frac{v^{(2)}}{2!} \nabla^2 y_n + \frac{v^{(3)}}{3!} \nabla^3 y_n + \ldots + \frac{v^{(r)}}{r!} \nabla^r y_n + \ldots + \frac{v^{(n)}}{n!} \Delta^n y_n$$

The above equation is known as **Gregory-Newton backward formula or Newton's backward interpolation formula.**

Note : 1. This formula is applicable only when the interval of difference is uniform.

2. This formula apply backward differences of y_n, hence this is used to interpolate the values of y nearer to the end of a set tabular values. (i.e., x lies between xn to xn- 1 and xn-1 to x_{n-2})

Example: Find the values of y at x = 28 from the following data.

x:	20	23	26	29
y	0.3420	0.3907	0.4384	0.4848

Solution.

Step 1. Since x = 28 is nearer to beginning of the table. Hence we apply Newton's backward formula.

Step 2. Construct the difference table

x	y	∇y_n	$\nabla^2 y_n$	$\nabla^3 y_n$
20	0.3420	(0.3420-0.3907)		
		0.0487	(0.0477-0.0487)	
23	0.3907		-0.001	
		0.0477		-0.0003
26	0.4384			
		0.0464	-0.0013	
29	0.4848			

Step 3. Write down the formula and put the various values :

$$P_3(x) = P_3 y(x_n + vh) = y_n + v^{(1)}\nabla y_n + \frac{v^{(2)}}{2!}\nabla^2 y_n + \frac{v^{(3)}}{3!}\nabla^3 y_n$$

Where $v^{(1)} = (x - x_n) / h = (28 - 29) / 3 = -0.3333$

$v^{(2)} = v(v+1) = (-0.333)(0.6666)$

$v^{(3)} = v(v+1)(v+2) = (-0.333)(0.6666)(1.6666)$

$$P_n(x=28) = y(28) = 0.4848 + (-0.3333)(0.0464) + (-0.3333)(0.6666)(-0.0013)/2$$

$$+(-0.3333)(0.6666)(1.6666)(-0.0003)/6$$

$$= 0.4848 - 0.015465 + 0.0001444 + 0.0000185$$

$$= \mathbf{0.4695}$$

Example: From the following table of half yearly premium for policies maturing at different ages, estimate the premium for policies maturing at age 63.

Age	x:	45	50	55	60	65
Premium	y:	114.84	96.16	83.32	74.48	68.48

Solution.

Step 1. Since $x = 63$ is nearer to beginning of the table and the values of x is equidistant i.e., $h = 5$.. Hence we apply Newton's backward formula.

Step 2. Construct the difference table

x	y	Δy_0	$\Delta^2 y0$	$\Delta^3 y_0$	$\Delta^4 y_0$
45	114.84				
		-18.68			
50	96.16		5.84		
		-12.84		-1.84	
55	83.12		4.00		-
				1.16	
		-8.84			
60	74.48		2.84		
		-6.00			
65	68.48				
					0.68

Step 3. Write down the formula and put the various values :

$$P_3(x) = P_3 y(x_n + vh) = y_n + v^{(1)} \Delta y_n + \frac{v^{(2)}}{2!} \Delta^2 y_n + \frac{v^{(3)}}{3!} \Delta^3 yn + \frac{v^{(4)}}{4!} \Delta^4 y_n$$

Where $v^{(1)} = (x - x_n) / h = (63 - 65) / 5 = -2/5 = -0.4$

$v(2) = v(v+1) = (-0.4)(1.6)$

$v(3) = v(v+1)(v+2) = (-0.4)(1.6)(2.6)$

$v(4) = v(v+1)(v+2))(v+3) = (-0.4)(1.6)(2.6)(3.6)$

$$P_4(x=63) = y(63) = 68.48 + [(-0.4)(-6.0)] + [(-0.4)(1.6)(2.84)/2]$$

$$+ [(-0.4)(1.6)(2.6)(-1.16)/6]$$

$$+ [(-0.4)(1.6)(2.6)(3.6)(0.68)/24]$$

= 68.48 +2.40 - 0.3408 +0.07424 – 0.028288

= **70.5852**

Example: From the following table , find the value of tan 49^0 15'

x^0:	45	46	47	48	49	50
tan x^0:	1.0	1.03553	1.07237	1.11061	1.15037	1.19175

Solution.

Step 1. Since $x = 49^o\ 45'$ is nearer to beginning of the table and the values of x is equidistant i.e., $h = 1$. Hence we apply Newton's backward formula.

Step 2. Construct the difference table to find various Δ's

x	y	∇y_0	$\nabla^2 y_0$	$\nabla^3 y_0$	$\nabla^4 y_0$	$\nabla^5 y_0$
45^0	1.0000					
		0.03553				
46	1.03553		0.00131			
		0.03684		0.00009		
47^0	1.07237		0.00140		0.00003	
		0.03824		0.00012		-0.00005
48^0	1.11061		0.00152		-0.00002	
		0.03976		0.00010		
49^0	1.15037		0.00162			
		0.04138				
50^0	1.19175					

Step 3. Write down the formula and substitute the various values :

$$P_5(x) = P_5 y(x_n + vh) = y_n + v^{(1)}\nabla y_n + \frac{v^{(2)}\nabla^2 y_n}{2!} + \frac{v^{(3)}\nabla^3 y_n}{3!} + \frac{v^{(4)}\nabla^4 y_n}{4!} + \frac{v^{(5)}\nabla^5 y_n}{5!}$$

Where $v = (49^o\ 45' - 50^0) / 1^0$

$= - 15' / 1^0$

$= -0.25$ ……………(since $1^0 = 60$ ‘)

$v(2) = v(v+1) \qquad = (-0.25))(0.75)$

$= (-0.25)(0.75)(1.75)$

$v(3) = v(v+1)(v+2)$

$v(4) = v(v+1)(v+2))(v+3) \quad = (-0.25)(0.75)(1.75)(2.75)$

$y(x=49^o 15') = P_5(49^o 15') = 1.19175 + (-0.25)(0.04138) + (-0.25)(0.75)(0.00162)/2$

$+(-0.25)(0.75)(1.75)(0.0001)/6$

$+(-0.25)(0.75)(1.75)(2.75)(-0.0002)/24$

$+(-0.25)(0.75)(1.75)(2.75)(3.75)(-0.00005)/120$

$= 1.19175 - 0.010345 - 0.000151875 + 0.000005 + \ldots$

$= \mathbf{1.18126}$

Lagrange's Interpolation Formula

Interpolation means the process of computing intermediate values of a function a given set of tabular values of a function. Suppose the following table represents a set of values of x and y.

x:	x_0	x_1	x_2	$x3$	………..	x_n
y:	y_0	y_1	y_2	$y3$	………..	y_n

We may require the value of $y = y_i$ for the given $x = x_i$, where x lies between x_0 *to* x_n

Let y = *f(x)* be a function taking the values $y_0, y_1, y_2, \ldots\ y_n$ corresponding to the *values* $x_0,\ x_1, x2,$ ……….. x_n. Now we are trying to find $y = y_i$ for the given $x = x_i$ under assumption that the function f(x) is not known. In such cases, x_i *'s* are not equally spaced we use *Lagrange's interpolation formula.*

Newton's Divided Difference Formula:

The divided difference $f[x_0, x_1, x_2, \ldots, x_n]$, sometimes also denoted $[x_0, x_1, x_2, \ldots, x_n]$, on $n+1$ points

X$_0$, x_1, ..., x_n of a function $f(x)$ is defined by $f[x_0] \equiv f(x_0)$ and

$$f[x_0, x_1, \ldots, x_n] = \frac{f[x_0, \ldots, x_{n-1}] - f[x_1, \ldots, x_n]}{x_0 - x_n}$$

for $n \geq 1$. The first few differences are

$$f[x_0, x_1] = \frac{f_0 - f_1}{x_0 - x_1}$$

$$f[x_0, x_1, x_2] = \frac{f[x_0, x_1] - f[x_1, x_2]}{x_0 - x_2}$$

$$f[x_0, x_1, \ldots, x_n] = \frac{f[x_0, \ldots, x_{n-1}] - f[x_1, \ldots, x_n]}{x_0 - x_n}.$$

Defining

$\pi_n(x) \equiv (x-x_0)(x-x_1)\cdots(x-x_n)$ and taking the derivative

$\pi_n'(x_k) = (x_k-x_0)\cdots(x_k-x_{k-1})(x_k-x_{k+1})\cdots(x_k-x_n)$ gives the identity

$$f[x_0, x_1, \ldots, x_n] = \sum_{k=0}^{n} \frac{f_k}{\pi_n'(x_k)}.$$

Lagrange's interpolation formula (for unequal intervals)

Let $y = f(x)$ denote a function which takes the values $y_0, y_1, y_2 \ldots\ldots\ldots, y_n$ corresponding to the values $x_0, x_1, x_2 \ldots\ldots\ldots, x_n.$

Let suppose that the values of x *i.e.*, $x_0, x_1, x_2 \ldots\ldots\ldots, x_n$. are not equidistant .

$$y_I = f(x_i) \quad I = 0,1,2, \ldots. N$$

Now, there are (n+1) paired values$(x_i,y_i,)$, *I = 0,1,2, … n and hence f(x) can be represented by a polynomial function of degree n in x.*

Let us consider f(x) as follows

$$\begin{aligned} f(x) = \; & a_0(x-x_1)(x-x_2)(x-x_3)\ldots(x-x_n) \\ & + a_1(x-x_0)(x-x_2)(x-x_3)\ldots(x-x_n) \\ & + a_2(x-x_0)(x-x_3)(x-x_4)\ldots(x-x_n) \\ & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ & + a_n(x-x_0)(x-x_2)(x-x_3)\ldots(x-x_{n-1}) \quad \ldots\ldots\ldots(1) \end{aligned}$$

Substituting $x = x_0, y = y_0$, *in the above equation*

$$y_0 = a_0(x-x_1)(x-x_2)(x-x_3)\ldots(x-x_n)$$

which implies $a_0 = y_0 / (x_0 - x_1)(x_0 - x_2)(x_0 - x_3) \ldots (x_0 - x_n)$

Similarly $a_1 = y_1 / (x_1 - x_0)(x_1 - x_2)(x_1 - x_3) \ldots (x_1 - x_n)$

$a_2 = y_2 / (x_2 - x_0)(x_2 - x_1)(x_2 - x_3) \ldots (x_2 - x_n)$

…………………………………………………

$a_n = y_n (x_n - x_0)(x_n - x_2)(x_n - x_3) \ldots (x_n - x_{n-1})$

Putting these values in (1), we get

$$y = f(x) = \frac{(x - x_1)(x - x_2)(x - x_3) \ldots (x - x_n)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3) \ldots (x_0 - x_n)} y_0$$

$$+ \frac{(x - x_0)(x - x_2)(x - x_3) \ldots (x - x_n)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3) \ldots (x_1 - x_n)} y_1$$

$$+ \frac{(x - x_0)(x - x_1)(x - x_3) \ldots (x - x_n)}{(x_2 - x_0)(x_2 - x_2)(x_1 - x_3) \ldots (x_1 - x_n)} y_2$$

$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$+ \frac{(x - x_0)(x - x_2)(x - x_3) \ldots (x - x_{n-1})}{(x_n - x_0)(x_n - x_2)(x_n - x_3) \ldots (x_n - x_{n-1})} y_n$$

The above equation is called ***Lagrange's interpolation formula*** for unequal intervals.

Note : 1. This formula is will be more useful when the interval of difference is not uniform.

Example. Using Lagrange's interpolation formula, find *y(10) from the following table*

x : *5* *6* *9* *11*

y : 3 13 14 16

Solution:

Step 1. Write down the Lagrange's formula :

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)\ldots(x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_2)(x_1-x_3))} y_2 + \frac{(x-x_0)(x-x_2)(x-x_2)}{(x_3-x_0)(x_3-x_2)(x_3-x_3)} y_3$$

$$= \frac{(x-6)\ (x-9)\ (x-11)}{(5-6)(5-9)(5-11)} (12) + \frac{(x-5)\ (x-9)\ (x-11)}{(6-5)(6-9)(6-11)} + \frac{(x-5)\ (x-6)\ (x-11)\ (14)}{(9-5)(9-6)(9-11)}$$

(13)

$$+ \frac{(x-5)\ (x-6)\ (x-19)}{} (16)$$

(11 – 5) (11– 6) (11 – 9)

Putting x = 10 in the above equation

$$Y(10) = f(10) = \frac{(4)(1)(-1)\ (12)}{(-1)(-4)(-6)} + \frac{(5)(1)(-1)\ (13)}{(1)(-3)(-5)} + \frac{(5)(4)(1))\ (14)}{(4)(3)(-2)} + \frac{(5)(4)\ (1)\ (16)}{(6)(5)(2)}$$

= ***14.6666***

Possible Questions

Part- B (5x8=40 Marks)

1. The population of a town is as follows.

Year (x) :	1941	1951	1961	1971	1981	1991
Population in Lakhs (y) :	20	24	29	36	46	51

Estimate the population increase during the period 1946 to 1976.

2. Using inverse interpolation formula, find the value of x when y=13.5.

x :	93.0	96.2	100.0	104.2	108.7
y :	11.38	12.80	14.70	17.07	19.91

3. Find the polynomial of least degree passing the points (0, -1), (1, 1), (2, 1), (3, -2).

4. Find the values of y at X=21 and X=28 from the following data.

X :	20	23	26	29
Y :	0.3420	0.3907	0.4384	0.4848

5. From the data given below, find the number of students whose weight is between 60 and 70.

Weight in Ibs. :	0-40	40-60	60-80	80-100	100-120
No. of students :	250	120	100	70	50

6. Using Lagrange's interpolation formula find the value corresponding to x = 10 from the following table.

x :	5	6	9	11
y :	12	13	14	16

7. Find the missing value of the table given below. What assumption have you made to find it?

Year :	1917	1918	1919	1920	1921
Export(in tons) :	443	384	-	397	467

8. Using Newton's divided difference formula, find the values of f(2), f(8) and f(15) given the following table.

x :	4	5	7	10	11	13
f(x) :	48	100	294	900	1210	2028

9. From the following table of half-yearly premium for policies maturing at different ages. Estimate the premium for policies maturing at age 46 & 63.

Age x :	45	50	55	60	65
Premium y :	114.84	96.16	83.32	74.48	68.48

10. Write the procedure for Lagrange's Interpolation Fomula.

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DEPARTMENT OF MATHEMATICS

Subject: Numerical Methods	**Subject Code: 15MMU503**	**L T P C**
Class:III B.Sc-B	**Semester:V**	**4 1 0 4**

UNIT V

Numerical Differentiation and Integration: Newton's Forward and backward differences to compute derivatives – Trapezoidal rule, Simpson's 1/3 &3/8 rule. Solution of ordinary differential equations:R-K method (II order , III order and IV order).

TEXT BOOK

T1:Venkataraman .M.K., Fifth Edition, 2001. Numerical Methods in Science and Engineering National publishing Company ,Madras.

R3: Kandaswamy. P., Thilagavathy K., and K.Gunavathy., 2013 .Numerical Methods, S. Chand &Company Ltd., New Delhi.

UNIT-V

NUMERICAL DIFFERENTIATION AND INTEGRATION

Numerical differentiation

The problem of Interpolation is finding the value of y for the given value of x among (x_i , y_j) for i= 1 to n. Now we find the derivatives of the corresponding arguments . If the required value of y lies in the first half of the interval then we call it as Forward interpolation .If the required value of y (derivative value) lies in the second half of the interval we call it as Backward interpolation also if the derivative of y lies in the middle of of class interval then we solve by central difference.

Newton's forward formula for Interpolation :

$Y= y_0 +u\, \Delta y_0+u(u-1)/2!\, \Delta^2 Y_0 +u\, (u-1)(u-2) / 3!\, \Delta^3 Y_0+\ldots\ldots$

Where $u = (x-x_0)/h$

Differentiating with respect to x ,

$dy/dx = (dy/du). (du/dx) = (1/h) (dy / du)$

$(dy / dx)\ x \neq x_0 = (1 / h) [\Delta y_0 +(2u-1)/2\ \ \Delta^2 y_0 +(3u^2 –6u+2)/ 6\ \ \Delta^3 y_0 +\ldots\ldots\ldots]$

$(dy / dx)\ x = x_0 = (1 / h) [\Delta y_0 –(1/2)\ \ \Delta^2 y_0 +(1/ 3)\ \ \Delta^3 y_0 +\ldots\ldots\ldots]$

$(d^2y / dx^2)\ x \neq x_0= d/dx (dy / dx) = d/dx(dy / du. du / dx)$

$= (1/h^2) [\Delta^2 y_0 + 6(u-1) / 6\, \Delta^3 y_0 + (12u^2 – 36 u +22) / 2\, \Delta^4 y_0 +\ldots\ldots.]$

$(d^2y / dx^2)\ x= x_0=(1/h^2) [\Delta^2 y_0 - \Delta^3 y_0 +(11/12)\, \Delta^4 y_0 +\ldots\ldots.]$

Similarly,

$(d^3y / dx^3)\ x \neq x_0= (1/h^3) [\Delta^3 y_0 + (2u – 3) / 2\, \Delta^4 y_0 +\ldots\ldots.]$

$(d^2y / dx^2)\ x= x_0=(1/h^3) [\Delta^3 y_0 –(3/2)\Delta^4 y_0 +\ldots\ldots.].$

In a similar manner the derivatives using backward interpolation an also be found out.

Using backward interpolation .

$(dy / dx)\ x \neq x_n = (1 / h) [\nabla y_n +(2u+1)/2\ \ \nabla^2 y_n +(3u^2 +6u+2)/ 6\ \ \nabla^3 y_n +\ldots\ldots\ldots]$

$(dy / dx)\ x = x_n = (1 / h) [\nabla y_n –(1/2)\ \ \nabla^2 y_n +(1/ 3)\, \nabla^3 y_n +\ldots\ldots\ldots]$

$(d^2y / dx^2)\ x \neq x_0= (1/h^2) [\nabla^2 y_0 + 6(u-1) / 6\, \nabla^3 y_0 + (12u^2 – 36 u +22) / 2\, \nabla^4 y_0 +\ldots\ldots.]$

$(d^2y / dx^2)\ x= x_0=(1/h^2) [\nabla^2 y_0 - \nabla^3 y_0 +(11/12)\, \nabla^4 y_0 +.]$

Example

Find the first two derivatives of $x^{(1/3)}$ at x= 50 and x= 56, given the table below.

X: 50 51 52 53 54 55 56

Y: 3.6840 3.7084 3.7325 3.7563 3.7798 3.8030 3.8259

X	Y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
50	3.6840				
51	3.7084	0.0244			
52	3.7325	0.0241	-0.0003	0	
53	3.7563	0.0238	-0.0003	0	0
54	3.7798	0.0235	-0.0003	0	0
55	3.8030	0.0232	-0.0003	0	0
56	3.8259	0.0229	-0.0003		

At x= 50,

$(dy/dx)_{x = x0} = (1/h)[\Delta y_0 – (1/2)\Delta^2 y_0 + (1/3)\Delta^3 y_0 + \ldots\ldots\ldots]$

$= (1/1)[0.024-(1/2)(-0.0003)+0] = 0.02455$

$(d^2y/dx^2)_{x= x0} = (1/h^2)[\Delta^2 y_0 - \Delta^3 y_0 + (11/12)\Delta^4 y_0 + \ldots\ldots\ldots]$

$= (1/1)[-0.003-0] = -.0003$

At x=56,

$(dy/dx)_{x = xn} = (1/h)[\nabla y_n + (1/2)\nabla^2 y_n + (1/3)\nabla^3 y_n + \ldots\ldots\ldots]$

$= (1/1)[0.0229+(1/2)(-0.0003)+0] = 0.02275.$

$(d^2y/dx^2)_{x= xn} = (1/h^2)[\nabla^2 y_n + \nabla^3 y_n + (11/12)\nabla^4 y_n + \ldots]$

$= (1/1)[-0.003-0] = -0.0003.$

For the above ptroblem let us find the first two derivatives of x when x= 52 and x= 55.

When x=52, From Newton’s forward formula

$(dy / dx)\ x \neq x_0 = (1 / h)[\Delta y_0 + (2u-1)/2\ \Delta^2 y_0 + (3u^2 –$

$6u+2)/ 6\ \Delta^3 y_0 + \ldots\ldots\ldots],$

$= (1/1)[0.0244+(3/2)(-0.0003)+0] = 0.02395,$

Since here $u= (x-x_0) / h = (52-50)/1 = 2.$

$(d^2y / dx^2)\ x \neq x_0 = (1/h^2)[\Delta^2 y_0 + 6(u-1) / 6\ \Delta^3 y_0 + (12u^2 – 36u + 22) / 2\ \Delta^4 y_0 + \ldots\ldots\ldots]$

= (1/)m [-0.0003+0] = -0.0003.

When x= 55,from backward interpolation

$(dy / dx)\ x \neq x_n = (1 / h)\ [\nabla y_n + (2v+1)/2\ \ \nabla^2 y_n + (3v^2 + 6v+2)/ 6\ \ \nabla^3 y_n + \ldots\ldots\ldots]$

= (1/1) [0.0229+(-1/2)(-0.0003)+0] = 0.02305,

Since here $v = (x - x_n) / h = (55-56)/1 = -1$.

$(d^2y / dx^2)\ x \neq x_n = (1/h^2)\ [\nabla^2 y_n + 6(v+1) / 6\ \nabla^3 y_n + (12v^2 + 36v + 22) / 2\ \nabla^4 y_n + \ldots\ldots.]$

= (1/1) [0.0229+(-1/2)(-0.0003)+0] = 0.02305.

Numerical Integration:

We know that $\int_a^b f(x)dx$ represents the area between y = f(x), x – axis and the ordinates x = a and x = b. This integration is possible only if the f(x) is explicitly given and if it is integrable. The problem of numerical integration can be stated as follows: Given as set of (n+1) paired values (x_i, y_i), i = 0,1,2,…,n of the function y=f(x), where f(x) is not known explicitly, it is required to compute $\int_{x_0}^{x_n} y\, dx$.

A general quadrature formula for equidistant ordinates (or Newton – cote's formula)

For equally spaced intervals, we have Newton's forward difference formula as

$$y(x)=y(x_0+uh)=y_0+u\Delta y_0+\frac{u(u-1)}{2!}\Delta^2 y_0+\ldots \qquad \ldots\ldots(1)$$

Now, instead of *f(x)*, we will replace it by this interpolating formula of Newton.

Here, $u = \frac{x-x_0}{h}$ where *h* is interval of differencing.

Since $x_n = x_0 + nh$, and $u = \frac{x-x_0}{h}$ we have $\frac{x-x_0}{h} = n = u$.

$$\int_{x_0}^{x_n} f(x)dx = \int_{x_0}^{x_n+nh} f(x)dx$$

$$= \int_{x_0}^{x_n+nh} P_n(x)\, dx \text{ where } P_n(x) \text{ is interpolating polynomial}$$

$$=\int_0^n \left(y_0 + u\,\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \ldots..\right)(hdu)$$

Since $dx = hdu$, and when $x = x_0$, $u = 0$ and when $x = x_0+nh$, $u = n$.

$$= h\left[y_0(u)+ \frac{u^2}{2}\Delta y_0 + \frac{\left(\frac{u^3}{3}-\frac{u^2}{2}\right)}{2}\Delta^2 y_0 + \frac{1}{6}\left(\frac{u^4}{4} - u^3 + u^2\right)\Delta^3 y_0 + \cdots\right]_0^n$$

$$\int_{x_0}^{x_n} f(x)dx = h\, n\left[y_0 + \frac{n^2}{2}\Delta y_0 + \frac{1}{2}\left(\frac{n^3}{3} - \frac{n^2}{2}\right)\Delta^2 y_0 + \frac{1}{6}\left(\frac{n^4}{4} - n^3 + n^2\right)\Delta^3 y_0 + \ldots\right] \quad (2)$$

The equation (2), called Newton-cote's quadrature formula is a general quadrature formula. Giving various values for *n*, we get a number of special formula.

Trapezoidal rule:

By putting n = 1, in the quadrature formula (i.e there are only two paired values and interpolating polynomial is linear).

$\int_{x_0}^{x_n+nh} f(x)dx = h\left[1.y_0 + \frac{1}{2}\Delta y_0\right]$ since other differences do not exist if n = 1.

$$= \int_{x_0}^{x_n} f(x)dx = \int_{x_0}^{x_n+nh} f(x)dx$$

$$= \int_{x_0}^{x_0+h} f(x)dx + \int_{x_0+h}^{x_n+2h} f(x)dx + \ldots\ldots\ldots + \int_{x_0+(n-1)h}^{x_n+nh} f(x)dx$$

$$= \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+y_3+\cdots.+y_{n-1})\right]$$

$= \frac{h}{2}$ [(sum of the first and the last ordinates) + 2(sum of the remaining ordinates)]

This is known as Trapezoidal Rule and the error in the trapezoidal rule is of the order h^2.

Romberg's method

For an interval of size h, let the error in the trapezoidal rule be kh^2 where k is a constant. Suppose we evaluate $I = \int_{x_0}^{x_n} y\,dx$, taking two different values of h, say h_1 and h_2, then

$$I = I_1 + E_1 = I_1 + kh_1^2 \quad I = I_2 + E_2 = I_2 + kh_2^2$$

Where I_1, I_2 are the values of I got by two different values of h, by trapezoidal rule and E_1, E_2 are the corresponding errors.

$$I_1 + kh_1^2 = I_2 + kh_2^2$$

$$k = \frac{I_1 - I_2}{h_2^2 - h_1^2}$$

substituting in (1), $I = I_1 + \frac{I_1 - I_2}{h_2^2 - h_1^2} h_1^2$ & $I = \frac{I_1 h_2^2 - I_2 h_1^2}{h_2^2 - h_1^2}$

This I is a better result than either I_1, I_2.

If $h_1 = h$ and $h_2 = \frac{1}{2}h$, then we get

$$I = \frac{I_1\left(\frac{1}{4}h^2\right) - I_2 h^2}{\frac{1}{4}h^2 - h^2} \qquad = \frac{4I_2 - I_1}{3} = I_2 + \frac{1}{2}(I_2 - I_1), \quad I = I_2 + \frac{1}{2}(I_2 - I_1)$$

We got this result by applying trapezoidal rule twice. By applying the trapezoidal rule many times, every time halving h, we get a sequence of results A_1, A_2, A_3,…… we apply the formula given by (3), to each of adjacent pairs and get the resultants B_1, B_2, B_3 ….. (which are improved values). Again applying the formula given by (3), to each of pairs B_1, B_2, B_3 …..we get another sequence of better results C_1, C_2, C_3 ….continuing in this way, we proceed until we get two successive values which are very close to each other. This systematic improvement of Richardson's method is called Romberg method or Romberg integration.

Simpson's one-third rule:

Setting n = 2 in Newton- cote's quadrature formula, we have $\int_{x_0}^{x_n} f(x)dx = h \left[2y_0 + \frac{4}{2}\Delta y_0 + \frac{1}{2}\left(\frac{8}{3} - \frac{4}{2}\right)\Delta^2 y_0\right]$ (since other terms vanish)

$$= \frac{h}{3}(y_2 + y_1 + y_0)$$

Similarly, $\int_{x_2}^{x_4} f(x)dx = \frac{h}{3}(y_2 + 4y_3 + y_4)$

$$\int_{x_2}^{x_4} f(x)dx = \frac{h}{3}(y_i + 4y_{i+1} + y_{i+2})$$

If n is an even integer, last integral will be

$$\int_{x_{n-2}}^{x_n} f(x)dx = \frac{h}{3}(y_{n-2} + 4y_{n-1} + y_n)$$

Adding all the integrals, if n is an even positive integer, that is, the number of ordinates y_0, y_1, y_2….y_n is odd, we have

$$\int_{x_0}^{x_n} f(x)dx = \int_{x_0}^{x_2} f(x)dx + \int_{x_2}^{x_4} f(x)dx + \ldots\ldots\ldots + \int_{x_{n-2}}^{x_n} f(x)dx$$

$$= \frac{h}{3}\left[(y_0 + y_n) + 2(y_2 + y_4 + \ldots) + \ldots\ldots + 4(y_1 + y_3 + \ldots)\right]$$

$= \frac{h}{3}$ [(sum of the first and the last ordinates) + 2(sum of remaining odd ordinates) + 2(sum of even ordinates)]

Simpson's three-eighths rule:

Putting n = 3 in Newton – cotes formula

$$= \frac{3h}{8}\left[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + \ldots + y_{n-1}) + 2(y_3 + \right]$$

$y_6+y_9+....+y_n)$ .....(2)

Equation (2) is called *Simpson's three – eighths rule* which is applicable only when *n* is a multiple of 3.Truncation error in simpson's rule is of the order h

Example

Evaluate $\int_{-3}^{3} x^4 dx$ by using (1) trapezoidal rule (2)simpson's rule. Verify your results by actual integration.

Solution.

Here $y(x) = x^4$. Interval length$(b – a) = 6$. So, we divide 6 equal intervals with $h = \frac{6}{6} = 1$.

We form below the table

x	-3	-2	-1	0	1	2	3
y	81	16	1	0	1	16	81

(i) By trapezoidal rule:

$\int_{-3}^{3} y\,dx = \frac{h}{2}$ [(sum of the first and the last ordinates) +

2(sum of the remaining ordinates)]

$= \frac{1}{2}$ [(81+81)+2(16+1+0+1+16)]

=115

(ii) By simpson's one - third rule (since number of ordinates is odd)**:**

$\int_{-3}^{3} y\,dx = \frac{1}{3}$ [(81+81) + 2(1+1) + 4(16+0+16)]

= 98.

(iii) Since *n* = 6, (multiple of three), we can also use **simpson's three - eighths rule.** By this rule,

$\int_{-3}^{3} y\,dx = \frac{1}{3}$ [(81+81) + 3(16+1+1+16) + 2(0)]

= 99

(iv) By actual integration,

$\int_{-3}^{3} x^4 dx = 2* \left[\frac{x^5}{5}\right]_0^3 = \frac{2*243}{5} = 97.2$

From the results obtained by various methods, we see that simpson's rule gives better result than trapezoidal rule.

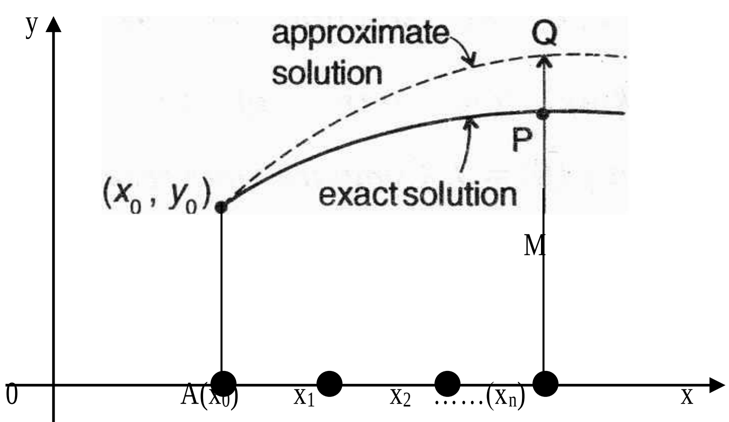
SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS

INTRODUCTION

Suppose we require to solve dy/dx=f(x,y) with the initial condition $y(x_0)=y_0$. By numerical solution of y at $x=x_0, x_1, x_2,..$ let y=y(x)be the exact solution. If we plot and draw the graph of y=y(x), (exact curve) and also draw the approximate curve by plotting $(x_0, y_0), (x_1,y_1), (x_2, y_2),\ldots$ we get two curves.

PM= exact value, QM=approximate value at $x=x_i$. Then

$QP=MQ-MP=y_i-y(x_i) = \varepsilon$ is called the truncation error at $x= x_i$



$QP=MQ-MP=y_i-y(x_i)= \varepsilon_i$ is called return error at $x=x_i$

RUNGE- KUTTA METHOD

Second order Runge-Kutta method (for first order O.D.E)

AIM : To solve dy / dx = f(x,y) given $y(x_0)=y_0$ ….(1)

Proof. By Taylor series, we have,

$y(x+h) = y(x)+ hy'(x)+ h^2/2!\, y''(x)+O(h^3)$ ……..(2)

Differentiating the equation (1) w.r.t.x,

$$y'' = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dx} = f_x + y' f_y = f_x + f f_y$$

Using the values of y' and y'' got from (1) and (3), in (2), we get,

$Y(x + h)-y(x) = hf + ½ h^2 [f_x + ff_y] + O(h^3)$

$\Delta y=hf+ ½ h^2(f_x+ff_y)+ O(h^3)$

Let $\Delta_1 y=k_1=f(x,y)$. $\Delta x=hf(x,y)$, $\Delta_2 y=k_2 =hf(x+mh,y+mk_1)$

and $\Delta y=ak_1 + bk_2$, Where a, b and m are constants to be determined to get the better accuracy of Δy. Expand k_2 and Δy in powers of h.

Expanding k_2, by Taylor series for two variables, we have

$K_2 =hf(x+mh,y+mk_1)$

$= h[f +mhf_x +mhff_y+\{(mh\partial/\partial x +mk_1 \partial/\partial y)^2 f / 2!\} +….]$ …(8)

$= hf+mh_2(f_x+ff_y)+….$ Higher powers of h ……….. (9)

Substituting k_1,k_2 in (7),

$\Delta y=ahf+b[hf+mh^2(f_x+ff_y)+O(h^3)]$

$=(a+b)hf+bmh^2(f_x+ff_y)+O(h^3)$ …… 10)

Equating Δy from (4) and (10), we get

$=hf+mh^2(f_x+ff_y)+……$ higher powers of h………. (9)

Substituting k_1, k_2 in (7),

$\Delta y = ahf+b[hf+mh^2(f_x+ff_y)+O(h^3)] =(a+b)hf+bmh^2(f_x+ff_y)+O(h^3)$ …………. (10)

Equating Δy from (4) and (10), we get

a+b=1 and bm= ½ ………… (11)

Now we have only two equations given by (1) to solve for three unknowns a,b,m.

From a+b=1, a=1-b and also m= 1/2b using (7),

$\Delta y=(1-b)k_1+bk_2,$ Where $k_1=hf(x,y)$

$K_2=hf(x+h/2b, y+hf/2b)$ Now $\Delta y=y(x+h)-y(x)$

$Y(x+h)=y(x)+(1-b)hf+bhf(x+h/2b,y+hf/2b)$

i.e., $y_{n+1}=y_n+(1-b)hf(x_n,y_n) +bhf(x_n+h/2b,y_n+h/2bf(x_n,y_n))+O(h^3)$

from this general second order Runge kutta formula, setting a=0, b=1, m=1/2, we get the second order Runge kutta algorithm as

$k_1=hf(x,y)$ & $k_2=hf(x+ ½ h, y + ½ k_1)$ and $\Delta y=k_2$ where $h=\Delta x$

Since the derivation of third and fourth order Runge Kutta algorithm are tedious, we state them below for use.

The third order Runge Kutta method algorithm is given below :

$K_1=hf(x,y)$

$K_2=hf(x+1/2h, y+1/2k_1)$

$K_3=hf(x+h,y+2k_2-k_1)$

and $\Delta y=1/6 (k_1+4k_2+k_3)$

The fourth order Runge Kutta method algorithm is mostly used in problems unless otherwise mentioned. It is

$K_1=hf(x,y)$

$K_2=hf(x+1/2h, y+1/2k_1)$

$K_3=hf(x+1/2h, y+1/2k_2)$

$K_4=hf(x+h, y+k_3)$

and $\Delta y= 1/6 (k_1+2k_2+2k_3+k_4)$

$y(x+h)=y(x)+ \Delta y$

Working Rule :

To solve $dy/dx = f(x,y)$, $y(x_0)=y_0$

Calculate $k_1=hf(x_0,y_0)$

$K_2=hf(x_0+1/2h,y_0+1/2k_1)$

$K_3= hf(x_0+1/2h,y_0+1/2k_2)$

$K_4=hf(x_0+h, y_0+k_3)$

and $\Delta y= 1/6 (k_1+2k_2+2k_3+k_4)$

where $\Delta x=h$

Now $y_1=y_0+ \Delta y$

Now starting from (x_1,y_1) and repeating the process, we get (x_2,y_2) etc.,

Example

Obtain the values of y at x=0.1, 0.2 using R.K method of (i) second order (ii) third order and (iii) fourth order for the differential equation y'=-y, given y(0)=1.

Solution :Here $f(x,y)=-y, x_0=0, y_0=1, x_1=0.1, x_2=0.2$

(i) Second Order:

$k_1=hf(x_0,y_0)=(0.1)(-y_0)= - 0.1$

$k_2=hf(x_0+ ½ h, y_0+ ½ k_1) = (0.1) f(0.05,0.95)$

$= -0.1(x0.95)= - 0.095= \Delta y$

$y_1=y_0+\Delta y=1-0.095=0.905$

$y_1=y(0.1)=0.905$

Again starting from (0.1, 0.905) replacing (x_0,y_0) by (x_1,y_1)we get

$k_1=(0.1) f(x_1,y_1)=(0.1) (-0.905)= - 0.0905$

$k_2=hf(x_1+ ½ h, y_1+ ½ k_1)$

$=(0.1)[f(0.15,0.85975)]=(0.1)(-0.85975)=-0.085975$

$\Delta y=k_2$ $y_2=y(0.2)=y_1+\Delta y=0.819025$

(ii) Third Order:

$k_1= hf(x_0, y_0) = -0.1$

$k_2= hf(x_0+ ½ h, y_0+ ½ k_1) = - 0.095$

$k_3=hf(x_0+h,y_0+2k_2-k_1)$

$= (0.1)f(0.1,0.9)=(0.1)(-0.9)= -0.0.9$

$\Delta y=1/6 (k_1+4k_2+k_3)$

$y(0.1)=y_1=y_0+\Delta y=1-0.09=0.91$

Again taking (x_1,y_1) has (x_0,y_0) repeat the process

$k_1=hf(x_1,y_1)=(0.1) (-0.91)=-0.091$

$k_2=hf(x_1+ ½ h, y_1+ ½ k_1)$

$= (0.1)f(0.15,0.865)=(0.1) (-0.865)= - 0.0865$

$k_3=hf(x_1+h, y_1+2k_2-k_1)$

$= (0.1)f(0.2,0.828) = -0.0828$

$y_2=y_1+\Delta y=0.91+1/6\ (k_1+4k_2+k_3)$

$= 0.91+1/6\ (-\ 0.091-0.3460 - 0.0828)$

$y(0.2)=0.823366$

(iii) Fourth order:

$k_1=hf(x_0,y_0)=(0.1)f(0.1)=-0.1$

$k_2=hf(x_0+ ½\ h,\ y_0+1/2k_1)=(0.1)f(0.05,\ 0.95) = -\ 0.095$

$k_3=hf(x_0+ ½\ h,\ y_0+ ½\ k_2) = (0.1)\ f(0.05,\ 0.9525) = -\ 0.09525$

$k_4=hf(x_0+h,y_0+k_3)=(0.1)f(0.1,0.90475) = -0.090475$

$\Delta y=1/6\ (k_1+2k_2+2k_3+k_4)$

$y_1=y_0+\Delta y=1+1/6\ (k_1+2k_2+2k_3+k_4)$

$y_1=y(0.1)=0.9048375$

Again start from this (x_1,y_1) and replace (x_0,y_0) and repeat

$k_1=hf(x_1,y_1)=(0.1)(-y1)= - 0.09048375$

$k_2=hf(x_1+1/2h,y_1+1/2k_1)$

$= (0.1)f(0.15,0.8595956) = -0.08595956$

$k_3=hf(x_1+ ½\ h,y_1+ ½\ \ k_2)$

$= (0.1)f(0.15,0.8618577)= - 0.08618577$

$k_4=hf(x_1+h,y_1+k_3)$

$= (0.1)f(0.2,0.8186517)= - 0.08186517$

$\Delta y=1/6(-0.09048375-2x\ 0.08595956- 2x\ 0.08618577 – 0.08186517) = -0.0861066067$

$y_2=y(0.2)=y_1+\Delta y=0.81873089$

Tabular values are:

x	Second order	Third order	Fourth order	Exact value Y=e-x
0.1	0.905	0.91	0.9048375	0.904837418
0.2	0.819025	0.823366	0.81873089	0.818730753

Part- B (5x8=40 Marks)

Possible Questions

1. The population of a certain town is given below, Find the rate of growth of population in 1931, 1941 1961 and 1971 .

year	1931	1941	1951	1961	1971
population	40.62	60.80	79.95	103.56	132.65

2. Using Romberg's method , evaluate $I = \int_0^1 dx / (1+ x)$ correct to 3 decimal places.

3. Find the first two derivatives of $(x)^{1/3}$ at x=50 and x=56 given the table below

X	50	51	52	53	54	55	56
$Y =((x)^{1/3})$	3.6840	3.7084	3.7086	3.7563	3.7798	3.8030	3.8259

4. Evaluate $\int_0^1 dx / (1+ x^2)$ using Trapezoidal rule with h = 0.2. Hence obtain the approximate value of π .

5. Evaluate $I= \int_0^6 dx / (1+ x)$ using both of the Simpson's rule.

6. Use Runge kutta method of fourth order find y for x = 0.1 and 0.2, given that $dy/dx = x + y$, $y(0) = 1$.

7. By dividing the range into the ten equal parts, evaluate $\int_0^\pi \sin x \, dx$ by Trapezoidal rule and Simpson's rule.

8. Using Runge kutta Method of fourth order , find y(0.8) correct to 4 decimal places if $y^1 =y- x^2$, y(0.6)=1.7379.

9. Using Romberg's method , evaluate , $I = \int_0^1 dx / (1+ x^2)$ correct to 3 decimal places.

10. Using Runge kutta Method of fourth order ,solve $dy/dx=y^2-x^2/y^2+x^2$ given y(0)=1 at x=0.2,0.4.

KARPAGAM ACADEMY OF HIGHER EDUCATION

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UNIT-I

SOLUTION OF ALGEBRAIC AND TRANSCENDENTAL EQUATIONS

Part-A(20X1=20 Marks)

(Question Nos. 1 to 20 Online Examinations)

Multiple Choice Questions

Question	Opt 1	Opt 2	Opt 3	Opt 4	Answer
-------- Method is based on the repeated application of the intermediate value theorem.	Gauss Seidal	Bisection	Regula Falsi	Newton Raphson	Bisection
The formula for Newton Raphson method is --	$x_{n+1} = f(x_n) / f'(xn)$	$x_{n+1} = xn + f(x_n) / f'(xn)$	$x_{n+1} = xn - f(x_n) / f'(xn)$	$x_{n+1} = xn - f'(x_n) / f(xn)$	$x_{n+1} = xn - f(x_n) / f'(xn)$
The order of convergence of Newton Raphson method is ----------	4	2	1	0	2
Graeffe's root squaring method is useful to find -----------------	complex roots	single roots	unequal roots	polynomial roots	polynomial roots
The approximate value of the root of f(x) given by the bisection method is ----	$x_0 = a + b$	$x_0 = f(a) + f(b)$	$x_0 = (a + b)/ 2$	$x_0 = (f(a) + f(b))/2$	$x_0 = (a + b)/ 2$
In Newton Raphson method, the error at any stage is proportional to the ------- of the error in the previous stage.	cube	square	square root	equal	square
The convergence of bisection method is -------.	linear	quadratic	slow	fast	slow
The order of convergence of Regula falsi method may be assumed to ----------.	1	1.618	0	0.5	1.618
----------------- Method is also called method of tangents.	Gauss Seidal	Secant	Bisection	Newton Raphson	Newton Raphson

If f (x) contains some functions like exponential, trigonometric, logarithmic etc., then f (x) is called -------------- equation.	Algebraic	transcendental	numerical	polynomial	transcendental
A polynomial in x of degree n is called an algebraic equation of degree n if -----	f (x) = 0	f (x) = 1	f (x) <1	f (x) >1	f (x) = 0
The method of false position is also known as ------------- method.	Gauss Seidal	Secant	Bisection	Regula falsi	Regula falsi
The Newton Rapson method fails if ---------------.	f ' (x) = 0	f (x) = 0	f (x) =1	f(x)≠0	f ' (x) = 0
The bisection method is simple but ------------------.	slowly divergent	fast convergent	slowly convergent	divergent	slowly convergent
________Method is also called as Bolzano method or interval having method.	Bisection	false position	Newton raphson	Horner's	Bisection
Graeffe's root squaring method has a great advantage over other methods in that it does not require prior information about the -----	initial value	approximate value	final value	zero	initial value
If we choose the initial approximation x_0 ------------ to the root then we get the root of the equation very quickly	close	far	average	equal	close
In Newton Rapson method when f '(x) is very large and the interval h will be --- then the root can be calculated in even less time.	small	large	Average of the roots	zero	small
The order of convergence in --------------- method is two.	Bisection	Regula falsi	False position	Newton raphson	Newton raphson
The approximate value $x_0 = (a + b)/ 2$ of the root of f (x) is given by the --------- method.	Bisection	Regula falsi	Newton raphson	Graffe's root squaring	Bisection
If f (x_1) and f (a) are of opposite signs, then the actual roots of the equation f(x)=0 in False position method lie between ----------------.	'a' and 'b'	'b' and 'x_1'	'a' and 'x_1'	'x_1' and 'x_2'	'a' and 'x_1'

The iterative procedure is repeated till the --------- is found to the desired degree of accuracy.	initial value	approximate value	root	final value	root
----------- is the method to find the root of algebraic or transcendental equation.	Graeffe's method	Regula falsi	Root squaring	Gauss Elimination	Regula falsi
----------- Method is the method to find the root of polynomial equation.	Graeffe's method	Regula falsi	bisection	Newton Raphson	Graeffe's method
The equation $3x - \cos x - 1 = 0$ is known as ------------- equation.	Polynomial	transcendental	algebraic	normal	transcendental
$x^4 + 2x - 1 = 0$ is --------- equation.	Polynomial	transcendental	algebraic	normal	algebraic
$x e^x - 3x + 1.2 = 0$ is known as --------- equation	Polynomial	transcendental	algebraic	normal	transcendental
If f(a) and f(b) have opposite signs then the root of f(x) = 0 lies between -------.	0&a	a & b	b & 0	1 & -1	a & b
The error at any stage is proportional to the square of the ------------------.	error in the previous stage	error in the next stage	error in the last stage	error in the first stage	error in the next stage
The convergence of iteration method is ----------------.	Zero	Polynomial	Quadratic	linear	linear
The method of successive Approximation is also called as -----------.	Bisection method	Iteration method	Regula falsi	Newton Raphson	Iteration method
The sufficient condition for convergence of iterations is ----------------.	$\vert f'(x) \vert = 1$	$\vert f'(x) \vert > 1$	$\vert f'(x) \vert < 1$	$\vert f'(x) \vert = 0$	$\vert f'(x) \vert < 1$
Assuming that a root of $x^3 - 9x + 1 = 0$ lies between 2 and 4. Find the initial approximation root value of bisection methodis -------.	2	3	4	3.5	3
In Newton Rapson method if -----------, then 'a' is taken as the initial approximation to the root.	$\vert f(a) \vert + \vert f(b) \vert$	$\vert f(a) \vert = \vert f(b) \vert$	$\vert f(a) \vert > \vert f(b) \vert$	$\vert f(a) \vert < \vert f(b) \vert$	$\vert f(a) \vert < \vert f(b) \vert$
In iteration method the given equation is taken in the form of -------------.	y = f(x)	x = f(x)	x = f(y)	x' = f(y)	x = f(x)
The sequence will converge rapidly in Iteration method, if $\vert f'(x) \vert$ is -------------.	Zero	Very large	Very small	one	Very small

In ----------- method, first find the integral part of the equation.	Bisection	Iteration	Regula falsi	Horner's	Horner's
If p = 2, then the convergence is -------------.	Cubic	Quadratic	Linear	Zero	Quadratic
In Iteration method if the convergence is linear then the convergence is of order ---	four	three	two	one	one
If the function f(x) is $e^x - 3x = 0$, then for Iteration method the variable x can be taken as ------------.	$e^x / 3$	$3 / e^x$	$e^x / 3x$	$e^{3x} / 3$	$e^x / 3$
By Regula Falsi method, the positive root of first approximation of $x^3 - 4x + 1 = 0$ lies between ---------------.	0 & 1	1 & 2	–1 and 2	0 and –1	1 & 2
In Iteration method if the convergence is ---------- then the convergence is of order one.	cubic	Quadratic	Linear	zero	Linear
The order of convergence of --------------- method may be assumed to 1.618.	Bisection	Regula falsi	Newton raphson	Graffe's root squaring	Regula falsi
In Newton Raphson method the choice of --------------- is very important for the convergence	initial value	final value	intermediate value	approximate value	initial value
If f(a) and f(b) are of opposite signs, a root of f(x) = 0 lies between 'a' and 'b'. This idea can be used to fix an ---------------------.	approximate root	actual root	intermediate root	real root	approximate root
In Graeffe root squaring method, if the roots of the given polynomial differ in magnitude, then the ------------ of the roots are separated widely for higher values of m	mth power	2^{m+1} th power	2^m th power	2^{m+2} th power	2^m th power
The polynomial equations is given in the form of $a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + an = 0$, where ai's are ----------.	Zero	one	complex	real	real
Newton Rapson method is also called method of ------------------.	Gauss Seidal	Regula Falsi	Bisection	tangents	tangents

If f (–1) and f (–2) are of opposite signs, then the negative roots of the equation f(x)=0 in False position method lie between ----------.	–1 and –2	–1 and 1	1 and –2	1 and 2	–1 and –2
The -------------- method fails if f ’(x) = 0.	Bisection	False Position	Newton Rapson	Iteration	Newton Rapson

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DEPARTMENT OF MATHEMATICS
UNIT-II
SOLUTIONS OF SIMULTANEOUS LINEAR ALGEBRAIC EQUATIONS
Part-A(20X1=20 Marks)
(Question Nos. 1 to 20 Online Examinations)
Multiple Choice Questions

Question	Opt 1	Opt 2	Opt 3	Opt 4	Answer
What are the types of solving linear system of equations-----------?	Direct and Iterative	differentiation	integration	interpolation	Direct and Iterative
Gauss elimination method is a -----------------.	indirect method	direct method	Iterative method	convergent	direct method
The rate of convergence in Gauss – Seidel method is roughly --------- times than that of Gauss Jacobi method.	2	3	4	0	2
Example for iterative method -------------.	Gauss elimination	Gauss Seidal	Gauss Jordan	Bisection	Gauss Seidal
In the absence of any better estimates, the initial approximations are taken as---	x = 0, y = 0, z = 0	x = 1, y = 1, z = 1	x = 2, y = 2, z = 2	x = 3, y = 3, z = 3	x = 0, y = 0, z = 0
When Gauss Jordan method is used to solve AX = B, A is transformed into -----	Scalar matrix	diagonal matrix	Upper triangular matrix	lower triangularmatrix	diagonal matrix
The modification of Gauss – Elimination method is called ---------------------	Gauss Jordan	Gauss Seidal	Gauss Jacobi	Crout's	Gauss Jordan
------------- Method produces the exact solution after a finite number of steps.	Gauss Seidal	Gauss Jacobi	Iterative method	Direct	Direct
In the upper triangular coefficient matrix, all the elements above the diagonal are --------------.	Zero	non-zero	unity	negative	non-zero

In the upper triangular coefficient matrix, all the elements below the diagonal are --------------.	Positive	non-zero	zero	negative	zero
Gauss Seidal method always --------- for a special type of systems.	converges	diverges	oscillates	equal	Converges
Condition for convergence of Gauss Seidal method is ---------------.	Coefficient matrix is diagonally dominant	pivot element is Zero	Coefficient matrix is not diagonally dominant	pivot element is one	Coefficient matrix is diagonally dominant
Modified form of Gauss Jacobi method is -------------------- method.	Gauss Jordan	Gauss Seidal	Regula falsi	Gauss Elimination	Gauss Seidal
In Gauss elimination method by means of elementary row operations, from which the unknowns are found by ----------------- method	Forward substitution	Backward substitution	random	equal to	Backward substitution
In -------------- iterative method, the current values of the unknowns at each stage of iteration are used in proceeding to the next stage of iteration.	Gauss Seidal	Gauss Jacobi	Gauss Jordan	Gauss Elimination	Gauss Seidal
The direct method fails if any one of the pivot elements become ----.	Zero	one	two	negative	Zero
In Gauss elimination method the given matrix is transformed into --------------.	Unit matrix	diagonal matrix	Upper triangular matrix	lower triangular matrix	Upper triangular matrix
Gauss Jordan method is a -----------------.	Direct method	Indirect method	iterative method	convergent	Direct method
Gauss Jacobi method is a -----------------.	Direct method	Indirect method	iterative method	convergent	Indirect method
The modification of Gauss – Jordan method is called ---------------------.	Gauss Jordan	Gauss Seidal	Gauss Jacobi	Gauss Seidal	Gauss Seidal
Gauss Seidal method always converges for -------- of systems.	Only the special type	all types	quadratic types	none	Only the special type
In solving the system of linear equations, the system can be written as ---	BX = B	AX = A	AX = B	AB = X	AX = B

In solving the system of linear equations, the augment matrix is --------	(A, A)	(B, B)	(A, X)	(A, B)	(A, B)
In the direct methods of solving a system of linear equations, at first the given system is written as ------------- form.	An augment matrix	a triangular matrix	constant matrix	Coefficient matrix	An augment matrix
The direct method fails if any one of the pivot elements become ----------------.	one	zero	two	negative	zero
In Gauss Jordan method, we get the solution ---------------------	without using back substitution method	By using back substitution method	by using forward substitution method	Without using forward substitution method	without using back substitution method
If the coefficient matrix is diagonally dominant, then ---------- method converges quickly.	Gauss elimination	Gauss jordan	Direct	Gauss Seidal	Gauss Seidal
The condition to apply Jocobi’s method to solve a system of equations---------	1st row is dominant	1st column is dominant	diagonally dominant	pivot element is zero	diagonally dominant
Iterative method is a ------------ method	Direct method	InDirect method	both 1st & 2nd	either 1st &2nd	InDirect method
------------- is also a self-correction method.	Iteration method	Direct method	Interpolation	none	Iteration method
The condition for convergence of Gauss Seidal method is that the ------ should be diagonally dominant	Constant matrix	unknown matrix	Coefficient matrix	Unit matrix	Coefficient matrix
In ----------- method, the coefficient matrix is transformed into diagonal matrix	Gauss elimination	Gauss jordan	Gauss jacobi	Gauss seidal	Gauss jordan
------------- Method takes less time to solve a system of equations comparatively than ' iterative method'	Direct method	Indirect method	Regula falsi	Bisection	Direct method
The iterative process continues till ------------- is secured.	convergency	divergency	oscillation	none	convergency
In Gauss elimination method, the solution is getting by means of ----------- from which the unknowns are found by back substitution.	Elementary operations	Elementary column operations	Elementary diagonal operations	Elementary row operations	Elementary row operations

The --------------- is reduced to an upper triangular matrix or a diagonal matrix in direct methods.	Coefficient matrix	Constant matrix	unknown matrix	Augment matrix	Augment matrix
The augment matrix is the combination of ------------------.	Coefficient matrix and constant matrix	Unknown matrix and constant matrix	Coefficient matrix and Unknown matrix	Coefficient matrix, constant matrix and Unknown matrix	Coefficient matrix and constant matrix
The given system of equations can be taken as in the form of -----------	A = B	BX= A	AX= B	AB = X	AX= B
Which is the condition to apply Gauss Seidal method to solve a system of equations?	1st row is dominant	1st column is dominant	diagonally dominant	last row dominant	diagonally dominant
Crout's method and triangularisation method are ---method.	Direct	Indirect	Iterative	Interpolation	Direct
The solution of simultaneous linear algebraic equations are found by using-----	Direct method	Indirect method	both 1st & 2nd	Bisection	InDirect method

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DEPARTMENT OF MATHEMATICS
UNIT-III
FINITE DIFFERENCES
Part-A(20X1=20 Marks)
(Question Nos. 1 to 20 Online Examinations)
Multiple Choice Questions

Question	Opt 1	Opt 2	Opt 3	Opt 4	Answer
The forward difference operator is denoted by ---------	D	E	δ	Δ	Δ
The backward difference operator is denoted by ------	D	E	∇	Δ	∇
The difference value y2 – y1 in a Newton's forward difference table is denoted by ----------.	Δy_0	Δy_1	Δy_2	Δy_0	Δy_1
The difference value $\Delta y_1 - \Delta y_0$ in a backward difference table is denoted by ---	$\Delta^2 y_0$	$\Delta^2 y_1$	Δy_1	Δy_0	$\Delta^2 y_0$
The backward difference operator is defined as -------	f(x)-f(x-h)	f(x)+f(x-h)	f(x-h)-f(x)	f(x+h)-f(x)	f(x)-f(x-h)
The forward difference operator is defined as ---------	f(x+h)+f(x)	f(x+h)-f(x)	f(x)-f(x-h)	f(x-h)-f(x)	f(x+h)-f(x)
The central difference operator is denoted by ---------	D	E	δ	Δ	δ
The shifting operator is also known as------- operator.	forward difference	backward difference	central difference	translation	translation
The operator D is -------- operator.	linear	non linear	normal	translation	linear
The operator is distributive over -------- .	subtraction	addition	multiplication	division	addition
The relation between Δ and E is Δ = --------	E-1	E+1	E+2	E-3	E-1
The relation between ∇ and E is ∇= --------	E – 1	$1 - E^{-1}$	$1 + E^{-1}$	$1 * E^{-1}$	$1 - E^{-1}$
The n th differences of a polynomial of the n th degree are --------	one	Zero	constant	three	constant
The (n+1) th and higher differences of a polynomial of the nth degree are --------	one	Zero	two	three	Zero
The interval of differencing(h) is defined by -------	$x_2 - x1$	$x_2 + x1$	$x_0 + x_2$	$x_0 - x_2$	$x_2 - x1$
The polynomial x(x-h)(x-2h)(x-3h)……(x-(n-1)h) is deined as ----------	difference of polynomial	factorial polynomial	forward difference	backward difference	factorial polynomial
The difference operator is denoted by --------	D	E	δ	Δ	D
The averaging operator is denoted by ------	D	E	δ	μ	μ

What will be the first difference of a polynomial of degree four?	Polynomial of degree one	Polynomial of degree two	Polynomial of degree three	Polynomial of degree four	Polynomial of degree three
The difference $D^3f(x)$ is called ------- differences f(x).	first	fourth	second	third	third
The differences Dy are called ------- differences f(x).	first	fourth	second	third	first
$\Delta^2 y_2$ = -----	$\Delta y_2 - \Delta y_3$	$\Delta y_1 - \Delta y_2$	$y_3 - y_2$	$\Delta y_3 - \Delta y_2$	$\Delta y_3 - \Delta y_2$
The second difference $\Delta^2 y_0$ is equal to	$y_2 + 2y_1 - y_0$	$y_2 - 2y_1 - y_0$	$y_2 - 2y_1 + y_0$	$y_2 + 2y_1 + y_0$	$y_2 - 2y_1 + y_0$
The second difference $\Delta^3 y_0$ is equal to	$y_3 - 3y_2 + 3y_1 - y_0$	$y_3 + 3y_2 + 3y_1 - y_0$	$y_3 + 3y_2 + 3y_1 + y_0$	$y_2 + 2y_1 + y_0$	$y_3 - 3y_2 + 3y_1 - y_0$
The differences Δy are called ------- differences f(x).	first	fourth	second	third	first

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DEPARTMENT OF MATHEMATICS
UNIT-IV
INTERPOLATION
Part-A(20X1=20 Marks)
(Question Nos. 1 to 20 Online Examinations)
Multiple Choice Questions

UNIT-IV

Question	Opt 1	Opt 2	Opt 3	Opt 4	Answer
In Newton's forward interpolation formula, the first two terms will give the -----	extrapolation	linear interpolation	parabolic interpolation	inter extra polation	linear interpolation
In Newton's forward interpolation formula, the three terms will give the ------	extrapolation	linear interpolation	parabolic interpolation	inter extra polation	parabolic interpolation
---------------- Formula can be used for unequal intervals.	Newton's forward	Newton's backward	Lagrange	Stirling	Lagrange
The process of computing the value of a function outside the range is called -----	interpolation	extrapolation	both	inverse interpolat ion	extrapolatio n
The process of computing the value of a function inside the range is called ------	Interpolation	extrapolation	both	inverse interpolat ion	Interpolation
The difference value $y_2 - y_1$ in a Newton's forward difference table is denoted by -----	Δy_0	Δy_1	Δy_2	Δy_0	Δy_1
------------- Formula can be used for interpolating the value of f(x) near the end of the tabular values.	Newton's forward	Newton's backward	Lagrange	Stirling	Newton's backward
The technique of estimating the value of a function for any intermediate value is --------	interpolation	extrapolation	forward method	backward method	interpolation
The values of the independent variable are not given at equidistance intervals, we use ------- formula.	Newton's forward	Newton's backward	Stirling	Lagrange	Lagrange
To find the unknown values of y for some x which lies at the --------- of the table, we use Newton's Backward formula	beginning	end	center	outside	end

To find the unknown values of y for some x which lies at the --------- of the table, we use Newton's Forward	beginning	end	center	outside	beginning
To find the unknown value of x for some y, which lies at the unequal intervals we use ------------------- formula.	Newton's Forward	Newton's Backward	Lagrange	Stirling	Lagrange
If the values of the variable y are given, then the method of finding the unknown variable x is called ----------------.	Newton's Forward	Newton's Backward	Interpolation	Inverse Interpolation	Inverse Interpolation
The divided difference operator is -----------	non-linear	normal	linear	zero	linear
The ------------- Formula can be used for interpolating the value of f(x) near the end of the tabular values	Newton's forward	Newton's backward	Lagrange	Stirling	Newton's backward
The values of the independent variable are not given at equidistance intervals, we use -------------- formula	Newton's forward	Newton's backward	Lagrange	Stirling	Lagrange
In Newton's forward interpolation formula, the value of u is calculated by -------.	$u = (x–x_n) / h$	$u = (x_n–x) / h$	$u = (x–x_0) / h$	$u = (x_0–x) / h$	$u = (x–x_0) / h$
In Newton's forward interpolation formula, the value x can be written as -----------.	$x_0–nh$	$x_n–nh$	$x_n + nh$	$x_0 + nh$	$x_0 + nh$
In Newton's backward interpolation formula, the value x can be written as ---------	$x_0–nh$	$x_n–nh$	$x_n + nh$	$x_0 + nh$	$x_n + nh$
In Newton's backward interpolation formula, the value of v is calculated by -------	$v = (x–x_n) / h$	$v = (x_n–x) / h$	$v = (x–x_0) / h$	$v = (x_0–x) / h$	$v = (x–x_n) / h$
The -----differences are symmetrical in all their arguments.	forward	backward	divided	central	divided
The value of any divided differences is ----------- of the order of the arguments.	independent	dependent	zero	one	independent

The n^{th} divided differences of a polynomial of the n th degree are --------	one	Zero	constant	three	constant

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DEPARTMENT OF MATHEMATICS
UNIT-V
NUMERICAL DIFFERENTIATION AND INTEGRATION

(Question Nos. 1 to 20 Online Examinations)

Question	Opt 1	Opt 2	Opt 3	Opt 4	Answer
When the function is given in the form of table of values instead of giving analytical expression we use	Bisection method	Numerical differentiation	Newton Rapson method	Numerical integration	Numerical differentiation
Numerical differentiation can be used only when the difference of some order are-----------	zero	one	costant	two	costant
The numerical forward differentiation of y with respect to x once is ------------.	= (1/h)* (Dy_0 +	(1/h)* (Dy_n + (2r–1)/2 *	= (1/h)* (Dy_n +	n Dy_0 + {n(n+1) /	= (1/h)* (Dy_0 +
The numerical backward differentiation of y w.r.t. x once is ------------.	= (1/h)* (Dy_0 +	Ñy_n + {n(n+1) /	= (1/h)* (Dy_n +	(1/h)* (Ñy_n +	(1/h)* (Ñy_n + (2r+1)/2
The second derivative of the Newton's forward differentiation is -------------.	y ” = $(1/h^2)$* {D^2y_0 –	y ” = $(1/h^2)$* {D^2y_0 +	y ” = (1/h)* {D^2y_0 +	y ” = (1/h)* {D^2y_0 –	y ” = $(1/h^2)$* {D^2y_0 –
The second derivative of the Newton's backward differentiation is ------	y ” = $(1/h^2)$* {D^2y_0 +	y ” = $(1/h^2)$* {D^2y_0 –	y ” = (1/h)* {D^2y_0 +	y ” = (1/h)* {D^2y_0 –	y ” = $(1/h^2)$* {D^2y_0 +
The order of error in Trapezoidal rule is ---.	h	h^3	h^2	h^4	h^2
The order of error in Simpson's rule is -----	h	h^3	h^2	h^4	h^4
Numerical evaluation of a definite integral is called --------.	Integration	Differentiation	Interpolation	Triangularization	Integration
Simpson's ⅜ rule can be applied only if the number of sub interval is in ------.	Equal	even	multiple of three	unequal	multiple of three
By putting n = 2 in Newton cote's formula we get ----------- rule.	Simpson's 1/3	Simpson's ⅜	Trapezoidal	Romberg	Simpson's 1/3

The Newton Cote's formula is also known as --------------- formula.	Simpson's 1/3	Simpson's 3/8	Trapezoidal	quadrature	quadrature
By putting n = 3 in Newton cote's formula we get ----------- rule.	Simpson's 1/3	Simpson's ⅜	Trapezoidal	Romberg	Simpson's ⅜
By putting n = 1 in Newton cote's formula we get ----------- rule.	Simpson's 1/3	Simpson's ⅜	Trapezoidal	newton's	Trapezoidal
The systematic improvement of Richardon's method is called--------- method	Simpson's 1/3	Simpson's ⅜	Trapezoidal	Romberg	Romberg
Simpson's 1/3 rule can be applied only when the number of interval is ------.	Equal	even	multiple of three	unequal	even
Simpson's rule is exact for a ------- ---------- even though it was derived for a Quadratic	cubic	less than cubic	linear	quadratic	linear
The accuracy of the result using the Trapezoidal rule can be improved by --------	Increasing the interval h	Decreasing the interval h	Increasing the number of iterations	altering the given function	Increasing the number of iterations
A particular case of Runge Kutta method of second order is ------------ -------.	Milne's method	Picard's method	Modified Euler method	Runge's method	Modified Euler method
Runge Kutta of first order is nothing but the --------------.	modified Euler method	Euler method	Taylor series	none of these	Euler method
In Runge Kutta second and fourth order methods, the values of k1 and k2 are ----	same	differ	always positive	always negative	same
The formula of Dy in fourth order Runge Kutta method is given by --- ------------.	$1/6 * (k_1 + 2k_2 + 3k_3 + 4k_4)$	$1/6 * (k_1 + k_2 + k_3 + k_4)$	$(k_1 + 2k_2 + 2k_3 + k_4)$	$1/6 * (k_1 + 2k_2 + 2k_3 + k_4)$	$1/6 * (k_1 + 2k_2 + 2k_3 + k_4)$
In second order Runge Kutta method the value of k2 is calculated by ---------	$h f(x + h/2 , y + k_1/2)$	$h f(x - h/2 , y - k_1/2)$	h f(x , y)	h f(0,0)	$h f(x + h/2 , y + k_1/2)$
__________values are calculated in Runge Kutta fourth order method.	k_1, k_2, k_3, k_4 and Dy	k_1, k_2 and Dy	k_1, k_2, k_3 and Dy	none of these	k_1, k_2, k_3, k_4 and Dy
The use of Runge kutta method gives ------------- to the solutions of the differential equation than Taylor's series method	Slow convergence	quick convergence	oscillation	divergence	quick convergence
In Runge – kutta method the value x is taken as ----------.	$x = x_0 + h$	$x_0 = x + h$	$h = x_0 + x$	$h = x_0 - x$	$x = x_0 + h$

In Runge – kutta method the value y is taken as -----.	$y = y_0 + h$	$y_0 = x_0 + h$	$y = y_0 - Dy$	$y = y_0 + Dy$	$y = y_0 + Dy$
In fourth order Runge Kutta method the value of k3 is calculated by ------------.	$h f(x - h/2, y - k_2/2)$	$h f(x + h/2, y + k_2/2)$	$h f(x, y)$	$h f(x - h/2, y - k_1/2)$	$h f(x + h/2, y + k_2/2)$
In fourth order Runge Kutta method the value of k4 is calculated by ------------.	$h f(x + h/2, y + k_1/2)$	$h f(x + h/2, y + k_2/2)$	$h f(x + h, y + k_3)$	$h f(x - h, y - k_3)$	$h f(x + h, y + k_3)$
___________is nothing but the modified Euler method.	Runge kutta method of second	Runge kutta method of third order	Runge kutta method of fourth	Taylor series method	Runge kutta method of second
In all the three methods of Rungekutta methods, the values ---------- are same.	k_4 & k_3	k_3 & k_2	k_2 & k_1	k_1, k_2, k_3 & k_4	k_2 & k_1
The formula of Δy in third order Runge Kutta method is given by ---------------.	$1/6 * (k_1 + 2k_2 + 3k_3 + 4k_4)$	$1/6 * (k_1 + 4k_2 + k_3)$	$1/6 * (4k_1 + 4k_2 + 4k_3)$	$1/6 * (k_1 + 2k_2 + 2k_3 + k_4)$	$1/6 * (k_1 + 4k_2 + k_3)$
The formula of Δy in second order Runge Kutta method is given by --------------.	k_1	k_2	k_3	k_4	k_2
In second order Runge Kutta method the value of k1 is calculated by ------------.	$h f(x + h/2, y + k_1/2)$	$h f(x + h/2, y + k_2/2)$	$h f(x, y)$	$h f(x - h/2, y - k_1/2)$	$h f(x, y)$
The Runge – Kutta methods are designed to give -------------- and they posses the advantage of requiring only the function values	greater accuracy	lesser accuracy	average accuracy	equal	greater accuracy
If dy/dx is a function x alone, then fourth order Runge – Kutta method reduces to --------------.	Trapezoidal rule	Taylor series	Euler method	Simpson method	Simpson method
In Runge Kutta methods, the derivatives of ----------- are not require and we require only the given function values at different	higher order	lower order	middle order	zero	higher order
The use of -------------- method gives quick convergence to the solutions of the differential equation than Taylor's series	Taylor series	Euler	Runge – Kutta	Simpson method	Runge – Kutta
If dy/dx is a function x alone, then ---------- Runge – Kutta method reduces to Simpson method	fourth order	third order	second order	first order	fourth order
If dy/dx is a function of ----------- then fourth order Runge – Kutta method reduces to Simpson method	x alone	y alone	both x and y	none	x alone

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KARPAGAM UNIVERSITY

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(For the candidates admitted from 2010 onwards)

B.Sc., DEGREE EXAMINATION, APRIL 2013

Sixth Semester

MATHEMATICS (COMPUTER APPLICATIONS)

NUMERICAL METHODS

Time: 3 hours Maximum : 60 marks

PART – A (20X ½ = 10 Marks)
Answer ALL the Questions

1. The bisection method is simple but ------------------.
   a. Slowly convergent b. fast convergent c. slowly divergent
   d. fast divergent

2. The convergence of iteration method is ----------------.
   a. zero b. polynomial c. quadratic d. linear

3. In Newton Raphson method, the error at any stage is proportional to the ----- of the error in the previous stage.
   a. Cubic b. square c. square root d. None of these.

4. The method of false position is also known as -------------- method.
   a. Gauss Seidal b. Secant c. Bisection d. Regula falsi

5. When Gauss Jordan method is used to solve AX = B, A is transformed into -----
   a. Scalar matrix b. diagonal matrix c. Upper triangular matrix
   d. lower triangular matrix

6. Condition for convergence of Gauss Seidal method is ---------------.
   a. Coefficient matrix is not diagonally dominant b. pivot element is Zero
   c. Coefficient matrix is diagonally dominant d. pivot element is non-zero.

7. Crout's method is also a --------- method.
   a. indirect b. direct c. iterative d. root.

8. Modified form of Gauss Jacobi method is -------------------- method.
   a. Gauss Jordan b. Gauss Seidal c. Gauss Jacobi d. Crout's.

9. Forward difference operator is denoted by the symbol --------
   a. Δ b. ∇ c. Σ d. Π

10. Shifting operator is also known as --------- operator.
    a. translation b. central c. forward d. backward.

11. Relation between E and ∇ is ∇ = ...
    a. $E - 1$ b. $1 - E^{-1}$ c. $1 + E^{-1}$ d. $1 * E^{-1}$

12. The n^{th} differences (forward, of a polynomial of the n^{th} degree are -----
    a. constant b. variable c. zero d. one

13. The process of computing the value of a function outside the range is called -----
    a. interpolation b. extrapolation c. both d. inverse interpolation

14. The difference value $\nabla y_1 - \nabla y_0$ in a Newton's forward difference table is denoted by ....
    a. $\nabla^2 y_0$ b. $\nabla^2 y_1$ c. ∇y_1 d. Δy_0

15. .... Interpolation formula can be used for equal and unequal intervals.
    a. Newton's forward b. Romberg c. Lagrange d. none

16. The divided difference operator is ....
    a. non-linear b. normal c. linear d. none

17. The order of error in Trapezoidal rule is ....
    a. h b. h^3 c. h^2 d. h^4

18. Simpson's ⅓ rule can be applied only if the number of sub interval is in ------.
    a. Equal b. even c. multiple of three d. unequal.

19. The systematic refinement of the values of I's is called -------- method.
    a. Romberg's b. Euler c. Runge – Kutta d. Simpson's

20. A particular case of Runge Kutta method of second order is ....
    a. Milne's method b. Picard's method c. Modified Euler method
    d. Taylor Series

PART B (5 X 4 = 20 Marks)
Answer ALL the Questions

21. a. Find the positive root of $x^3 - 2x - 5 = 0$ by the Regula Falsi Method correct to two decimal places.

Or

b. Find the positive root of $x^4 - x = 10$ correct to three decimal places using Newton Raphson Method.

22) a. Solve the system by Gauss – Elimination method.

$2x + 3y - z = 5$
$4x + 4y - 3z = 3$
$2x - 3y + 2z = 2$

Or

b. Solve the following system of equation by Gauss Jordan method

$5x + 4y = 15$
$3x + 7y = 12$

23) a. Find forward difference table for the following data

X :	0	5	10	15
Y :	14	379	1444	3584

Or

b. State any three properties of operator.

24) a. Using Newton's forward interpolation formula evaluate y at x = 5

X	:	4	6	8	10
Y	:	1	3	8	10

Or

b. Form the divided difference table for the following data.

x	:	1	2	4	7	12
f(x)	:	22	30	82	106	206

25) a. Evaluate $\int_0^1 \frac{dx}{1 + x^2}$ using Trapezoidal rule

Or

b. Write Simpson's 1/3 and 3/8[th] rule formula.

PART C (3 x 10 = 30 Marks)
Answer any THREE Questions

26) Given that $x^3 - 3x + 1 = 0$ has a root between 1 and 2. Find it to three decimal places using Horner's method.

27) Solve the system of equations

$2x - 3y + 20z = 25$
$20x + y - 2z = 17$
$3x + 20y - z = -18$

By Gauss – Jacobi iteration method (only two iteration)

28. Explain forward and backward difference operators.

29. Using Lagrange's formula find f(3)

X	:	0	1	2	5
F(x)	:	2	3	12	147

30) Write R-K method formula for II order, III order and IV order method.

Reg. No.................................

[10MCU603]

KARPAGAM UNIVERSITY

(Under Section 3 of UGC Act 1956)
COIMBATORE – 641 021
(For the candidates admitted from 2010 onwards)

B.Sc., DEGREE EXAMINATION, APRIL 2013

Sixth Semester

MATHEMATICS (COMPUTER APPLICATIONS)

NUMERICAL METHODS

Time: 3 hours | Maximum : 60 marks

PART – A (20X ½ = 10 Marks)

Answer ALL the Questions

1. The bisection method is simple but ------------------.
   a. Slowly convergent b. fast convergent c. slowly divergent
   d. fast divergent

2. The convergence of iteration method is ----------------.
   a. zero b. polynomial c. quadratic d. linear

3. In Newton Raphson method, the error at any stage is proportional to the ----- of the error in the previous stage.
   a. Cubic b. square c. square root d. None of these.

4. The method of false position is also known as ------------- method.
   a. Gauss Seidal b. Secant c. Bisection d. Regula falsi

5. When Gauss Jordan method is used to solve AX = B, A is transformed into -----
   a. Scalar matrix b. diagonal matrix c. Upper triangular matrix
   d. lower triangular matrix

6. Condition for convergence of Gauss Seidal method is ---------------.
   a. Coefficient matrix is not diagonally dominant b. pivot element is Zero
   c. Coefficient matrix is diagonally dominant d. pivot element is non-zero.

7. Crout's method is also a --------- method.
   a. indirect b. direct c. iterative d. root.

8. Modified form of Gauss Jacobi method is -------------------- method.
   a. Gauss Jordan b. Gauss Seidal c. Gauss Jacobi d. Crout's.

9. Forward difference operator is denoted by the symbol --------
   a. Δ b. ∇ c. Σ d. Π

10. Shifting operator is also known as ---------- operator.
    a. translation b. central c. forward d. backward.

11. Relation between E and ∇ is ∇ = ----------
    a. $E - 1$ b. $1 - E^{-1}$ c. $1 + E^{-1}$ d. $1 * E^{-1}$

12. The n^{th} differences (forward, of a polynomial of the n^{th} degree are -----
    a. constant b. variable c. zero d. one

13. The process of computing the value of a function outside the range is called -----
    a. interpolation b. extrapolation c. both d. inverse interpolation

14. The difference value $\nabla y_1 - \nabla y_0$ in a Newton's forward difference table is denoted by -----------------
    a. $\nabla^2 y_0$ b. $\nabla^2 y_1$ c. ∇y_1 d. Δy_0

15. ------------- Interpolation formula can be used for equal and unequal intervals.
    a. Newton's forward b. Romberg c. Lagrange d. none

16. The divided difference operator is -----------
    a. non-linear b. normal c. linear d. none

17. The order of error in Trapezoidal rule is -------------.
    a. h b. h^3 c. h^2 d. h^4

18. Simpson's ⅜ rule can be applied only if the number of sub interval is in ------.
    a. Equal b. even c. multiple of three d. unequal.

19. The systematic refinement of the values of I's is called --------- method.
    a. Romberg's b. Euler c. Runge – Kutta d. Simpson's

20. A particular case of Runge Kutta method of second order is ------------------.
    a. Milne's method b. Picard's method c. Modified Euler method
    d. Taylor Series

PART B (5 X 4 = 20 Marks)

Answer ALL the Questions

21. a. Find the positive root of $x^3 - 2x - 5 = 0$ by the Regula Falsi Method correct to two decimal places.

Or

b. Find the positive root of $x^4 - x = 10$ correct to three decimal places using Newton Raphson Method.

22. a. Solve the system by Gauss – Elimination method.

$2x + 3y - z = 5$
$4x + 4y - 3z = 3$
$2x - 3y + 2z = 2$

Or

b. Solve the following system of equation by Gauss Jordan method

$5x + 4y = 15$
$3x + 7y = 12$

23. a. Find forward difference table for the following data

X :	0	5	10	15
Y :	14	379	1444	3584

Or

b. State any three properties of operator.

24. a. Using Newton's forward interpolation formula evaluate y at $x = 5$

X	:	4	6	8	10
Y	:	1	3	8	10

Or

b. Form the divided difference table for the following data.

x	:	1	2	4	7	12
f(x)	:	22	30	82	106	206

25. a. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Trapezoidal rule

Or

b. Write Simpson's 1/3 and 3/8[th] rule formula.

PART C (3 x 10 = 30 Marks)
Answer any THREE Questions

26. Given that $x^3 - 3x + 1 = 0$ has a root between 1 and 2. Find it to three decimal places using Horner's method.

27. Solve the system of equations

$2x - 3y + 20z = 25$
$20x + y - 2z = 17$
$3x + 20y - z = -18$

By Gauss – Jacobi iteration method (only two iteration)

28. Explain forward and backward difference operators.

29. Using Lagrange's formula find f(3)

X	:	0	1	2	5
F(x)	:	2	3	12	147

30. Write R-K method formula for II order, III order and IV order method.

Reg. No..................................

[13MMU604]

KARPAGAM UNIVERSITY

Karpagam Academy of Higher Education
(Established Under Section 3 of UGC Act 1956)
COIMBATORE – 641 021
(For the candidates admitted from 2013 onwards)

B.Sc., DEGREE EXAMINATION, APRIL 2016

Sixth Semester

MATHEMATICS

NUMERICAL METHODS

Time: 3 hours　　　　Maximum : 60 marks

PART – A (20 x 1 = 20 Marks) (30 Minutes)
(Question Nos. 1 to 20 Online Examinations)

PART B (5 x 8 = 40 Marks) (2 ½ Hours)
Answer ALL the Questions

21. a) Find the positive root of $x - \cos x = 0$ by using bisection method.

Or

b) Solve the equation $x^3 + x^2 - 1 = 0$ for the positive root by iteration method.

22. a) Solve the following system of equations by Gauss Elimination method

$$10x + y + z = 12$$
$$x + 10y + z = 12$$
$$x + y + 10z = 12$$

Or

b) Solve the following system by triangularisation method.

$$5x - 2y + z = 4$$
$$7x + y - 5z = 8$$
$$3x + 7y + 4z = 10$$

23. a) Evaluate i) $\Delta^n(e^{ax+b})$ ii) $\Delta^n[\sin(ax+b)]$ iii) $\Delta^n[\cos(ax+b)]$ iv) $\Delta^n[\log(ax+b)]$

Or

b) If $y = f(x)$ is a polynomial of degree 3 and the following table gives the values of x & y. Locate and correct the wrong values of y.

x	0	1	2	3	4	5	6
y	4	10	30	75	160	294	490

24. a) The population of a town is as follows.

Year (x)	1941	1951	1961	1971	1981	1991
Population in Lakhs (y)	20	24	29	36	46	51

Estimate the population increase during the period 1946 to 1976.

Or

b) Using inverse interpolation formula, find the value of x when y=13.5.

x	93.0	96.2	100.0	104.2	108.7
y	11.38	12.80	14.70	17.07	19.91

25. a) The population of a certain town is given below, Find the rate of growth of population in 1931, 1941 1961 and 1971 .

Year	1931	1941	1951	1961	1971
Population	40.62	60.80	79.95	103.56	132.65

Or

b) Using Romberg's method , evaluate $I = \int_0^1 \frac{dx}{(1+x)}$ correct to 3 decimal places.

Hence evaluate $\log_e 2$.

Reg. No.................................

[14MMU604]

KARPAGAM UNIVERSITY

Karpagam Academy of Higher Education
(Established Under Section 3 of UGC Act 1956)
COIMBATORE – 641 021
(For the candidates admitted from 2014 onwards)

B.Sc., DEGREE EXAMINATION, APRIL 2017

Sixth Semester

MATHEMATICS

NUMERICAL METHODS

Time: 3 hours | Maximum : 60 marks

PART – A (20 x 1 = 20 Marks) (30 Minutes)
(Question Nos. 1 to 20 Online Examinations)

PART B (5 x 8 = 40 Marks) (2 ½ Hours)
Answer ALL the Questions

21. a) Find the root of the equation lies between 0 and 1 of the equation $x^3 - 6x + 4 = 0$ using Newton Raphson method .

(Or)

b) Show that the root of the equation lies between 1 and 2 of the equation $x^3 - 3x + 1 = 0$ using Horner's method .

22. a) Solve by Gauss Jordan method

$$2x - 3y + z = 1$$
$$x + 4y + 5z = 25$$
$$3x - 4y + z = 2$$

(Or)

b) Solve by Gauss Jacobi method

$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110$$

23. a) Find the 7^{th} term of the sequence 2,9,28,65,126,217 and also find the general term.

(Or)

b) Prove that n^{th} differences of a polynomial of the n^{th} degree are constants.

24. a) Using Newton's divided difference formula, find the values of f (2), f (8) and f (15) from the following table:

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

(Or)

b) Using Lagrange's interpolation formula, find y(27) from the following table.

X	14	17	31	35
Y	68.7	64.0	44.0	39.1

25. a) Using fourth order Runge kutta method to find an approximate value of y when x = 0.1, 0.2 given that $y' = x + y$, $y(0) = 1$.

(Or)

b) Use Romberg method to compute $\int_0^1 \frac{dx}{1+x^2}$ correct to 4 decimal places. Hence find Π.

Reg.No-------------

(15MMU503)

Karpagam Academy of Higher Education
Coimbatore-21
Department of Mathematics
Fifth Semester
I Internal Test-July'17
Numerical Methods

Date: 19.07.17(FN) **Time: 2 Hours**

Class:III B.Sc Mathematics A&B **Maximum Marks:50**

PART-A(20X1=20 Marks)

Answer all the Questions:

1. The equation 3x – cosx – 1 = 0 is known as ------------- equation.
   a) polynomial b)transcedental
   c) algebraic d) exponential

2. -------------- method is also called as Bolzano method or interval having method.
   a) Bisection b) False position
   c) Newton Rapsond) Euler

3. The convergence of iteration -------------.
   a) zero b) polynomial
   c) quadratic d) linear

4. The order of convergence of Regula falsi method may be assumed to --------.
   a) 1.618 b) 1 c) 1.234 d) 0.5

5. The method of false position is also known as ------------- method.
   a) Gauss Seidal b) Secant
   c) Bisection d) Regula falsi

6. The order of convergence of Newton Raphson method is ---
   a) 4 b) 2 c) 1 d) 0

7. Graeffe's root squaring method is useful to find -----------.
   a) complex roots b) single root
   c) unequal roots d) polynomial roots

8.------------method is also called method of tangents.
   a) Gauss Seidal b) Secant
   c) Bisection d) Newton Rapson

9. The sufficient condition for convergence of iterations is -----.
   a) $| \phi'(x) | = 1$ b) $| \phi'(x) | > 1$
   c) $| \phi'(x) | < 1$ d) $| \phi'(x) | < 0$

10. The Newton Rapson method fails if -----.
   a) $f'(x) = 0$ b) $f(x) = 0$
   c) $f(x) = 1$ d) $f'(x) = 1$

11. The augment matrix is the combination of ------------------.
   a) Coefficient matrix and constant matrix
   b) Unknown matrix and constant matrix
   c) Coefficient matrix and unknown matrix
   d) Coefficient matrix,constant matrix and unknown matrix

12. Gauss elimination method is a -----------.
a) indirect method b) direct method
c) iterative method d) convergent

13. When Gauss Jordan method is used to solve AX = B, A is transformed into -----
a) Scalar matrix b) Diagonal matrix
c) Upper triangular matrix d) Lower triangular matrix

14. In the upper triangular coefficient matrix, all the elements below the diagonal are --------------.
a) positive b) non zero
c) zero d) negative.

15. Method of triangularization is also a ----- method.
a) indirect b) direct
c) iterative d) root.

16. The modification of Gauss – Elimination method is called ---------------.
a) Gauss Jordan b) Gauss Seidal
c) Gauss Jacobi d) Crout's method.

17. In Gauss elimination method the given matrix is transformed into --------------.
a) Unit matrix b) diagonal matrix
c) Upper triangular matrix
d) Lower triangular matrix

18. In ------- method the coefficient matrix is decomposed into upper and lower triangular matrices.
a) Gauss Jordan b) Triangularization
c) Gauss Jacobi d) Gauss Seidal

19. Crout's method is also a --------- method.
a) indirect b) direct c) iterative d) root.

20. In crout's method, the auxiliary matrix is also known as -------- matrix.
a) square b) diagonal c) upper triangular d) derived

PART-B(3 X 10 = 30 Marks)

Answer all the Questions:

21.a) Find an approximate root of $x \log_{10} x = 1.2$ by False position method.

(OR)

b) Find the positive root of the equation $x^3 + 3x - 1 = 0$ correct to 2decimal places by Horner's method.

22.a) Find all the roots of the equation $x^3 - 9x^2 + 18x - 6 = 0$ by Graeffe's method (root squaring, three times).

(OR)

b) Find the positive root of $f(x) = 2x^3 - 3x - 6 = 0$ by Newton Raphson method correct to four decimal places.

23.a) Solve the following system by Gauss Jordan method

$$10x + y + z = 12$$
$$2x + 10y + z = 13$$
$$x + y + 5z = 7.$$

(OR)

b) Solve the following system by triangularisation method.

$$5x - 2y + z = 4$$
$$7x + y - 5z = 8$$
$$3x + 7y + 4z = 10$$

Reg.No---------------

(15MMU503)

Karpagam Academy of Higher Education
Coimbatore-21
Department of Mathematics
Fifth Semester
II Internal Test-Aug'17
Numerical Methods

Date: 09.08.17(AN) **Time: 2 Hours**
Class: III B.Sc Mathematics **Maximum Marks:50**

PART-A(20X1=20 Marks)
Answer all the Questions:

1. Modified form of Gauss Jacobi method is ----------- method.
   a) Gauss Jordan b) Gauss Seidal
   c) Gauss Jacobi d) Crout's

2. ------------- is also a self-correction method.
   a) Direct method b) indirect method
   c) interpolation d) Gauss Elimination

3. Condition for convergence of Gauss Seidal method is --------
   a) Coefficient matrix is not diagonally dominant
   b) pivot element is Zero
   c) Coefficient matrix is diagonally dominant
   d) pivot element is non-zero.

4. The rate of convergence in Gauss – Seidel method is roughly --------- times than that of Gauss Jacobi method.
   a) 0 b) 3 c) 4 d) 2

5. Example for iterative method -------------.
   a) Gauss elimination b) Gauss Seidal
   c) Gauss Jordon d) Bisection

6. Backward difference operator is denoted by the symbol ------
   a) Δ b) ∇ c) $\sum$ d) $\prod$

7. Shifting operator is also known as ---------- operator.
   a) translation b) central c) forward d) backward.

8. Relation between E and ∇ is ∇ = ----------
   a) $E - 1$ b) $1 - E^{-1}$ c) $1 + E^{-1}$ d) $1 * E^{-1}$

9. The inverse operator E^{-1} is defined by $E^{-1} f(x)$ = -------
   a) f(x-h) b) f(x+h) c) f(x+h/2) d) f(x-h/2)

10. The operator D is -------- operator.
    a) linear b) non linear c) normal d) translation

11. In difference, f(x+h) – f(x) = --------
    a) $\Delta f(x)$ b) $\nabla f(x)$ c) $\Delta^2 f(x)$ d) h(x)

12. The operators are distributive over ------------
    a) subtraction b) multiplication
    c) division d) addition

13. The difference value $y_2 - y_1$ in a Newton's forward difference table is denoted by ------ .
    a) Δy_0 b) Δy_1 c) Δy_2 d) Δy_4

14. The central difference operator is denoted by ---------

a) D b) δ c) ∇ d) Δ

15. The polynomial x(x-h)(x-2h)(x-3h)……(x-(n-1)h) is defined as ----------

a) difference of polynomial b) factorial polynomial
c) forward difference d) backward difference

16. The difference value $\nabla y_2 - \nabla y_1$ in a backward difference table is denoted by ----

a) $\nabla^2 y_2$ b) $\nabla^2 y_0$ c) ∇y_2 d) ∇y_0

17. The $(n+1)^{th}$ and higher differences of a polynomial of degree n are -----

a) constant b) variable c) zeros d) one

18. The averaging operator is denoted by ------

a) D b) E c) δ d) μ

19. Shifting operator E is defined by Ef(x) = -------

a) f(x-h) b) f(x+h) c) f(x+h/2) d) f(x-h/2)

20. Relation between E and δ is δ = ------------

a) $E^{1/2} - E^{-1/2}$ b) $E^{1/2} + E^{-1/2}$
c) $E^{1/2} * E^{-1/2}$ d) $E^{1/2} - E^{1/2}$

PART-A(3 X 10 = 30 Marks)

Answer all the Questions:

21.a) Solve the following system of equations by Crout's method.

x + y + z = 3
2x - y + 3z = 16
3x + y - z = -3

(OR)

b) Solve the following system of equations by Gauss-Jacobi method

10x – 5y -2z = 3
4x -10y +3z = -3
x + 6y +10z = -3

22. a) Prove that n^{th} difference of a polynomial of the n^{th} degree are constants.

(OR)

b) Evaluate i) $\Delta^n(e^{ax+b})$ ii) Δ^n[sin(ax+b)]

iii) Δ^n[cos(ax+b)] iv) Δ[log(ax+b)]

23.a) i) Write the relation between Δ and E.

ii) Express $x^3 + x^2 + x + 1$ in factorial polynomials and get their successive differences taking h = 1.

(OR)

b) Estimate the production for 1964 & 1966 from the following data.

x :	1	2	3	4	5	6	7
y :	2	4	8	-	32	64	128

Reg.No--------------

(15MMU503)

KARPAGAM UNIVERSITY

Karpagam Academy of Higher Education

Coimbatore-21

Department of Mathematics

Fifth Semester

Model Examination-Sep'17

Numerical Methods

Date: 6.10.17(FN) Time: 3 Hours

Class: III B.Sc Mathematics Maximum Marks: 60

PART-A(20X1=20 Marks)

Answer all the Questions:

1. If f (x) contains some functions like exponential, trigonometric, logarithmic etc., then f (x) is called ------- equation.
   a) Algebraic b) transcendental
   c) numerical d) polynomial

2. The bisection method is simple but ------------------.
   a) slowly convergent b) fast convergent
   c) slowly divergent d) fast divergent

3. In Newton Raphson method, the error at any stage is Proportional to the ------- of the error in the previous stage.
   a) cube b) square c) square root d) equal

4. In --------- method, first find the integral part of the equation.
   a) Iteration b) Regula Falsi c) Bisection d) Horner's

5. In the upper triangular coefficient matrix, all the elements above the diagonal are -------------.
   a) Zero b) non – zero c) unity d) negative.

6. The direct method fails if any one of the pivot elements become ----------------.
   a) zero b) one c) two d) negative

7. Method of Triangularisation is also known as ------.
   a) factorization b) false position
   c) Bolzano d) iteration

8. Condition for convergence of Gauss Seidal method is --------
   a) Coefficient matrix is not diagonally dominant
   b) pivot element is Zero
   c) Coefficient matrix is diagonally dominant
   d) pivot element is non-zero.

9. In difference, f(x+h) – f(x) = --------.
   a) $\Delta f(x)$ b) $\nabla f(x)$ c) $\Delta^2 f(x)$ d) h(x)

10. Shifting operator is also known as --------- operator.
    a) translation b) central c) forward d) backward.

11. The operators are distributive over ------------
    a) subtraction b) multiplication c) division d) addition

12. Relation between Δ and E is Δ = ----------
    a) E – 1 b) E + 1 c) E * 1 d) 1 – E

13. The process of computing the value of a function outside the range is called -----.
    a) interpolation b) extrapolation
    c) both d) inverse interpolation

14. ------------- Interpolation formula can be used for equal and un equal intervals.

a) Newton's forward b) Newton's forward

c) Lagrange d) Romberg

15. The divided difference operator is -----------

a) non-linear b) normal c) linear d) translation

16. In Newton's forward interpolation formula, the three terms will give the ------

a) extrapolation b) linear interpolation

c) parabolic interpolation d) inter extra polation

17. The Newton Cote's formula is also known as -------- formula.

a) Simpson's 1/3 b) Simpson's 3/8

c) Trapezoidal d) quadrature

18. By putting n = 3 in Newton cote's formula we get ----------- rule.

a) Simpson's 1/3 rule b) Simpson's ⅜ rule

c) Trapezoidal rule d) Romberg

19. The order of error in Trapezoidal rule is -------------.

a) h b) h^3 c) h^2 d) h^4

20. In Runge Kutta second and fourth order methods, the values of k_1 and k_2 are ----.

a) same b) differ

c) always positive d) always negative

PART -B(5x8= 40 Marks)

Answer all the questions:

21. a) Find the positive root of $e^x = 3x$ by using Bisection method.

(OR)

b) Solve the equation $x^3 + x^2 - 1 = 0$ for the positive root by iteration method.

22.a) Solve the following system by triangularisation method.

$5x - 2y + z = 4,\ 7x + y - 5z = 8,\ 3x + 7y + 4z = 10$

(OR)

b) Solve the following system of equations by Gauss-Seidel method.

$8x - 3y + 2z = 20, 4x + 11y - z = 33,\ 6x + 3y + 12z = 35$

23. a) Find the 7^{th} term of the sequence 2,9,28,65,126,217 and also. Find the General term.

(OR)

b) If $y = f(x)$ is a polynomial of degree 3 and the following table gives the values of x & y. Locate and correct the wrong values of y.

x : 0	1	2	3	4	5	6
y : 4	10	30	75	160	294	490

24. a) From the following table of half-yearly premium for policies maturing at different ages. Estimate the premium for policies maturing at age 46 & 63.

Age (x):	45	50	55	60	65
Premium(y):	114.84	96.16	83.32	74.48	68.48

(OR)

b) Using Newton's divided difference formula, find the values of f(2), f(8) and f(15) given the following table.

x :	4	5	7	10	11	13
f(x) :	48	100	294	900	1210	2028

25. a) Evaluate $I = \int_0^6 dx/(1+x)$ using both of the Simpson's rule.

(OR)

b) Use Runge kutta method of fourth order find y for x = 0.1 and 0.2, given that $dy/dx = x + y$, $y(0) = 1$.