KARPAGAM ACADEMY OF HIGHER EDUCATION

(Deemed to be University Established Under Section 3 of UGC Act 1956)

Coimbatore – 641 021.

LECTURE PLAN
DEPARTMENT OF MATHEMATICS

STAFF NAME: Dr. K. KALIDASS
SUBJECT NAME: THEORY OF EQUATIONS
SUB.CODE:17MMU202
SEMESTER: II CLASS: I B.Sc., MATHEMATICS

S. No	Lecture Duration Hour	Topics To Be Covered	Support Materials
UNIT-I			
1	1	Introduction to polynomials	T: Ch 2, 33
2	1	Theorem relating to polynomials when the variable receives large value	T: Ch 1, 5
3	1	Theorem when the variable receives small values	T: Ch 1, 6
4	1	Continuity of a rational integral function	T: Ch 1, 10
5	1	Continuity of a rational integral function	T: Ch 1, 11
6	1	Tutorial	
7	1	Form of the quotient and remainder when a polynomial is divided by a Binomial	T: Ch 1, 11
8	1	Form of the quotient and remainder when a polynomial is divided by a Binomial	T: Ch 1, 11
9	1	Tabulation of functions.	T: Ch 1, 12
10	1	Tabulation of functions	T: Ch 1, 12
11	1	Tabulation of functions.	T: Ch 1, 13
12	1	Tutorial	
13	1	Graphic representation of a polynomial	T: Ch 1, 14
14	1	Graphic representation of a polynomial	T: Ch 1, 15
15	1	Graphic representation of a polynomial	R1: Ch
16	1	Graphic representation of a polynomial	T: Ch 1, 17
17	1	Maximum and minimum values of polynomials	T: Ch 1, 18
18	1	Tutorial	
19	1	Maximum and minimum values of polynomials	T: Ch 1, 18
20	1	Maximum and minimum values of polynomials	T: Ch 1, 18
21	1	Recapitulation and Discussion of possible questions	
Total No of Hours Planned For Unit 1 21 hours			

UNIT-II			
1	1	Theorems relating to the real roots of equations	T: Ch 2, 19
2	1	Theorems relating to the real roots of equations	T: Ch 2, 20
3	1	Theorems relating to the real roots of equations	T: Ch 2, 20
4	1	Existence of a root in the general equation	T: Ch 2, 21
5	1	Existence of a root in the general equation	T: Ch 2, 21
6	1	Tutorial	
7	1	Imaginary roots	T: Ch 2, 22
8	1	Imaginary roots	T: Ch 2, 22
9	1	Imaginary roots	T: Ch 2, 22
10	1	Theorem determining the number of roots of an equation	T: Ch 2, 22
11	1	Theorem determining the number of roots of an equation	T: Ch 2, 23
12	1	Tutorial	
13	1	Descartes' rule of signs for positive roots	T: Ch 2, 28
14	1	Descartes' rule of signs for negative roots	T: Ch 2, 28
15	1	Use of Descartes' rule in proving the existence of imaginary roots	R2: Ch
16	1	Use of Descartes' rule in proving the existence of imaginary roots	T: Ch 2, 30
17	1	Use of Descartes' rule in proving the existence of imaginary roots	T: Ch 2, 30
18	1	Tutorial	
19	1	Theorem relating to the substitution of two given numbers for the variable	T: Ch 2, 31
20	1	Theorem relating to the substitution of two given numbers for the variable	T: Ch 2, 32
21	1	Recapitulation and Discussion of possible questions	
Total No of Hours Planned For Unit II 21 hours			
Unit III			
1	1	Relations between the roots and coefficients	T: Ch 3,35
2	1	Relations between the roots and coefficients	T: Ch 3, 36
3	1	Applications of the theorem	T: Ch 3, 37
4	1	Applications of the theorem	T: Ch 3,37-38
5	1	Applications of the theorem	T: Ch 3,39
6	1	Tutorial	
7	1	relation exists between two of its roots	T: Ch 3,40
8	1	relation exists between two of its roots	T: Ch 3,41-42
9	1	relation exists between two of its roots	T: Ch 3,43

10	1	The cube roots of unity	T: Ch 3,44
11	1	The cube roots of unity	T: Ch 3,45
12	1	Tutorial	
13	1	Symmetric functions of the roots	T: Ch 3,46-47
14	1	Symmetric functions of the roots	T: Ch 3,47
15	1	Symmetric functions of the roots	T: Ch 3,48
16	1	Examples	T: Ch 3,49
17	1	Examples	T: Ch 3,49
18	1	Tutorial	
19	1	Theorems relating to symmetric functions	R3: Ch
20	1	Theorems relating to symmetric functions	T: Ch 3, 51-53
21	1	Recapitulation and Discussion of possible questions	
Total No of Hours Planned For Unit 1II 21 hours			
UNIT-IV			
1	1	Transformation of equations	T: Ch 4, 60
2	1	Transformation of equations	T: Ch 4, 60
3	1	Roots with signs changed	T: Ch 4, 62
4	1	Roots with signs changed	T: Ch 4, 63
5	1	Roots multiplied by a given quantity	T: Ch 4, 64
6	1	Tutorial	
7	1	Roots multiplied by a given quantity	T: Ch 4, 65
8	1	Reciprocal roots and reciprocal equations	T: Ch 4, 65-66
9	1	Reciprocal roots and reciprocal equations	T: Ch 4, 67
10	1	increase or diminish the roots by a given quantity	T: Ch 4, 68-69
11	1	increase or diminish the roots by a given quantity	T: Ch 4, 70
12	1	Tutorial	
13	1	Removal of terms	T: Ch 4, 70
14	1	Binomial coefficients	T: Ch 4, 70
15	1	Reciprocal equations	T: Ch 4, 71
16	1	Binomial equations	T: Ch 4, 72-73
17	1	Propositions embracing their leading general Properties	T: Ch 4, 73
18	1	Tutorial	
19	1	The special roots of the equation - Solution of binomial equations by circular functions	T: Ch 4, 75
20	1	The special roots of the equation - Solution of binomial equations by circular functions	T: Ch54, 82
21	1	Recapitulation and Discussion of possible questions	
Total No of Hours Planned For Unit IV 21 Hours			
Unit V			

1	1	The algebraic solution of equations	T: Ch 6,101
2	1	The algebraic solution of equations	T: Ch 6,102-103
3	1	The algebraic solution of the cubic equation	T: Ch 6,104
4	1	The algebraic solution of the cubic equation	T: Ch 6,105
5	1	Application to numerical equations	T: Ch 6,105-106
6	1	Tutorial	
7	1	Expression of the cubic as the difference of two cubes	T: Ch 6,107
8	1	Expression of the cubic as the difference of two cubes	T: Ch 6,108-109
9	1	Solution of the cubic by symmetric functions of the roots	T: Ch 6,109-110
10	1	Graphic representation of the derived function	T: Ch 6,111
11	1	Theorem relating to the maxima and minima of a polynomial	T: Ch 7,146-147
12	1	Tutorial	
13	1	Theorem relating to the maxima and minima of a polynomial	T: Ch 7, 147
14	1	Rolle's Theorem	T: Ch 7,148
15	1	Rolle's Theorem	T: Ch 7,148
16	1	Corollary	T: Ch 7,149
17	1	Constitution of the derived functions	T: Ch 7,149
18	1	Tutorial	
19	1	Constitution of the derived functions	
20	1	Recapitulation and Discussion of possible questions	
21	1	Recapitulation and Discussion of possible questions	
Total	**21 Hours**		

Total no. of Hours for the Course: 60 hours

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TEXT BOOK

T: Burnside W.S., and Panton A.W.,(1954). The Theory of Equations, Eighth Edition , Dublin University Press.

REFERENCES

R1: Leonard Eugene Dickson (2012). First Course in the theory of Equations. J. Wiley & sons, London: Chapman & Hall, Limited, New York.

R2: Turnbull,H.W (2013)., Theory Of Equations, Fourth Edition, Published In Great Britain Bt, Oliver And Boyd Ltd., Edinburgh.

R3. James Víctor Uspensky., (2005). Theory of Equations, McGraw-Hill Book Co, New York.

R4. MacDuffee C.C., (1962). Theory of Equations, John Wiley & Sons Inc., New York.

UNIT-I

SYLLABUS

General properties of polynomials:Theorem relating to polynomials when the variable receives large values, similar theorem when the variable receives small values.

Continuity of a rational integral function - Form of the quotient and remainder when a polynomial is divided by a Binomial - Tabulation of functions - Graphic representation of a polynomial - Maximum and minimum values of polynomials

Theorem.—*If in the polynomial*

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$$

the value $\dfrac{a_k}{a_0} + 1$, *or any greater value, be substituted for* x, *where* a_k *is that one of the coefficients* $a_1, a_2, \ldots a_n$ *whose numerical value is greatest, irrespective of sign, the term containing the highest power of* x *will exceed the sum of all the terms which follow.*

The inequality

$$a_0x^n > a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x' + a_n$$

is satisfied by the following :—

$$a_0x^n > a_k(x^{n-1} + x^{n-2} + \ldots + x + 1),$$

where a_k is the greatest among the coefficients

$$a_1, a_2, \ldots a_{n-1}, a_n,$$

without regard to sign.

This leads, summing the geometric series, to the condition

$$a_0x^n > a_k\frac{x^n - 1}{x - 1}, \quad \text{or} \quad x^n > \frac{a_k}{a_0(x-1)}(x^n - 1),$$

which is satisfied if $a_0(x-1)$ be > or = a_k,

or

$$x > \text{or} = \frac{a_k}{a_0} + 1;$$

which proves the theorem.

Theorem.—*If in the polynomial*

$$a_0x^n + a_1x^{n-1} + \ldots + a_{n-1}x + a_n$$

the value $\dfrac{a_n}{a_n + a_k}$, *or any smaller value, be substituted for* x, *where* a_k *is the greatest coefficient exclusive of* a_n, *the term* a_n *will be numerically greater than the sum of all the others.*

1

To prove this, let $x = \frac{1}{y}$; then by the theorem of Art. 4, a_k being now the greatest among the coefficients $a_0, a_1, \ldots a_{n-1}$, without regard to sign, the value $\frac{a_k}{a_n} + 1$, or any greater value of y, will make

$$a_n y^n > a_{n-1} y^{n-1} + a_{n-2} y^{n-2} + \ldots + a_1 y + a_0,$$

that is, $$a_n > a_{n-1}\frac{1}{y} + a_{n-2}\frac{1}{y^2} + \ldots a_0 \frac{1}{y^n};$$

hence the value $\frac{a_n}{a_n + a_k}$, or any less value of x, will make

$$a_n > a_{n-1}x + a_{n-2}x^2 + \ldots + a_0 x^n.$$

6. **Change of form of a Polynomial corresponding to an increase or diminution of the Variable. Derived Functions.**—We shall now examine the form assumed by the polynomial when $x+h$ is substituted for x.

If we suppose h essentially positive, the resulting form will correspond to an increase of the variable; and by changing the sign of h in the result we obtain the form corresponding to a diminution of x.

When x is changed to $x+h$, $f(x)$ becomes $f(x+h)$, or

$$a_0(x+h)^n + a_1(x+h)^{n-1} + a_2(x+h)^{n-2} + \ldots + a_{n-2}(x+h)^2 + a_{n-1}(x+h) + a_n.$$

Let each term of this expression be expanded by the binomial theorem, and the result arranged according to ascending powers of h. We then have

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-2}x^2 + a_{n-1}x + a_n$$

$$+ h\left\{na_0x^{n-1} + (n-1)a_1x^{n-2} + (n-2)a_2x^{n-3} + \ldots + 2a_{n-2}x + a_{n-1}\right\}$$

$$+ \frac{h^2}{1.2}\left\{n(n-1)a_0x^{n-2} + (n-1)(n-2)a_1x^{n-3} + \ldots + 2a_{n-2}\right\}$$

$$+ \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$

$$+ \frac{h^n}{1.2.3\ldots n}\left\{n.n-1\ldots 2.1\right\}a_0.$$

It will be observed that the part of this expression independent of h is $f(x)$: a result which is obvious *à priori;* and that the subsequent coefficients of the different powers of h are functions of x of degrees diminishing by unity. It will further be observed that the coefficient of h may be derived from $f(x)$ in the following manner :—Let each term in $f(x)$ be multiplied by the exponent of x in that term, and let the exponent of x in the term be diminished by unity, the sign being retained; the sum of all the terms of $f(x)$ treated in this way will constitute a polynomial of dimensions one degree lower than those of $f(x)$. This polynomial is called the *first derived function* of $f(x)$. It is usual to represent this function by the notation $f'(x)$.

The coefficient of $\frac{h^2}{1\,.\,2}$ may be derived from $f'(x)$ by a process the same as that employed in deriving $f'(x)$ from $f(x)$, or by the operation twice performed on $f(x)$. It is represented by $f''(x)$. Thus $f''(x)$ is called the *second derived function* of $f(x)$; and in like manner the succeeding coefficients may all be derived by successive operations of this character; so that, employing the notation here indicated, we may write the result thus :—

$$f(x+h) = f(x) + f'(x)h + \frac{f''(x)}{1\,.\,2}h^2 + \frac{f'''(x)}{1\,.\,2\,.\,3}h^3 + \ldots + a_0 h^n.$$

We may observe that, since the interchange of x and h does not alter $f(x+h)$, the expansion may also be written in the form

$$f(x+h) = f(h) + f'(h)x + \frac{f''(h)}{1\,.\,2}x^2 + \frac{f'''(h)}{1\,.\,2\,.\,3}x^3 + \ldots + a_0 x^n.$$

EXAMPLES.

1. Find the quotient and remainder when $3x^4 - 5x^3 + 10x^2 + 11x - 61$ is divided by $x - 3$.

The calculation is arranged as follows:—

$$\begin{array}{rrrrr} 3 & -5 & 10 & 11 & -61. \\ & 9 & 12 & 66 & 231. \\ \hline & 4 & 22 & 77 & 170. \end{array}$$

Thus the quotient is $3x^3 + 4x^2 + 22x + 77$, and the remainder 170.

2. Find the quotient and remainder when $x^3 + 5x^2 + 3x + 2$ is divided by $x - 1$.

Ans. $Q = x^2 + 6x + 9$, $R = 11$.

3. Find Q and R when $x^5 - 4x^4 + 7x^3 - 11x - 13$ is divided by $x - 5$.

[N. B.—When any term in a polynomial is absent, care must be taken to supply the place of its coefficient by zero in writing down the coefficients of $f(x)$. In this example, therefore, the series in the first line will be

$$1 \quad -4 \quad 7 \quad 0 \quad -11 \quad -13.]$$

Ans. $Q = x^4 + x^3 + 12x^2 + 60x + 289$, $R = 1432$.

4. Find the quotient and remainder when $x^9 + 3x^7 - 15x^2 + 2$ is divided by $x - 2$.

Ans. $Q = x^8 + 2x^7 + 7x^6 + 14x^5 + 28x^4 + 56x^3 + 112x^2 + 209x + 418$, $R = 838$.

5. Find the quotient and remainder when $x^5 + x^2 - 10x + 113$ is divided by $x + 4$.

Ans. $Q = x^4 - 4x^3 + 16x^2 - 63x + 242$; $R = -855$.

For example, the result of substituting 3 for x in the polynomial of Ex. 1, Art. 8, viz.,

$$3x^4 - 5x^3 + 10x^2 + 11x - 61,$$

is 170, this being the remainder after division by $x - 3$. The student can verify this by actual substitution.

Again, the result of substituting -4 for x in

$$x^5 + x^2 - 10x + 113$$

is -855, as appears from Ex. 5, Art. 8. We saw in Art. 7 that as x receives a continuous series of values increasing from $-\infty$ to $+\infty$, $f(x)$ will pass through a corresponding continuous series. If we substitute in succession for x, in a polynomial whose coefficients are given numbers, a series of numbers such as

$$-\infty, \ldots -3, -2, -1, \quad 0, \quad 1, \quad 2, \quad 3, \ldots +\infty,$$

and calculate, and note down, the corresponding values of $f(x)$, the process may be called the *tabulation of the function*.

1. Tabulate the trinomial $2x^2 + x - 6$, for the values of x

$$-4, -3, -2, -1, \quad 0, \quad 1, \quad 2, \quad 3, \quad 4.$$

Values of x,	-4	-3	-2	-1	0	1	2	3	4
Values of $f(x)$,	22	9	0	-5	-6	-3	4	15	30

2. Tabulate the polynomial $10x^3 - 17x^2 + x + 6$ for the values of x

$$-4, -3, -2, -1, \quad 0, \quad 1, \quad 2, \quad 3, \quad 4.$$

Values of x,	-4	-3	-2	-1	0	1	2	3	4
Values of $f(x)$	-910	-420	-144	-22	6	0	20	126	378

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Pollachi Main Road, Eachanari (Po),
Coimbatore –641 021

Subject: THEORY OF EQUATIONS **Subject Code: 17MMU202**

Class : I B.Sc Mathematics **Semester : II**

UNIT -I

PART A (20x1=20 Marks)

(Question Nos. 1 to 20 Online Examinations)

Possible Questions

Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer
Let $f(x) = x^2 - 7x - 6$. Then $f(-1) =$	0	1	2	3	
Let $f(x) = x^2 - 7x - 6$. Then $f(0) =$	-6	-7	-8	0	
Let $f(x) = x^2 - 7x + 6$. Then $f(1) =$	0	1	2	3	
Let $f(x) = x^2 + 7x + 6$. Then $f(1) =$	11	12	13	14	
Root of $x^4 + 2x^2 + 1 = 0$ is	i	-1	1	0	
Number of positive roots of $x^4 + 2x^2 + 1 = 0$ is	0	1	2	3	
Number of negative roots of $x^4 + 2x^2 + 1 = 0$ is	0	1	2	3	
Number of imaginary roots of $x^4 + 2x^2 + 1 = 0$ is	1	2	3	4	
Number of terms involving in a complete equation of degree n is	n	n+1	n+2	n+2	

Number of terms involving in a complete equation of degree 100 is	100	101	102	103	
$x^4 + 2x^2 + 1 = 0$ is a ----- equation	complete	incomplete	algebraical	all the above	
$x^4 + 2x^2 + 1 = 0$ is a ----- equation	complete	numerical	algebraical	all the above	
$ax^4 + bx^2 + c = 0$ is a ----- equation	complete	numerical	algebraical	all the above	
Number of terms involving in an incomplete equation of degree n is -------- n	less than	less than or equal	greater than	greater than or equal	
Number of terms involving in an incomplete equation of degree 100 is ----- 100	less than	less than or equal	greater than	greater than or equal	
$x^4 + 2x^2 + 1$ is a ----- polynomial	quadratic	biquadratic	cubic	quintic	
$x^4 + 2x^2 + 1$ is a ----- polynomial	quadratic	biquadratic	cubic	quintic	
$x^5 + 2x^2 + 1$ is a ----- polynomial	quadratic	biquadratic	cubic	quintic	
$x^2 + 2x + 1$ is a ----- polynomial	quadratic $\int_c \frac{f(z)}{z-a}\,dz$	biquadratic	cubic $\int_c \frac{f(z)}{z-a}\,dz$	quintic	$\int_c \frac{f(z)}{z-a}\,dz$
$x^4 + 2x^2 + 1 = 0$ is a ----- equation	quadratic	biquadratic	cubic	quintic	
$x^4 + 2x^2 + 1 = 0$ is a ----- equation	quadratic	biquadratic	cubic	quintic	
$x^5 + 2x^2 + 1 = 0$ is a ----- equation	quadratic	biquadratic	cubic	quintic	
$x^2 + 2x + 1 = 0$ is a ----- equation	quadratic	biquadratic	cubic	quintic	
First derived function of 4^{th} degree polynomial is a ---- polynomial	quadratic	biquadratic	cubic	quintic	
First derived function of biquadratic polynomial is a ---- polynomial	quadratic	biquadratic	cubic	quintic	
First derived function of 5^{th} degree polynomial is a ---- polynomial	quadratic	biquadratic	cubic	quintic	
First derived function of quintic polynomial is a ---- polynomial	quadratic	biquadratic	cubic	quintic	

First derived function of 3rd degree polynomial is a ---- polynomial	quadratic	biquadratic	cubic	quintic	
The simple arc is also known as ….	multiple	Jordan	double	multiple	Jordan
The derivative of an analytic function is also …	analytic	continuous	derivative	bounded	continuous
The function $(z-i)^2$ have a zero i of order…..	2	1	0	3	2

2
-6
0
14
i
0
0
4
n+1

101
incomplet e
numerical
complete
less than or equal
less than or equal
biquadrati c
cubic
quintic
quadratic
biquadrati c
cubic
quintic
quadratic
cubic
cubic
biquadrati c
biquadrati c

quadratic

UNIT-II

SYLLABUS

General properties of equations: Theorems relating to the real roots of equations - Existence of a root in the general equation. Imaginary roots - Theorem determining the number of roots of an equation.

Descartes' rule of signs for positive roots - Descartes' rule of signs for negative roots - Use of Descartes' rule in proving the existence of imaginary roots - Theorem relating to the substitution of two given numbers for the variable

Theorem.—*If two real quantities a and b be substituted for the unknown quantity x in any polynomial $f(x)$, and if they furnish results having different signs, one plus and the other minus; then the equation $f(x) = 0$ must have at least one real root intermediate in value between a and b.*

This theorem is an immediate consequence of the property of the continuity of the function $f(x)$ established in Art. 7; for since $f(x)$ changes continuously from $f(a)$ to $f(b)$, i. e. passes through all the intermediate values, while x changes from a to b; and since one of these quantities, $f(a)$ or $f(b)$, is positive, and the other negative, it follows that for some value of x intermediate between a and b, $f(x)$ must attain the value zero which is intermediate between $f(a)$ and $f(b)$.

Corollary.—*If there exist no real quantity which, substituted for x, makes $f(x) = 0$, then $f(x)$ must be positive for every real value of x.*

13. **Theorem.**—*Every equation of an odd degree has at least one real root of a sign opposite to that of its last term.*

This is an immediate consequence of the theorem in the last Article. Substitute in succession $-\infty$, 0, ∞ for x in the polynomial $f(x)$. The results are, n being odd (see Art. 4),

$x = -\infty$, $f(x)$ is negative;

$x = 0$, sign of $f(x)$ is the same as that of a_n;

$x = +\infty$, $f(x)$ is positive.

If a_n is positive, the equation must have a real root between $-\infty$ and 0, *i.e.* a real negative root; and if a_n is negative, the equation must have a real root between 0 and ∞, *i.e.* a real positive root. The theorem is thus proved.

14. **Theorem.**—*Every equation of an even degree, whose last term is negative, has at least two real roots, one positive and the other negative.*

The results of substituting $-\infty$, 0, ∞ are in this case

$$\begin{array}{rc} -\infty, & +, \\ 0, & -, \\ +\infty, & +; \end{array}$$

hence there is a real root between $-\infty$ and 0, and another between 0 and $+\infty$; *i.e.*, there exist at least one real negative, and one real positive root.

In this simple instance we observe that, in the absence of any real values, there are two imaginary expressions which reduce the polynomial to zero. The general proposition of which this is a very particular illustration is, that *every rational integra equation*

$$a_0x^n + a_1x^{n-1} + a_2x^{n-1} + \ldots + a_{n-1}x + a_n = 0$$

must have a root of the form

$$\alpha + \beta\sqrt{-1},$$

α and β being real finite quantities. This proposition includes both real and imaginary roots, the former corresponding to the value $\beta = 0$.

16. **Theorem.**—*Every equation of n dimensions has n roots, and no more.*

We first observe that if any quantity h is a root of the equation $f(x) = 0$, then $f(x)$ is divisible by $x - h$ without a remainder. This is evident from Art. 9; for if $f(h) = 0$, *i.e.* if h is a root of $f(x) = 0$, R must be $= 0$.

The converse of this is also obviously true.

Let, now, the given equation be

$$f(x) = x^n + p_1x^{n-1} + p_2x^{n-2} + \ldots + p_{n-1}x + p_n = 0.$$

This equation must have a root, real or imaginary (see Art. 15), which we shall denote by the symbol a_1. Let the quotient, when $f(x)$ is divided by $x - a_1$, be $\phi_1(x)$; we have then the identical equation

$$f(x) \equiv (x - a_1)\,\phi_1(x).$$

Again, the equation $\phi_1(x)=0$, which is of $n-1$ dimensions, must have a root, which we represent by a_2. Let the quotient obtained by dividing $\phi_1(x)$ by $x-a_2$ be $\phi_2(x)$. Hence

$$\phi_1(x) = (x-a_2)\,\phi_2(x),$$

$$\text{and} \quad \therefore f(x) = (x-a_1)\,(x-a_2)\,\phi_2(x),$$

where $\phi_2(x)$ is of $n-2$ dimensions.

Proceeding in this manner, we prove that $f(x)$ consists of the product of n factors, each containing x in the first degree, and a numerical factor $\phi_n(x)$. Comparing the coefficients of x^n, it is plain that $\phi_n(x)=1$. Thus we prove the identical equation

$$f(x) \equiv (x-a_1)(x-a_2)(x-a_3)\,\ldots\ldots\,(x-a_{n-1})\,(x-a_n).$$

It is evident that the substitution of any one of the quantities $a_1, a_2, \ldots a_n$ for x in the right-hand member of this equation will reduce that member to zero, and will therefore reduce $f(x)$ to zero; that is to say, the equation $f(x)=0$ has for roots the n quantities $a_1,\ a_2, a_3 \ldots a_{n-1},\ a_n$. And it can have no other roots; for if any quantity other than one of the quantities $a_1,\ a_2, \ldots a_n$ be substituted in the right-hand member of the above equation, the factors will be all different from zero, and therefore the product cannot vanish.

Corollary.—*Two polynomials of the n^{th} degree cannot be equal to one another for more than n values of the variable without being completely identical.*

1. Find the equation whose roots are

$$-3, \quad -1, \quad 4, \quad 5.$$

Ans. $x^4-5x^3-13x^2+53x+60=0$.

2. The equation

$$x^4-6x^3+8x^2-17x+10=0$$

has a root 5; find the equation containing the remaining roots.

[N. B.—Use the method of division of Art. 8.]

Ans. $x^3-x^2+3x-2=0$.

3. Solve the equation

$$x^4-16x^3+86x^2-176x+105=0,$$

two roots being 1 and 7.

Ans. The other two roots are 3, 5.

4. Form the equation whose roots are

$$-\frac{3}{2}, \quad 3, \quad \frac{1}{7}.$$

Ans. $14x^3-23x^2-60x+9=0$.

5. Solve the cubic equation

$$x^3-1=0.$$

Here it is evident that $x=1$ satisfies the equation. Divide by $x-1$, and solve the resulting quadratic. The two roots are found to be

$$-\frac{1}{2}+\frac{1}{2}\sqrt{-3}, \quad -\frac{1}{2}-\frac{1}{2}\sqrt{-3}.$$

It can be easily shown that if either of these imaginary roots is squared, the other results. It is usual to represent these roots by ω and ω^2. They are called the two *imaginary cube roots of unity*. We have the identical equation

$$x^3-1 \equiv (x-1)(x-\omega)(x-\omega^2).$$

18. **Imaginary Roots enter Equations in Pairs.**—The proposition we have to prove may be stated as follows:—*If an equation $f(x) = 0$, whose coefficients are all real quantities, have for a root the imaginary expression $\alpha + \beta\sqrt{-1}$, it must also have for a root the conjugate imaginary expression $\alpha - \beta\sqrt{-1}$.*

The product

$$(x - \alpha - \beta\sqrt{-1})(x - \alpha + \beta\sqrt{-1}) = (x - \alpha)^2 + \beta^2.$$

Let the polynomial $f(x)$ be divided by the second member of this identity, and if possible let there be a remainder $Rx + R'$. We have then the identical equation

$$f(x) = \{(x - \alpha)^2 + \beta^2\}\, Q + Rx + R',$$

where Q is the quotient, of $n - 2$ dimensions in x. Substitute in this identity $a+\beta\sqrt{-1}$ for x. This, by hypothesis, causes $f(x)$ to vanish. It also causes $(x-a)^2+\beta^2$ to vanish. Hence

$$R\,(a+\beta\sqrt{-1})+R'=0.$$

From this we obtain the two equations

$$Ra+R'=0, \qquad R\beta=0,$$

since the real and imaginary parts cannot destroy one another; hence

$$R=0, \qquad R'=0.$$

Thus the remainder $Rx+R'$ vanishes; and, therefore, $f(x)$ is divisible without remainder by the product of the two factors

$$x-a-\beta\sqrt{-1}, \qquad x-a+\beta\sqrt{-1}.$$

The equation has, consequently, the root $a-\beta\sqrt{-1}$ as well as the root $a+\beta\sqrt{-1}$.

Thus the total number of imaginary roots in an equation with real coefficients will always be even; and every polynomial may be regarded as composed of real factors, each pair of imaginary roots producing a real quadratic factor, and each real root producing a real simple factor. The actual resolution of the polynomial into these factors constitutes the complete solution of the equation.

19. **Descartes' Rule of Signs—Positive Roots.**—This rule, which enables us, by the mere inspection of a given equation, to assign a superior limit to the number of its positive roots, may be enunciated as follows :—*No equation can have more positive roots than it has changes of sign from + to −, and from − to +, in the terms of its first member.*

We shall content ourselves for the present with the proof which is usually given, and which is more a verification than a general demonstration of this celebrated theorem of Descartes. It will be subsequently shown that this rule of Descartes, and other similar rules which were discovered by early investigators relative to the number of the positive, negative, and imaginary roots of equations, are immediate deductions from the more general theorems of Budan and Fourier.

Let the signs of a polynomial taken at random succeed each other in the following order :—

$$+ + - + - - - + + - + -.$$

20. **Descartes' Rule of Signs—Negative Roots.**—In order to give the most advantageous statement to Descartes' rule in the case of negative roots, we first prove that if $-x$ be substituted for x in the equation $f(x) = 0$, the resulting equation will have the same roots as the original except that their signs will be changed. This follows from the identical equation of Art. 16

$$f(x) \equiv (x - a_1)(x - a_2)(x - a_3) \ldots (x - a_n),$$

from which we derive

$$f(-x) \equiv (-1)^n (x + a_1)(x + a_2)(x + a_3) \ldots (x + a_n).$$

From this it is evident that the roots of $f(-x) = 0$ are

$$-a_1, \quad -a_2, \quad -a_3, \ldots \ldots -a_n.$$

Hence the negative roots of $f(x)$ are positive roots of $f(-x)$, and we may enunciate Descartes' rule for negative roots as follows:—*No equation can have a greater number of negative roots than there are changes of sign in the terms of the polynomial $f(-x)$.*

22. **Theorem.**—We shall close this chapter with the following theorem, which defines fully the conclusions which can be drawn as to the roots of an equation from the signs furnished by its first member when two given numbers are substituted for x:—*If two numbers a and b, substituted for x in the polynomial $f(x)$, give results with contrary signs, an odd number of real roots of the equation $f(x) = 0$ lies between them; and if they give results with the same sign, either no real root or an even number of real roots lies between them.*

We proceed to prove the first part of this proposition: the second is proved in an exactly similar manner.

Let the following m roots $\alpha_1, \alpha_2, \ldots . \alpha_m$, and no others, of the equation $f(x) = 0$ lie between the quantities a and b, of which, as usual, we take a to be the lesser.

Let $\phi(x)$ be the quotient when $f(x)$ is divided by the product of the m factors $(x - \alpha_1)(x - \alpha_2) \ldots . (x - \alpha_m)$. We have, then, the identical equation

$$f(x) = (x - \alpha_1)(x - \alpha_2) \ldots . (x - \alpha_m)\,\phi(x).$$

Putting in this successively $x = a$, $x = b$, we obtain

$$f(a) = (a - \alpha_1)(a - \alpha_2) \ldots . (a - \alpha_m)\,\phi(a),$$
$$f(b) = (b - \alpha_1)(b - \alpha_2) \ldots . (b - \alpha_m)\,\phi(b).$$

Now $\phi(a)$ and $\phi(b)$ have the same sign; for if they had different signs there would be, by Art. 12, one root at least of the equation $\phi(x) = 0$ between them. By hypothesis, $f(a)$ and $f(b)$ have different signs; hence the signs of the products

$$(a - \alpha_1)(a - \alpha_2) \ldots . (a - \alpha_m),$$
$$(b - \alpha_1)(b - \alpha_2) \ldots . (b - \alpha_m),$$

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Class : I B.Sc Mathematics | **Semester : II**

UNIT -II

PART A (20x1=20 Marks)

(Question Nos. 1 to 20 Online Examinations)

Possible Questions

Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer
Let $f(x) = x^2 - 7x - 6$ First derived function of cubic polynomial is a ---- polynomial	quadratic	biquadratic	cubic	quintic	quadratic
If first derived function of f(x) is positive at a then f(x) is ------ $f(-1) =$	decreasing	increasing	both increasing and decreasing	neither increasing nor decreasing	increasing
If first derived function of f(x) is negative at a then f(x) is ------	decreasing	increasing	both increasing and decreasing	neither increasing nor decreasing	decreasing
Let $f(x) = x^2 - 7x - 6$ second derived function of 4^{th} degree polynomial is a ---- polynomial	quadratic	biquadratic	cubic	quintic	quadratic
second derived function of biquadratic polynomial is a ---- polynomial $f(0) =$	quadratic	biquadratic	cubic	quintic	quadratic
second derived function of 5^{th} degree polynomial is a ---- polynomial	quadratic	biquadratic	cubic	quintic	cubic
Let $f(x) = x^2 - 7x + 6$ second derived function of quintic polynomial is a ---- polynomial	quadratic	biquadratic	cubic	quintic	cubic
second derived function of 6^{th} degree polynomial is a ---- polynomial $f(1) =$	quadratic	biquadratic	cubic	quintic	biquadratic
second derived function of cubic polynomial is a ---- polynomial	quadratic	biquadratic	cubic	quintic	biquadratic

$Let\ f(x) = x^2 + 7x + 6$ If the number of points in which the curve of f(x) cuts is less than the degree of the polynomial then f(x)=0 has ------- roots	real	positive real	negative real	imaginary	imaginary
If the number of points $f(b) =$ in which the curve of f(x) cuts is equal to the degree of the polynomial then f(x)=0 has no ------- roots	real	positive real	negative real	imaginary	imaginary
If the number of points in which the curve of f(x) cuts is less than the degree of the polynomial then f(x)=0 has ------ number of imaginary roots	odd	even	3	5	even
If the number of points in which the curve of f(x) cuts is equal to the degree of the polynomial then f(x)=0 has ------- number of imaginary roots	0	1	2	3	0
$x^4 + 2x^2 + 1 = 0$ If f(a) and f(b) are having opposite signs then f(x)=0 has ------ one real root between a and b	at least	at most	exactly	no	at least
If f(a) and f(b) are having opposite signs then f(x)=0 has at least ----- real root between a and b	1	2	3	4	1
$x^4 + 2x^2 + 1 = 0$ If f(a) and f(b) are having ------- signs then f(x)=0 has at least one real root between a and b	opposite	same	both opposite and same	neither opposite nor same	opposite
If f(a) and f(b) are having opposite signs then f(x) must attain the value ----- between a and b	0	1	2	3	0
$x^4 + 2x^2 + 1 = 0$ If f(a) and f(b) are having -------- signs then f(x) must attain the value 0 between a and b	opposite	same	both opposite and same	neither opposite nor same	opposite
If f(x)=0 has no real root then f (x) must be	negative	positive	both positive and negative	neither positive nor negative	positive
$x^4 + 2x^2 + 1 = 0$ If f(x)=0 has no ------ root then f (x) must be positive	real	both real and imaginary	neither real nor imaginary	imaginary	real
Every equation of an ----- degree has at least one real root of a sign opposite to that of its last term	odd	even	both odd and even	neither odd nor even	odd
Every equation of an odd degree has ----- one real root of a sign opposite to that of its last term	at least	at most	exactly	more than	at least
$x^4 + 2x^2 + 1 = 0$ Every equation of an odd degree has at least ----- real root of a sign opposite to that of its last term	1	2	3	4	1

$x^4 + 2x^2 + 1 = 0$ Every equation of an odd degree has at least one real root of a sign ------- to that of its last term	opposite	same	both opposite and same	neither opposite nor same	opposite
$ax^4 + bx^2 + c = 0$ Every equation of an odd degree has at least one real root of a sign opposite to that of its ---- term	first	second	third	last	last
------ equation of an odd degree has at least one real root of a sign opposite to that of its last term	Every	No	Few	Finite	Every
Every equation of an ------- degree whose last term is negative has at least two real roots	odd	even	both odd and even	neither odd nor even	even
$x^4 + 2x^2 + 1$ Every equation of an even degree whose ------ term is negative has at least two real roots	first	second	third	last	last
$x^4 + 2x^2 + 1$ Every equation of an even degree whose last term is ------- has at least two real roots	negative	positive	both positive and negative	neither positive nor negative	negative
$x^5 + 2x^3 + 1$ Every equation of an even degree whose last term is negative has -------- two real roots	at least	at most	exactly	more than	at least
$x^2 + 2x + 1$ Every equation of an even degree whose last term is negative has at least ---- real roots	2	3	4	5	2
$x^4 + 2x^2 + 1 = 0$ ------ equation of an even degree whose last term is negative has at least two real roots	Every	No	Few	Finite	Every
$x^4 + 2x^2 + 1 = 0$ Every equation of an ------- degree whose last term is negative has at least one positive and one negative root	odd	even	both odd and even	neither odd nor even	even
$x^5 + 2x^3 + 1 = 0$ Every equation of an even degree whose ------ term is negative has at least one positive and one negative root	first	second	third	last	last
$x^2 + 2x + 1 = 0$ Every equation of an even degree whose last term is ------- has at least one positive and one negative root	negative	positive	both positive and negative	neither positive nor negative	negative
Every equation of an even degree whose last term is negative has -------- one positive and one negative root	at least	at most	exactly	more than	at least
Every equation of an even degree whose last term is negative has at least ---- positive and one negative root	1	2	3	4	1
------- equation of an even degree whose last term is negative has at least one positive and one negative root	Every	No	Few	Finite	Every

Every equation of an even degree whose last term is negative has at least –one positive and ---- negative root	1	2	3	4	1
Every equation of n degree has ----- roots and no more	n	n+1	n+2	n+3	

n

UNIT-III

SYLLABUS

Relations between the roots and coefficients-Theorem - Applications of the theorem - Depression of an equation when a relation exists between two of its roots - The cube roots of unity - Symmetric functions of the roots – Examples - Theorems relating to symmetric functions - Examples

23. **Relations between the Roots and Coefficients.**— Taking for simplicity the coefficient of the highest power of x as unity, and representing, as in Art. 16, the n roots of an equation by $\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_n$, we have the following identity:—

$$x^n + p_1 x^{n-1} + p_2 x^{n-2} + \ldots + p_{n-1}x + p_n$$
$$= (x - \alpha_1)(x - \alpha_2)(x - \alpha_3) \ldots (x - \alpha_n). \quad (1)$$

When the factors of the second member of this identity are multiplied together, the product will consist, as is proved in elementary works on Algebra, of a highest term x^n; plus a term x^{n-1} multiplied by the factor

$$-(\alpha_1 + \alpha_2 + \alpha_3 + \ldots + \alpha_n),$$

i. e. the sum of the roots with their signs changed; plus a term x^{n-2} multiplied by the factor

$$\alpha_1\alpha_2 + \alpha_1\alpha_3 + \alpha_2\alpha_3 + \ldots + \alpha_{n-1}\alpha_n,$$

i. e. the sum of the products of the roots taken in pairs; plus a term x^{n-3} multiplied by the factor

$$-(\alpha_1\alpha_2\alpha_3 + \alpha_1\alpha_2\alpha_4 + \ldots + \alpha_{n-2}\alpha_{n-1}\alpha_n),$$

i. e. the sum of the products of the roots with their signs changed taken three by three; and so on. It is plain that the sign of each coefficient will be negative or positive according as the number of roots in each product is odd or even. The last term is

$$\pm \alpha_1\alpha_2\alpha_3 \ldots \alpha_{n-1}\alpha_n,$$

the sign being $-$ if n is odd, and $+$ if n is even. Equating the coefficients of x on each side of the identity (1), we have the following series of equations:—

$$\left.\begin{aligned}
p_1 &= -(a_1 + a_2 + a_3 + \ldots + a_n), \\
p_2 &= (a_1 a_2 + a_1 a_3 + a_2 a_3 + \ldots + a_{n-1} a_n), \\
p_3 &= -(a_1 a_2 a_3 + a_1 a_3 a_4 + \ldots + a_{n-2} a_{n-1} a_n), \\
&\quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
p_n &= (-1)^n a_1 a_2 a_3 \ldots a_{n-1} a_n,
\end{aligned}\right\} \qquad (2)$$

which furnish us with the following

Theorem.—*In every algebraic equation, the coefficient of whose highest term is unity, the coefficient p_1 of the second term with its sign changed is equal to the sum of the roots.*

The coefficient p_2 of the third term is equal to the sum of the products of the roots taken two by two.

The coefficient p_3 of the fourth term with its sign changed is equal to the sum of the products of the roots taken three by three; and so on, the signs of the coefficients being taken alternately negative and positive, and the number of roots multiplied together in each term of the corresponding function of the roots increasing by unity, till finally that function is reached which consists of the product of the n roots.

Cor. 1.—Every root of an equation is a divisor of the last term.

Cor. 2.—If the roots of an equation be all positive, the coefficients will be alternately positive and negative; and if the roots be all negative, the coefficients will be all positive. This is obvious from the equations (2) [cf. Arts. 19 and 20].

1. Solve the equation

$$x^3 - 5x^2 - 16x + 80 = 0,$$

the sum of two of its roots being equal to nothing.

Let the roots be α, β, γ. We have, then,

$$\alpha + \beta + \gamma = 5,$$
$$\alpha\beta + \alpha\gamma + \beta\gamma = -16,$$
$$\alpha\beta\gamma = -80.$$

Taking $\beta + \gamma = 0$, we have, from the first of these, $\alpha = 5$, and from either second or third we obtain $\beta\gamma = -16$. We find for β and γ the values 4 and -4. the three roots are 5, 4, -4.

2. Solve the equation

$$x^3 - 3x^2 + 4 = 0,$$

two of its roots being equal.

Let the roots be α, α, β. We have

$$2\alpha + \beta = 3,$$
$$\alpha^2 + 2\alpha\beta = 0,$$

from which we find $\alpha = 2$, and $\beta = -1$. The roots are 2, 2, -1.

3. The equation

$$x^4 + 4x^3 - 2x^2 - 12x + 9 = 0$$

has two pairs of equal roots; find them.

Let the roots be α, α, β, β; we have

$$2\alpha + 2\beta = -4,$$
$$\alpha^2 + \beta^2 + 4\alpha\beta = -2,$$

from which we obtain for α and β the values 1 and -3.

4. Solve the equation

$$x^3 - 9x^2 + 14x + 24 = 0,$$

two of whose roots are in the ratio of 3 to 2.

25. **Depression of an Equation when a relation exists between two of its Roots.**—The examples given under the preceding Article illustrate the use of the equations connecting the roots and coefficients in determining the roots in particular cases when known relations exist among them. The object of the present Article is to show that, in general, *if a relation of the form $\beta = \phi(\alpha)$ exist between two of the roots of an equation $f(x) = 0$, the equation may be depressed two dimensions.*

Let $\phi(x)$ be substituted for x in the identity

$$f(x) \equiv a_0x^n + a_1x^{n-1} + \ldots + a_n,$$

then $f(\phi(x)) \equiv a_0(\phi(x))^n + a_1(\phi(x))^{n-1} + \ldots + a_{n-1}\phi(x) + a_n.$

We represent, for convenience, the second member of this identity by $F(x)$. Substitute α for x, then

$$F(\alpha) = f(\phi(\alpha)) = f(\beta) = 0;$$

hence α satisfies the equation $F(x) = 0$, and it also satisfies the equation $f(x) = 0$; hence the polynomials $f(x)$ and $F(x)$ have a common measure $x - \alpha$; thus α can be determined, and from it $\phi(\alpha)$ or β, and the given equation can be depressed two dimensions.

Examples.

1. The equation

$$x^3 - 5x^2 - 4x + 20 = 0$$

has two roots whose difference $= 3$: find them.

Here $\beta - \alpha = 3$, $\beta = 3 + \alpha$; substitute $x + 3$ for x in the given polynomial $f(x)$; it becomes $x^3 + 4x^2 - 7x - 10$; the common measure of this and $f(x)$ is $x - 2$; from which $\alpha = 2$, $\beta = 5$; the third root is -2.

2. The equation

$$x^4 - 5x^3 + 11x^2 - 13x + 6 = 0$$

has two roots connected by the relation $2\beta + 3\alpha = 7$: find all the roots.

Ans. 1, 2, $1 \pm \sqrt{-2}$.

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UNIT -III

PART A (20x1=20 Marks)

(Question Nos. 1 to 20 Online Examinations)

Possible Questions

Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer
Every equation of ------- degree has n roots and no more	n	n+1	n+2	n+3	n
----- equation of n degree has n roots and no more	Every	No	Few	Finite	Every
The total number of imaginary roots in an equation with real coefficients will always be	odd	even	both odd and even	neither odd nor even	even
The total number of ----- roots in an equation with real coefficients will always be even	real	positive real	negative real	imaginary	imaginary
---- equation can have more positive roots than it has changes of signs in the terms of its first member	Every	No	Few	Finite	no
No equation can have more ----- roots than it has changes of signs in the terms of its first member	negative	positive	both positive and negative	neither positive nor negative	positive
$Let\ f(x) = x^2 - 7x + 6$ If the signs of the terms of an equation be all -------, it cannot have a positive root	negative	positive	both positive and negative	neither positive nor negative	positive
If the signs of the terms of an equation be all positive, it cannot have a ----- root	negative	positive	both positive and negative	neither positive nor negative	positive
If the signs of the terms of an equation be --- positive, it cannot have a positive root	all	no	few	finite	all
If the signs of the terms of an equation be all positive, then number of positive roots of an equation is	0	1	2	3	0

If the signs of the terms of an equation be all positive, it may have a ----- root	negative	positive	both positive and negative	neither positive nor negative	negative
If the signs of the terms of any complete equation be alternatively positive and negative cannot have a ------ root	negative	positive	both positive and negative	neither positive nor negative	negative
If an equation involve only even powers of x and if all the coefficients have positive signs, cannot have a --- root	real	positive real	negative real	all the above	all the above
$x^4 + 2x^2 + 1 = 0$ If an equation involve only ------ powers of x and if all the coefficients have positive signs, cannot have a real root	odd	even	both odd and even	neither odd nor even	even
If an equation involve only even powers of x and if all the coefficients have ------ signs, cannot have a real root	negative	positive	both positive and negative	neither positive nor negative	positive
$x^4 + 2x^2 + 1 = 0$ If an equation involve only odd powers of x and if all the coefficients have positive signs, then number of real root is	1	2	3	4	1
If an equation involve only odd powers of x and if all the coefficients have positive signs, the real root is	0	1	2	3	0
$x^4 + 2x^2 + 1 = 0$ If an equation involve only odd powers of x and if all the coefficients have ------- signs, has the root zero	negative	positive	both positive and negative	neither positive nor negative	positive
If an equation involve only ------ powers of x and if all the coefficients have positive signs, has the root zero	odd	even	both odd and even	neither odd nor even	odd
$x^4 + 2x^2 + 1 = 0$ x-a is a factor of $x^n - a^n$ if n is	2	3	4	6	3
x-a is a factor of $x^n - a^n$ if n is $x^4 + 2x^2 + 1 = 0$	2	4	6	odd	odd
$x^4 + 2x^2 + 1 = 0$ ----- is a factor of $x^n - a^n$ if n is odd $ax^4 + bx^2 + c = 0$	x-a	x	x+a	a	x-a
If n is even then number of real roots of $x^n - 1 = 0$ is	1	2	3	4	2
$x^4 + 2x^2 + 1$ If n is even then number of positive real roots of $x^n - 1 = 0$ is $x^4 + 2x^2 + 1$	1	2	3	4	1

If n is even then number of negative real roots of $x^n - 1 = 0$ is	1	2	3	4	1
If n is even then number of imaginary roots of $x^n - 1 = 0$ is	n	n-1	n-2	n-3	n-2
If n is even then one of the real root of $x^n - 1 = 0$ is	1	2	3	4	1
If n is even then one of the real root of $x^n - 1 = 0$ is	-1	-2	-3	-4	-1
If n is odd then number of real roots of $x^n - 1 = 0$ is	1	2	3	4	1
If n is odd then number of positive real roots of $x^n - 1 = 0$ is	1	2	3	4	1

UNIT-IV

SYLLABUS

Transformation of Equations: Transformation of equations - Roots with signs changed - Roots multiplied by a given quantity - Reciprocal roots and reciprocal equations - To increase or diminish the roots by a given quantity - Removal of terms - Binomial coefficients.

Solution of reciprocal and binomial equations: Reciprocal equations - Binomial equations. Propositions embracing their leading general Properties - The special roots of the equation - Solution of binomial equations by circular functions – Examples.

29. **Transformation of Equations.**—We can, without knowing the roots of an equation, transform it into another whose roots shall have certain assigned relations to those of the proposed. The utility of this process consists in the fact that the discussion of the transformed equation will often be more simple than that of the original. We proceed to explain the most important transformations of equations.

30. **Roots with Signs changed.**—To transform an equation into another whose roots are those of the given equation with contrary signs, let $\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_n$ be the roots of

$$x^n + p_1x^{n-1} + p_2x^{n-2} + \ldots p_{n-1}x + p_n = 0;$$

then

$$x^n + p_1x^{n-1} + p_2x^{n-2} + \ldots + p_{n-1}x + p_n \equiv (x-\alpha_1)(x-\alpha_2)\ldots(x-\alpha_n);$$

change x into $-y$; we have, then, whether n be even or odd,

$$y^n - p_1y^{n-1} + p_2y^{n-2} - \ldots \pm p_{n-1}y \mp p_n = (y+\alpha_1)(y+\alpha_2)\ldots(y+\alpha_n).$$

The polynomial in y equated to zero is an equation whose roots are $-\alpha_1, -\alpha_2, \ldots -\alpha_n$; and to effect the required transformation we have only to *change the signs of every alternate term of the given equation beginning with the second.*

31. **Roots Multiplied by a Given Quantity.**—To transform an equation whose roots are $a_1, a_2, \ldots a_n$ into another whose roots are $ma_1, ma_2, \ldots ma_n$, we change x into $\frac{y}{m}$ in the identity of the preceding article. We have then, after multiplication by m^n,

$$y^n + mp_1y^{n-1} + m^2p_2y^{n-2} + \ldots + m^{n-1}p_{n-1}y + m^np_n$$
$$= (y - ma_1)(y - ma_2) \ldots . (y - ma_n).$$

Hence, to multiply the roots of an equation by a given quantity m, *we have only to multiply the successive coefficients, beginning with the second, by* $m, m^2, m^3, \ldots m^n$.

The present transformation is useful for getting rid of the coefficient of the first term of an equation when it is not unity; and generally for removing fractional coefficients from an equation. If there is a coefficient a_0 of the first term, we form the equation whose roots are $a_0a_1, a_0a_2, \ldots a_0a_n$; the transformed equation will be divisible by a_0, and after such division the coefficient of x^n will be unity.

When there are fractional coefficients, we can get rid of them by multiplying the roots by a quantity m, which is the least common multiple of all the denominators of the fractions. In many cases, multiplication by a quantity less than the least common multiple will be sufficient for this purpose, as will appear in the following examples:—

1. Change the equation

$$3x^4 - 4x^3 + 4x^2 - 2x + 1 = 0$$

into another the coefficient of whose highest term will be unity. We multiply the roots by 3.

Ans. $x^4 - 4x^3 + 12x^2 - 18x + 27 = 0.$

32. **Reciprocal Roots.**—To transform an equation into one whose roots are the reciprocals of the roots of the proposed equation, we change x into $\frac{1}{y}$ in the identity of Art. 30. This gives, after certain easy reductions,

$$\frac{1}{y^n}+\frac{p_1}{y^{n-1}}+\frac{p_2}{y^{n-2}}+\ldots+\frac{p_{n-1}}{y}+p_n=\frac{p_n}{y^n}\left(y-\frac{1}{a_1}\right)\left(y-\frac{1}{a_2}\right)\ldots\left(y-\frac{1}{a_n}\right),$$

or

$$y^n+\frac{p_{n-1}}{p_n}y^{n-1}+\frac{p_{n-2}}{p_n}y^{n-2}+\ldots+\frac{p_1}{p_n}y+\frac{1}{p_n}=\left(y-\frac{1}{a_1}\right)\left(y-\frac{1}{a_2}\right)\ldots\left(y-\frac{1}{a_n}\right);$$

hence, if in the given equation we replace x by $\frac{1}{y}$, and multiply by y^n, the resulting polynomial in y equated to zero will have for roots $\frac{1}{a_1}, \frac{1}{a_2}, \ldots \frac{1}{a_n}$.

EXAMPLE

33. **Reciprocal Equations.**—There is a certain class of equations which remain unaltered when x is changed into $\frac{1}{x}$. These are called *reciprocal equations.* The conditions which must obtain among the coefficients of an equation in order that it should be one of this class are, from the preceding Article, plainly the following:—

$$\frac{p_{n-1}}{p_n} = p_1, \ \frac{p_{n-2}}{p_n} = p_2, \ \&c. \ldots \frac{p_1}{p_n} = p_{n-1}, \ \ \frac{1}{p_n} = p_n.$$

The last of these conditions gives $p_n^2 = 1$, or $p_n = \pm 1$. Reciprocal equations are divided into two classes, according as p_n is equal to $+1$, or to -1.

(1). In the first case

$$p_{n-1} = p_1, \ p_{n-2} = p_2, \ \ldots \ p_1 = p_{n-1};$$

and we have the *first class of reciprocal equations, in which the coefficients of the corresponding terms taken from the beginning and end are equal in magnitude and have the same signs.*

(2). In the second case, when $p_n = -1$,

$$p_{n-1} = -p_1, \quad p_{n-2} = -p_2, \ \&c. \ldots p_1 = -p_{n-1};$$

and we have the *second class of reciprocal equations, in which corresponding terms counting from the beginning and end are equal in magnitude but different in sign.* It is to be observed that in this case when the degree of the equation is even, say $n = 2m$, one of the conditions becomes $p_m = -p_m$, or $p_m = 0$; so that in reciprocal equations of the second class, whose degree is even, the middle term is absent.

1. Find the equation whose roots are those of

$$x^4 - 5x^3 + 7x^2 - 17x + 11 = 0,$$

each diminished by 4.

The operation is best exhibited as follows :—

1	-5	7	-17	11
	4	-4	12	-20
	-1	3	-5	**-9**
	4	12	60	
	3	15	**55**	
	4	28		
	7	**43**		
	4			
	11			

Here the first division of the given polynomial by $x-4$ gives the remainder -9 ($=A_4$), and the quotient $x^3 - x^2 + 3x - 5$ (cf. Art. 8). Dividing this again by $x-4$, we get the remainder 55 ($=A_3$), and the quotient $x^2+3x+15$. Dividing this again, we get the remainder 43 ($=A_2$), and quotient $x+7$; and dividing this we get $A_1 = 11$, and $A_0 = 1$; hence the required transformed equation is

$$y^4 + 11y^3 + 43y^2 + 55y - 9 = 0.$$

KARPAGAM ACADEMY OF HIGHER EDUCATION
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Subject: THEORY OF EQUATIONS **Subject Code: 17MMU202**

Class : I B.Sc Mathematics **Semester : II**

UNIT -IV

PART A (20x1=20 Marks)

(Question Nos. 1 to 20 Online Examinations)

Possible Questions

Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer
If n is odd then number of negative real roots of $x^n + 1 = 0$ is	0	1	2	3	1
If n is odd then number of imaginary roots of $x^n + 1 = 0$ is	n	n-1	n-2	n-3	n-1
If n is odd then one of the real root of $x^n + 1 = 0$ is	-1	-2	-3	-4	-1
The sum of products of the roots of $x^n + 1 = 0$ taken one at a time, is	0	1	2	3	0
The sum of products of the roots of $x^n + 1 = 0$ taken in pairs, is	0	1	2	3	0
The sum of products of the roots of $x^n + 1 = 0$ taken three by three, is	0	1	2	3	0
The sum of products of the roots of $x^n + 1 = 0$ taken four by four , is	0	1	2	3	0
The sum of products of the roots of $x^n + 1 = 0$ taken five by five, is	0	1	2	3	0
The sum of products of the roots of $x^n + 1 = 0$ taken six by six, is	0	1	2	3	0
If n is even, the products of roots of $x^n + 1 = 0$ is	0	-2	1	-1	1
If n is odd, the products of roots of $x^n + 1 = 0$ is	0	-2	1	-1	-1

UNIT-V

SYLLABUS

Algebraic Solution Of the Cubic and Biquadratic: On the algebraic solution of equations - The algebraic solution of the cubic equation - Application to numerical equations - Expression of the cubic as the difference of two cubes - Solution of the cubic by symmetric functions of the roots – Examples .

Properties of the Derived Functions: Graphic representation of the derived function - Theorem relating to the maxima and minima of a polynomial - Rolle's Theorem. Corollary - Constitution of the derived functions

(1). *First method of solution : by resolving into factors.* Let it be required to resolve the quadratic $x^2 + Px + Q$ into its simple factors. For this purpose we put it under the form

$$x^2 + Px + Q + \theta - \theta,$$

and determine θ so that

$$x^2 + Px + Q + \theta$$

may be a perfect square, *i.e.* we make

$$\theta + Q = \frac{P^2}{4}, \quad \text{or} \quad \theta = \frac{P^2 - 4Q}{4};$$

whence, putting for θ its value, we have

$$x^2 + Px + Q = \left(x + \frac{P}{2}\right)^2 - \left(0x + \frac{\sqrt{P^2 - 4Q}}{2}\right)^2.$$

Thus we have reduced the quadratic to the form $u^2 - v^2$; and its simple factors are $u + v$, and $u - v$.

Subsequently we shall reduce the cubic to the form

$$(lx + m)^3 - (l'x + m')^3, \quad \text{or} \quad u^3 - v^3,$$

and obtain its solution from the simple equations

$$u - v = 0, \quad u - \omega v = 0, \quad u - \omega^2 v = 0.$$

It will be shown also that the biquadratic may be reduced to either of the forms

$$(lx^2 + mx + n)^2 - (l'x^2 + m'x + n')^2,$$
$$(x^2 + px + q)(x^2 + p'x + q'),$$

by solving a cubic equation ; and, consequently, the solution of the biquadratic completed by solving two quadratics, viz., in the first case, $lx^2 + mx + n = \pm (l'x^2 + m'x + n')$; and in the second case, $x^2 + px + q = 0$, and $x^2 + p'x + q' = 0$.

(2). *Second method of solution : by assuming for a root a general form involving radicals.*

Assuming $x = p + \sqrt{q}$ to be a root of the equation $x^2 + Px + Q = 0$, and rationalizing the equation $x = p + \sqrt{q}$, we have

$$x^2 - 2px + p^2 - q = 0.$$

Now, if this equation be identical with $x^2 + Px + Q = 0$, we have

$$2p = -P, \quad p^2 - q = Q,$$

giving $$x = p + \sqrt{q} = \frac{-P \pm \sqrt{P^2 - 4Q}}{2},$$

which is the solution of the quadratic equation.

In the case of the cubic equation we shall find that

$$x = \sqrt[3]{p} + \frac{A}{\sqrt[3]{p}}$$

is the proper form to represent a root ; this formula giving precisely three values for x, in consequence of the manner in which the cube root enters into it.

In the case of the biquadratic equation we shall find that

$$\sqrt{p} + \sqrt{q} + \frac{A}{\sqrt{p}\sqrt{q}}, \quad \sqrt{q}\sqrt{r} + \sqrt{r}\sqrt{p} + \sqrt{p}\sqrt{q}$$

are forms which represent a root ; these formulas each giving four, and only four, values of x when the square roots receive their double signs.

(3). *Third method of solution : by symmetric functions of the roots.*

Consider the quadratic equation $x^2 + Px + Q = 0$, of which the roots are α, β.

Then
$$\alpha + \beta = -P,$$
$$\alpha\beta = Q.$$

If we attempt to determine α and β by these equations, we fall back on the original equation (see Art. 24); but if we could obtain a second equation between the roots and coefficients, of the form $l\alpha + m\beta = f(P, Q)$, we could easily find α and β by means of this equation and the equation $\alpha + \beta = -P$.

Now in the case of the quadratic there is no difficulty in finding the required equation; for, obviously,

$$(\alpha - \beta)^2 = P^2 - 4Q; \text{ and, therefore, } \alpha - \beta = \sqrt{P^2 - 4Q}.$$

In the case of the cubic equation $x^3 + Px^2 + Qx + R = 0$, we require *two* simple equations of the form

$$l\alpha + m\beta + n\gamma = f(P, Q, R),$$

in addition to the equation $\alpha + \beta + \gamma = -P$, to determine the roots α, β, γ. It will subsequently be proved that the functions

$$(\alpha + \omega\beta + \omega^2\gamma)^3, \quad (\alpha + \omega^2\beta + \omega\gamma)^3$$

may be expressed in terms of the coefficients by solving a *quadratic* equation; and when their values are known the roots of the cubic may be easily found.

In the case of the biquadratic equation

$$x^4 + Px^3 + Qx^2 + Rx + S = 0$$

we require *three* simple equations of the form

$$l\alpha + m\beta + n\gamma + r\delta = f(P, Q, R, S),$$

in addition to the equation

$$\alpha + \beta + \gamma + \delta = -P,$$

56. The Algebraic Solution of the Cubic Equation.—Let the general cubic equation

$$ax^3 + 3bx^2 + 3cx + d = 0$$

be put under the form

$$z^3 + 3Hz + G = 0,$$

where $z = ax + b$, $H = ac - b^2$, $G = a^2d - 3abc + 2b^3$ (see Art. 37).

To solve this equation, assume*

$$z = \sqrt[3]{p} + \sqrt[3]{q};$$

hence, cubing,

$$z^3 = p + q + 3\sqrt[3]{p}\,\sqrt[3]{q}\,(\sqrt[3]{p} + \sqrt[3]{q}),$$

therefore

$$z^3 - 3\sqrt[3]{p}\,\sqrt[3]{q}\,.\,z - (p + q) = 0.$$

Now, comparing coefficients, we have

$$\sqrt[3]{p}\,.\,\sqrt[3]{q} = -H, \quad p + q = -G;$$

from which equations we obtain

$$p = \tfrac{1}{2}(-G + \sqrt{G^2 + 4H^3}), \quad q = \tfrac{1}{2}(-G - \sqrt{G^2 + 4H^3});$$

58. **Expression of the Cubic as the Difference of two Cubes.**—Let the given cubic

$$ax^3 + 3bx^2 + 3cx + d = \phi(x)$$

be put under the form

$$z^3 + 3Hz + G,$$

where $z = ax + b$.

Now assume

$$z^3 + 3Hz + G = \frac{1}{\mu - \nu}\{\mu(z+\nu)^3 - \nu(z+\mu)^3\}, \qquad (1)$$

where μ and ν are quantities to be determined; the second side of this identity becomes, when reduced,

$$z^3 - 3\mu\nu z - \mu\nu(\mu+\nu).$$

Comparing coefficients,

$$\mu\nu = -H, \quad \mu\nu(\mu+\nu) = -G;$$

therefore

$$\mu + \nu = \frac{G}{H}, \quad \mu - \nu = \frac{a\sqrt{\Delta}}{H};$$

where $a^2\Delta = G^2 + 4H^3$, as in Art. 41;

also
$$(z+\mu)(z+\nu) = z^2 + \frac{G}{H}z - H. \qquad (2)$$

Whence, putting for z its value, $ax + b$, we have from (1)

$$a^3\phi(x) = \left(\frac{G + a\Delta^{\frac{1}{2}}}{2\Delta^{\frac{1}{2}}}\right)\left(ax + b + \frac{G - a\Delta^{\frac{1}{2}}}{2H}\right)^3 - \left(\frac{G - a\Delta^{\frac{1}{2}}}{2\Delta^{\frac{1}{2}}}\right)\left(ax + b + \frac{G + a\Delta^{\frac{1}{2}}}{2H}\right)^3,$$

which is the required expression of $\phi(x)$ as the difference of two cubes.

The function (2), when transformed and reduced, becomes

$$\frac{a^2}{H}\{(ac - b^2)x^2 + (ad - bc)x + (bd - c^2)\},$$

which contains the two factors $ax + b + \mu$, $ax + b + \nu$.

The expression of the roots of this quadratic in terms of the roots of the given cubic may be seen on referring to Ex. 23, p. 57.

60. **Homographic Relation between two Roots of a Cubic.**—Before proceeding to the discussion of the biquadratic we prove the following important proposition relative to the cubic :—

The roots of the cubic are connected in pairs by a homographic relation in terms of the coefficients.

Referring to Example 13, Art. 27, we have the relations

$$a_0^2\{(\beta-\gamma)^2+(\gamma-\alpha)^2+(\alpha-\beta)^2\}=18(a_1^2-a_0a_2),$$
$$a_0^2\{\alpha(\beta-\gamma)^2+\beta(\gamma-\alpha)^2+\gamma(\alpha-\beta)^2\}=9(a_0a_3-a_1a_2),$$
$$a_0^2\{\alpha^2(\beta-\gamma)^2+\beta^2(\gamma-\alpha)^2+\gamma^2(\alpha-\beta)^2\}=18(a_2^2-a_1a_3).$$

We adopt the notation

$$a_0a_2-a_1^2=H,\quad a_0a_3-a_1a_2=2H_1,\quad a_1a_3-a_2^2=H_2.$$

Now, multiplying the above equations by $\alpha\beta$, $-(\alpha+\beta)$, 1, respectively, and adding, since

$$\alpha^2-\alpha(\alpha+\beta)+\alpha\beta=0,\quad \beta^2-\beta(\alpha+\beta)+\alpha\beta=0,$$

we have

$$a_0^2(\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)^2=18\{H\alpha\beta+H_1(\alpha+\beta)+H_2\};$$

but

$$a_0^4(\beta-\gamma)^2(\gamma-\alpha)^2(\alpha-\beta)^2=-27\Delta=108(HH_2-H_1^2)$$

(see Art. 41); whence

$$\pm\sqrt{-\frac{\Delta}{3}}\left(\frac{\alpha-\beta}{2}\right)=H\alpha\beta+H_1(\alpha+\beta)+H_2,$$

and, therefore,

$$H\alpha\beta+\left(H_1+\frac{1}{2}\sqrt{-\frac{\Delta}{3}}\right)\alpha+\left(H_1-\frac{1}{2}\sqrt{-\frac{\Delta}{3}}\right)\beta+H_2=0,$$

which is the required homographic relation (see Art. 39).

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Subject: THEORY OF EQUATIONS | **Subject Code: 17MMU202**

Class : I B.Sc Mathematics | **Semester : II**

UNIT -V

PART A (20x1=20 Marks)

(Question Nos. 1 to 20 Online Examinations)

Possible Questions

Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer
The sum of products of the roots of $x^{100}+x^{99}+x^{98}+\cdots+1=0$ taken one at a time, is	99	100	0	-1	-1
The sum of products of the roots of $x^{100}+x^{99}+x^{98}+\cdots+1=0$ taken in pairs, is	99	100	-1	1	1
The sum of products of the roots of $x^{100}+x^{99}+x^{98}+\cdots+1=0$ taken three by three, is	99	100	1	-1	-1
The sum of products of the roots of $x^{100}+x^{99}+x^{98}+\cdots+1=0$ taken four by four ,is	99	100	1	-1	1
The sum of products of the roots of $x^{100}+x^{99}+x^{98}+\cdots+1=0$ taken in pairs, is	99	100	1	-1	-1
The absolute term of $x^{100}+x^{99}+x^{98}+\cdots+1=0$ is	99	100	1	-1	1
The leading coefficient of $x^{100}+x^{99}+x^{98}+\cdots+1=0$ is	99	100	1	-1	1
The number of terms involving in $x^{100}+x^{99}+x^{98}+\cdots+1=0$ is	99	100	101	102	101
The number of negative roots of $x^{100}-x^{99}+x^{98}-\cdots+1=0$ is	100	99	1	0	0
The number of real roots of $x^{100}+x^{98}+x^{96}+\cdots+1=0$ is	100	99	1	0	0
The number of imaginary roots of $x^{99}+x^{97}+x^{95}+\cdots+x=0$ is	98	99	1	0	98

Root of $x^{99} + x^{97} + x^{95} + \cdots + x = 0$ is	0	1	2	3	0
Every root of an equation is a divisor of the ------term	first	second	third	last	last
---- root of an equation is a divisor of the last term	Every	Not every	No	positive	Every
If all roots be negative, all coefficients will be	negative	positive	both positive and negative	neither positive nor negative	positive
If all roots be --------, all coefficients will be	negative	positive	both positive and negative	neither positive nor negative	negative

Reg. No..................

17MMU202

Karpagam Academy of Higher Education
Coimbatore-21
Department of Mathematics
Second Semester- I Internal test
Theory of equations

Date: 19.01.2018(AN) Time: 2 hours
Class: I B.Sc Mathematics Max Marks: 50

Answer ALL questions
PART - A ($20 \times 1 = 20$ marks)

1. The remainder when $3x^4 - 5x^3 + 10x^2 + 11x - 61$ is divided by $(x - 3)$, is

   a. 173 b. 172
   c. 171 d. 170

2. The quotient when $x^5 + x^2 - 10x + 113$ is divided by $(x + 4)$, is

   a. $x^4 - 4x^3 + 16x^2 - 63x + 242$
   b. $x^4 + 4x^3 - 16x^2 - 63x + 242$
   c. $x^4 - 4x^3 - 16x^2 - 63x + 242$
   d. $x^4 - 4x^3 - 16x^2 - 63x - 242$

3. The curve of $2x^2 + x + 2$ lies

   a. entirely above x-axis b. entirely below x-axis
   c. both a and b d. neither a nor b

4. Roots of $2x^2 + x - 6$ are

   a. -2 b. $\frac{3}{2}$
   c. both a and b d. neither a nor b.

5. Roots of $2x^2 + x + 2$ are

   a. real b. complex
   c. both a and b d. neither a nor b.

6. Roots of $2x^2 + x - 6$ are

   a. real b. complex
   c. both a and b d. neither a nor b.

7. The number of real zeros of the polynomial function $x^2 + 1$ is

   a. 2 b. 3
   c. 1 d. 0.

8. $x^{1001} + 1001 = 0$ has —— one real root

   a. atmost b. atleast
   c. exactly d. all the above.

9. $x^{1002} - 1001 = 0$ has —— two real roots

   a. atmost b. atleast
   c. exactly d. all the above.

10. $x^{102} - 102 = 0$ has ——positive root

    a. 1 b. 2
    c. 3 d. 0.

11. A polynomial equation in x of degree n always have

    a. n distinct roots b. n real roots
    c. n complex roots d. $n - 1$ complex roots

12. If $-2-3i$ is a root of the polynomial equation $p(x)=0$, then another root is

a. $-2+3i$ b. $2+3i$
c. $2-3i$ d. neither a or b

13. A zero of the polynomial x^3+2x-1 equals:

a. 1 b.i
c. $-i$ d. $1+i$

14. The equation $x^3+2x+3=0$ has

a. one positive real root b. three real roots
c. one negative real root d. four real roots

15. The graph of $x^{100}+2x-3$ intersect the x axis at

a. atleast two points b. one positive point
c. both a and b d. neither a nor b.

16. An even degree equation whose last term is positive, may have no —- root at all

a. real b. complex
c. both a and b d. neither a nor b.

17. An odd degree equation whose last term is positive has atleast one —- root at all

a. positive b. negative
c. both a and b d. neither a nor b.

18. Root of $x^3-1=0$ is

a. -1 b. $\frac{-1+i\sqrt{3}}{2}$
c. $\frac{-1-i\sqrt{3}}{2}$ d. all the above

19. If the signs of the terms of an equation be all positive, it cannot have —–

a. positive b. negative
c. both a and b d. neither a nor b.

20. If the signs of the terms of a complete equation be alternatively positive, it cannot have —–

a. positive b. negative
c. both a and b d. neither a nor b.

Part B-($3\times 2=6$ marks)

21. Find Q and R when $x^5-4x^4+7x^3-11x-13$ is divided by $x-5$

22. Form the equation whose roots are $-\frac{3}{2}, 3, \frac{1}{7}$

23. State Descartes rule of signs for positive roots

Part C-($3\times 8=24$ marks)

24. a) If in the polynomial $a_0x^n+a_1x^{n-1}+\cdots+a_{n-1}x+a_n$, the value $\frac{a_k}{a_0}+1$, or any greater value, be substituted for x, where a_k , is that one of the coefficients $a_1,\cdots,a_n$ whose numerical value is greatest, irrespective of sign, then prove that the term containing the highest power of x will exceed the sum of all the terms which follow.

OR

b) i Find Q and R when $x^9+3x^7-15x^2+2$ is divided by $x-2$

ii Tabulate polynomial $x^9+3x^7-15x^2+2$ for values of x

25. a) Prove that an odd degree equation has atleast one real root of a sign opposite to that of its last term

OR

b) Prove that every equation of n dimensions has n roots

26. a) i State and prove Descartes rule of signs for negative roots

ii Form a rational equation which shall have for two of is roots $1 + 5\sqrt{-1}$ and $5 - \sqrt{-1}$

OR

b) Prove that if two numbers a and b, substituted for x in tle polynomial $f(x)$, give results with contrary signs, an odd rnumber of real roots of the equation $f(x) = 0$ lies between them. Also prove that if they give results with the same sign, either no real root or an even number of real roots lies between them.