KARPAGAM ACADEMY OF HIGHER EDUCATION

(Deemed to be University Established Under Section 3 of UGC Act 1956)

Coimbatore – 641 021.

Semester – VI

16ITU603A **NUMERICAL METHODS** **3H – 3C**

Instruction Hours / week:L: 3 T: 0 P: 0 **Marks:**Int :**40** Ext : **60** Total: **100**

SCOPE

It exposes the students to study numerical techniques as powerful tools in scientific computing

OBJECTIVES

This course a deep knowledge to the learners to understand the basic concepts of numerical methods which utilize computers to solve Engineering problems that are not easily solved or even impossible to solve by analytical means.

UNIT I

Floating point representation and computer arithmetic – significant digits. Errors: round-off error – local truncation error – global truncation error – order of a method convergence and terminal conditions – efficient computations – bisection method – secant methods – Regula-Falsi method – Newton – Raphson method – Newton's method for solving non-linear systems.

UNIT II

Gauss elimination method (with row pivoting) and Gauss-Jordan method – Gauss thomas method for tridiagonal systems. Interative methods: Jacobi and Gauss-seidalinterative methods.

UNIT III

Interpolation: Lagrange's form and Newton's form – Finite difference operators – Gregory Newton forward and backward differences inerpolation Piecewise polynomial interpolation: Linear interpolation – Cubic spline interpolation (only method).

UNIT IV

Numerical differentiation: First derivatives and second order derivates – Richardson extrapolation. Numerical integration: traphezoid rule – simpson's rule (only method) – newton – Cotes open formulas.

UNIT V

Extrapolation method : Romberg integration- Cosine quadrature. Ordinary differential equations: Euler's method modified Euler's methods – Heun method and mid-point method – Runge-kutta second methods – Heun method without iteration – mid-point method and Ralston's method – classical 4^{th} order Runge-Kutta method.

Suggested Readings

1. Laurence V. Fausett (2012). Applied Numerical analysis using MATLAB. Pearson.
2. M.K.Jain, S.R.K.Iyengar, R.K.Jain (2012). Numerical methods for scientific and engineering computation. New Age International Publisher.
3. Steven C. Chopra (2010). Applied Numerical methods with MATLAB for Engineers and Scientists. Tata McGraw Hill.

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LECTURE PLAN
DEPARTMENT OF MATHEMATICS

Staff name: M.Indhumathi
Subject Name: Numerical Methods **Sub.Code:16ITU603A**
Semester: VI **Class: III B.Sc (IT)**

S.No	Lecture Duration Period	Topics to be Covered	Support Material/Page Nos
UNIT-I			
1.	1	Introduction: Floating Point Representation & Computer Arithmetic & Significant Digits. Explanation of Example problems.	T_1:Chap1 P.No:1-7
2.	1	Errors: Round off Errors & Local Truncation Error and Global Truncation Error.	T_1:Chap1 P.No:7-8
3.	1	Order of a Method Convergence and Problems on Terminal Conditions , Efficient Computations & Bisection Methods, Problems on Secant Methods & Regula – Falsi method.	T_1:Chap2 P.No:20-26,
4.	1	Newton-Raphson Method, Problems on Newton's Method for Solving non-Linear System.	T1:Chap2 P.No:26-29
5.	1	Recapitulation and Discussion of possible questions	
Total No of Hours Planned For Unit I=5			
UNIT-II			
1.	1	Problems on Gauss Elimination Method (With Row Pivoting).	T_1:Chap 3 P.No:114-118
2.	1	Problems Gauss – Jordan Method & Gauss Thomas Method for Tridiagonal Systems.	T_1:Chap 3 P.No:119-120,

3.	1	Interative Methods: Problems on Jacobi Interative Method.	T_1:Chap 3 P.No:146-150
4.	1	Problems on Gauss- Seidal Interative methods.	T_1:Chap 3 P.No:150-152
5.	1	Recapitulation and Discussion of possible questions	
Total No of Hours Planned For Unit II=5			
UNIT-III			
1.	1	Interpolation : Lagrange's Form & Newton's Form and Finite Difference Operator.	T_1:Chap 4 P.No:213-214
2.	1	Problems on Linear Interpolation.	T_1:Chap 4 P.No:214-216
3.	1	Problems on Gregory Newton Forward & Backward Differences, Interpolations Piecewise Polynomial Interpolation.	T_1:Chap 4 P.No:235-236
4.	1	Problems on Cubic Spline Interpolation (Only Method)	T_1:Chap 4 P.No:260-266
5.	1	Recapitulation and Discussion of possible questions	
Total No of Hours Planned For Unit III=5			
UNIT-IV			
1.	1	Numerical Differentiation: Problems on First Derivatives and Second Order Derivatives.	T_1:Chap 4 P.No:267-271
2.	1	Problems on Richardson Extrapolation.	R_2:Chap 19 P.No:455
3.	1	Numerical Integration: Problems on Traphezoidal Rule.	T_1:Chap 5 P.No:387
4.	1	Problems on Simpson's Rule (Only Method)	T_1:Chap 5 P.No:388-390
5.	1	Newton – Cotes Open Formulas.	R_2: Chap 17 P. No: 396-398
6.	1	Recapitulation and Discussion of possible questions	
Total No of Hours Planned For Unit IV=6			
UNIT-V			
1.	1	Extrapolation Method: Problems on Romberg Integration and Cosine Quadrature.	T_1:Chap 5 P.No:390-392

2.	1	Ordinary Differential Equations: Problems on Euler's Method Modified Euler's Methods.	T_1:Chap 6 P.No: 425-431
3.	1	Problems on Heun Method without Iteration	T_1:Chap 6 P.No:431-433
4.	1	Mid – Point Method & Runge- Kutta Second Methods.	T_1:Chap 6 P.No:440-442
5.	1	Problems on Ralston's Method & Classical Fourth Order Runge-Kutta Method.	T_1:Chap 6 P.No:451-456
6.	1	Recapitulation and Discussion of possible questions	
7.	1	Discuss on Previous ESE Question Papers	
8.	1	Discuss on Previous ESE Question Papers	
9.	1	Discuss on Previous ESE Question Papers	
Total No of Hours Planned for unit V=9			

Total Planned Hours	**30**

SUGGESTED READINGS

TEXT BOOK

1. M.K.Jain, S.R.K.Iyengar,R.K.Jain(2012). Numerical Methods for Scientific and engineering computation. New Age International Publisher.

REFERENCES

1. Laurence V. Fausett(2012). Applied Numerical analysis using MATLAB. Pearson.
2. Steven C. Chopra (2010). Applied Numerical methods with MATLAB for Engineers and Scientists. Tata McGraw Hill.

UNIT-I

SYLLABUS

Floating point representation and computer arithmetic – significant digits. Errors: round-off error – local truncation error – global truncation error – order of a method convergence and terminal conditions – efficient computations – bisection method – secant methods – Regula-Falsi method – Newton – Raphson method – Newton's method for solving non-linear systems.

UNIT - I

SOLUTION OF EQUATIONS

3.1 Bisection Method

After reading this unit , you should be able to:

1. *follow the algorithm of the bisection method of solving a nonlinear equation,*
2. *use the bisection method to solve examples of finding roots of a nonlinear equation, and*
3. *enumerate the advantages and disadvantages of the bisection method.*

Bisection method

Since the method is based on finding the root between two points, the method falls under the category of bracketing methods.

Since the root is bracketed between two points, x_ℓ and x_u, one can find the mid-point, x_m between x_ℓ and x_u. This gives us two new intervals

1. x_ℓ and x_m, and
2. x_m and x_u.

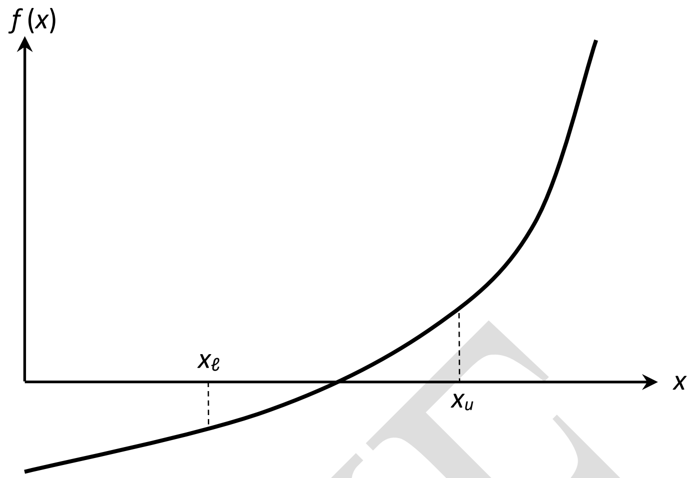

Figure 1 At least one root exists between the two points if the function is real, continuous, and changes sign.

Formula for false position (or) Regula falsi method:

$$x = \frac{a\,f(b) - bf(a)}{f(b) - f(a)}$$

Gauss Seidal method:

Let the rearranged form of a given set of equation be

$$x = \frac{1}{a_1}\left(d_1 - b_1 y - c_1 z\right)$$

$$y = \frac{1}{b_2}\left(d_2 - a_2 x - c_2 z\right)$$

$$z = \frac{1}{c_3}\left(d_3 - a_3 x - b_3 y\right)$$

we start with the initial values $y^{(0)}$, $z^{(0)}$ for y and z get $x^{(1)}$ from $x^{(1)} = \frac{1}{a_1}\left(d_1 - b_{1y}{}^{(0)} - c_{1z}{}^{(0)}\right)$

Condition for Gauss – Jacobi method of converges:

Let the given equation be

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

The sufficient condition is
$$|a_1| \geq |b_1| + |c_1|$$
$$|b_2| \geq |a_2| + |c_2|$$
$$|c_3| \geq |a_3| + |b_3|$$

Newton's algorithm for finding the Pth root of a Number N:

The Pth root of the a positive number N is the root of the equation.

$$x^p - N = 0$$
$$f(x) = x^p - N$$
$$f^1(x) = px^{p-1}$$

By Newton's algorithm

$$x_k = 1 - \frac{f(x)}{f'(x)}$$

$$x_k = \left(\frac{x_k^P - N}{Px_k^{p-1}}\right) + 1$$

$$= \frac{Px_k{}^p - x_k^P - N}{Px_{k^{p-1}}}$$

$$= \frac{(p-1)x_k^P + N}{Px_k^{p-1}}$$

Newton Raphson formula for cube root of a positive number k.

$$x = \sqrt[3]{k}$$
$$f(x) = x^3 - k = 0$$
$$f^1(x) = 3x^2$$
$$x_{n+1} = x_n - \frac{x_n^3 - k}{3x_n^2}$$
$$= \frac{1}{3}\left[2x_n + \frac{k}{x_n^2}\right]$$

Gauss – elimination method to solve Ax = B:

In this method the given system is transformed into an equivalent system with upper – triangular co efficient matrix i.e. a matrix in which all elements below the diagonal elements are zero which can be solved by back substitution.

Newtons – Raphson – formula $\sqrt{a}$ or $\sqrt{N}$ Or

x_{n+1} = $\frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$ **n = 0, 1, 2, …..**

Let

$x = \sqrt{a}, \quad x^2 - a = 0$

$$f(x) = x^2 - a$$

$$f^1(x) = 2x$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f^1(x_n)}$$

$$= x_n - \frac{x_n^2 - a}{2x_n}$$

$$= \frac{x_n^2 + a}{2x_n} = \left[x_n + \frac{a}{x_n}\right] \frac{1}{2}$$

Example:02

Evaluate $\sqrt{12}$ applying Newton formula.

Solution:

Let x = $\sqrt{12}$

$x^2 = 12 \Rightarrow x^2 - 12 = 0$

$f(x) = x^2 - 12$

f(3) = -ve,

f(4) = +ve

take $x_0 = 3$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f^1(x_0)} = 3 - \frac{f(3)}{f^1(3)} = 3.5$$

$$x_2 = x_1 - \frac{f(x_1)}{f^1(x_1)} = 3.5 - \frac{(3.5)^2 - 12}{2(3.5)} = 3.464$$

The root is 3.464

Numerical Examples:

01. Find the square root of 8. (by Newton – Raphson).

Solution:

Given N = 8 Clearly $2 < \sqrt{8} < 3$ taking $x_0 = 2.5$ we get

$$x_1 = \frac{1}{2}\left[x_0 + \frac{N}{x_0}\right] = \frac{1}{2}\left[2.5 + \frac{8}{2.5}\right] = 2.85$$

$$x_2 = \frac{1}{2}\left[x_1 + \frac{N}{x_1}\right] = \frac{1}{2}\left[2.85 + \frac{8}{2.85}\right] = 2.8285$$

$$x_3 = \frac{1}{2}\left[x_2 + \frac{N}{x_2}\right] = \frac{1}{2}\left[2.828 + \frac{8}{2.828}\right] = 2.8284$$

$$x_4 = \frac{1}{2}\left[x_3 + \frac{N}{x_3}\right] = \frac{1}{2}\left[2.8284 + \frac{8}{2.8284}\right] = 2.8284$$

$$\therefore \sqrt{8} = 2.8284$$

02. By applying Newton's method twice, find the real root near 2 of the equation $x^4 - 12x + 7 = 0$

Solution:

Let $f(x) = x^4 - 12x + 7$

$f(x) = 4x^3 - 12$

Put $x_0 = 2$, $f(x_0) = -1$

$f(x_0) = 20$

$$x_1 = x_0 - \frac{f(x_0)}{f^1(x_0)} = 2 - \frac{(-1)}{20} = \frac{41}{20} = 2.05$$

$$x_2 = x_1 - \frac{f(x_1)}{f^1(x_1)} = 2.05 - \frac{(2.05)^4 - 12(2.05) + 1}{4(2.05)^3 - 12}$$

$$= 2.6706$$

The root of the equation is 2.6706.

03. Find the approximately value of the root of equation $x^3 + x - 1 = 0$ near x = 1, by the method of false using the formula twice.

Solution:

$$f(x) = x^3 + x - 1$$
$$f(0.5) = -0.675,\ f(1) = 1$$
$$\text{Hence the root lies between } 0.5 \,\&\, 1$$
$$a = 0.5,\ b = 1$$
$$x_0 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = 0.64$$
$$f(0.64) = -0.097a < 0$$
$$\text{The root lies between } 0.64 \,\&\, 1$$
$$x_1 = \frac{(0.64) - (1)(-0.0979)}{1 - (-0.0979)} = 0.672$$

04.Use the iteration method to find a root of the equation x = ½ + sin x?

Solution:

Let f(x) = sin x – x + ½

f(1) = sin 1 – 1 + ½ = 0.84 – 0.5 = +ve

f(2) = sin 2 – 2 + ½ = 0.9.9 – 1.5 = -ve.

A root lies between 1 and 2. The given equation can be written as

x = sin x + ½ = ϕ(x)

$$\left|\phi^1(x)\right| = \left|\cos x\right| < 1 \text{ in } (1,2).$$

Hence the iteration method can be applied. Le the approximation be x_0 = 1.

The successive approximation are as follows:

$x_1 = \phi(x_0)$ = sin 1 + ½ = 0.8414 + 0.5 = 1.3414

$x_2 = \phi(x_1)$ = sin (1.3414) + ½ = 0.9738 + 0.5 = 1.4738

$x_3 = \phi(x_2) = \sin(1.4738) + ½ = 0.9952 + 0.5 = 1.4952$

$x_4 = \phi(x_3) = \sin(1.4952) + ½ = 0.9971 + 0.5 = 1.4971$

$x_5 = \phi(x_4) = \sin(1.4971) + ½ = 0.9972 + 0.5 = 1.4972$

Since x_4 and x_5 are almost equal the required root is 1.497.

05. If an approximate root of the equation x (l- logx) = 0.5 lies between 0.1 and 0.2 find the value of the root correct to three decimal places.

Solution:

Given f(x) = x(1- log x) -0.5

$f^1(x) = (1- \log x) + x\left(-\frac{1}{x}\right)$

$= -\log x$

f(0.1) = 0.1 [1-log (0.1)] – 0.5 = 0.1697 (-ve)

f(0.2) = 0.2 [1-log (0.2)] – 0.5 = 0.02188 (+ve)

$x_0 = 0.9$

$$x_1 = x_0 - \frac{f(x_0)}{f^1(x_0)}$$
$$= 0.2 - \frac{0.2(1-\log(0.2)-0.5}{-\log(0.2)}$$
$$= 0.2 - \frac{0.02188}{1.6094} = 0.1864$$

$$x_2 = x_1 - \frac{f(x_1)}{f^1(x_1)}$$
$$= 0.1864 - \frac{0.1864(1-\log(0.1864)-0.5}{-\log(0.1864)}$$
$$= 0.1864 + \frac{0.0004666}{1.6799} = 0.1866$$

$$x_3 = x_1 - \frac{f(x_2)}{f^1(x_2)}$$
$$= 0.1866 - \frac{0.1866(1 - \log(0.1866) - 0.5}{-\log(0.1866)}$$
$$= 0.1866$$

Hence the approximate root is 0.1866.

06. Find the root between (2, 3) of $x^3 - 2x - 5 = 0$ by regula falsi method

Solution:

Given $f(x) = x^3 - 2x - 5$

$f(2) = 8 - 4 - 5 = -1$

$f(3) = 27 - 6 - 5 = 16$

Let us take a = 2, b = 3

The first approximation to the root is x_1 and is given by

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{2f(3) - 3f(2)}{f(3) - f(2)}$$
$$= \frac{2 \times 16 - 3(-1)}{16 - (-1)} = 2.058$$
$$f(2.058) = (2.058)^3 - 2(2.058) - 5$$
$$= -0.4$$

The root lies between 2.058 and 3

Taking a = 2.058 and b = 3 we have the second approximation to the root given by

$$x_2 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$= \frac{(2.058) \times 16 - 3(-0.4)}{16 - (-0.4)}$$

$$= 2.081$$

$$f\ (2.081) = (2.081)^3 - 2(2.081) - 5$$

$$= -0.15$$

The root lies between 2.081 and 3

Take a = 2.081, b = 3

The third approximation to the root is given by

$$x_3 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$= \frac{2.081 \times 16 - 3(-0.15)}{16 - (-0.15)}$$

$$= 2.089$$

Now

$$f\ (2.089) = (2.089)^3 - 2(2.089) - 5$$

$$= -0.062$$

The root lies between 2.089 and 3

Take a = 2.089, b = 3

$$x_1 = \frac{2.089 \times 16 - 3(-0.062)}{16 - (-0.062)}$$

$$= 2.093$$

The required root is 2.09.

07. Find the approximate root of $xe^x = 3$ by Newton's Raphson method correct to three decimal places.

Solution:

Given $f(x) = xe^x - 3$

$f^1(x) = xe^x + e^x$

$f(1) = 1e^{-1} - 3 = 2.7182 - 3 = -0.2817$ (-ve)

$f(1.5) = 1.5e^{1.5} - 3 = 3.7223$ (+ve)

Here f(1) is –ve (Negative) and f(1.5) is +ve (positive). Therefore the root lies between 1 ad 1.5. Since the magnitude of f(1) < f(1.5) we can take the initial approximate $x_0 = 1$. The first approximation is

$$x_1 = x_0 \frac{f(x_0)}{f^1(x_0)}$$
$$= 1 - \frac{-0.2817}{5.4363} = 1.0518$$

The second approximation

$$x_2 = x_1 - \frac{f(x_1)}{f^1(x_1)}$$
$$= 1.0518 - \frac{-0.0111}{5.8739}$$
$$= 1.0499$$

The third approximation is

$$x_3 = x_2 - \frac{f(x_2)}{f^1(x_2)}$$
$$= 1.0499 - \frac{1.0499\, e^{1.0499} - 3}{1.0499\, e^{1.0499} + e^{1.04999}}$$
$$= 1.0499$$

Hence the root of xe^x is 1.0499

UNIT II

1. Solve the following system by Gaussian elimination method

$$x_1 - x_2 + x_3 = 1$$
$$-3x_1 + 2x_2 - 3x_3 = -6$$
$$2x_1 - 5x_2 + 4x_3 = 5$$

Solution:

Write the given system as in the matrix form

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ -3 & 2 & -3 & -6 \\ 2 & -5 & 4 & 5 \end{array}\right)$$

From the first column with non – zero component select the component with the large absolute value this component is called the pivot.

$$\rightarrow \left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ -3 & 2 & -3 & -6 \\ 2 & -5 & 4 & 5 \end{array}\right)$$

Rearrange the row to move the pivot to the top eg.

First column. Here we interchange the first and second row.

$$\rightarrow \begin{pmatrix} -3 & 2 & -3 & -6 \\ 1 & -1 & 1 & 1 \\ 2 & -5 & 4 & 5 \end{pmatrix}$$

Make the pivot as 1, by dividing the first row by the pivot.

$$= \left(\begin{array}{ccc|c} -1 & -2/3 & 1 & +2 \\ 1 & -1 & 1 & 1 \\ 2 & -5 & 4 & 5 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 1 & -2/3 & 1 & 2 \\ 0 & -1/3 & 0 & -1 \\ 0 & -11/3 & 2 & 1 \end{array}\right) \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 \to 2R_1 \end{array}$$

Delete the first row and first column and perform turn the above procedure.

$$= \begin{pmatrix} 1 & -2/3 & 1 & 2 \\ 0 & -1/3 & 0 & -1 \\ 0 & -11/3 & 2 & 1 \end{pmatrix} \leftarrow \text{New pivot}$$

$$= \begin{pmatrix} 1 & -2/3 & 1 & 2 \\ 0 & -11/3 & 2 & 1 \\ 0 & -1/3 & 0 & -1 \end{pmatrix} R_2 \leftrightarrow R_3$$

$$= \begin{pmatrix} 1 & -2/3 & 1 & 2 \\ 0 & 1 & -6/11 & -3/11 \\ 0 & 0 & -2/11 & -12/11 \end{pmatrix} R_2 \to R_2 \times \frac{-3}{11}$$

$$R_3 \to R_3 + \frac{1}{11} R_2$$

Delete first two row's and first two columns

$$\begin{pmatrix} 1 & -2/3 & 1 & 2 \\ 0 & 1 & -6/11 & -3/11 \\ 0 & 0 & -2/11 & -12/11 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2/3 & 1 & 2 \\ 0 & 1 & -6/11 & -3/11 \\ 0 & 0 & 1 & 6 \end{pmatrix}$$

$\boxed{x_3 = 6}$ $x_2 - 6/11 \; x_3 = -3/11$

$\boxed{x_2 = 3}$

$x_1 - 2/3 \; x_2 + x_3 = 2 \Rightarrow \boxed{x_1 = -2}$

09. Using the Gauss – Jordan method solve the following equation.

10x + y + z = 12

2x + 10y +z = 13

x + y + 5z = 7

Solution:

$$\text{Step 1} \Rightarrow \begin{pmatrix} 10 & 1 & 1 & 12 \\ 2 & 10 & 1 & 13 \\ 1 & 1 & 5 & 7 \end{pmatrix}$$

$$\text{Step 2} \Rightarrow \begin{pmatrix} 1 & \frac{1}{10} & \frac{1}{10} & 12/10 \\ 2 & 10 & 1 & 13 \\ 1 & 1 & 5 & 7 \end{pmatrix} R_1 \to \frac{R_1}{10}$$

$$\text{Step 3} \Rightarrow \begin{pmatrix} 1 & \frac{1}{10} & \frac{1}{10} & 12/10 \\ 0 & \frac{49}{5} & \frac{4}{5} & \frac{106}{10} \\ 0 & \frac{9}{10} & \frac{49}{10} & \frac{58}{10} \end{pmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\text{Step 4} \Rightarrow \begin{pmatrix} 1 & \frac{1}{10} & \frac{1}{10} & 12/10 \\ 0 & 1 & \frac{4}{49} & \frac{53}{49} \\ 0 & \frac{9}{10} & \frac{49}{10} & \frac{58}{10} \end{pmatrix} R_2 \to R_2 \div \frac{49}{5}$$

$$\text{Step 5} \Rightarrow \begin{pmatrix} 1 & 0 & 0.0918 & 1.0918 \\ 0 & 1 & \frac{4}{49} & \frac{53}{49} \\ 0 & 0 & 4.8265 & 4.8265 \end{pmatrix} \begin{matrix} R_3 \to R_3 - 9/10 R_2 \\ R_1 \to R_1 - 1/10 R_2 \end{matrix}$$

$$\text{Step 6} \Rightarrow \begin{pmatrix} 1 & 0 & 0.0918 & 1.0918 \\ 0 & 1 & \frac{4}{49} & \frac{53}{49} \\ 0 & 0 & 1 & 1 \end{pmatrix} R_2 \to R_3 \div 4.8265$$

$$\text{Step 7} \Rightarrow \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{matrix} R_2 \to R_2 - 0.0918 R_3 \\ R_2 \to R_2 - 4/49 R_3 \end{matrix}$$

The matrix finally reduced to the form

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\therefore x = y = z = 1$$

3. Solve the following equation using Jacobi iteration method:-

20x + y – 2z = 17

3x + 20y – z = - 18

2x – 3y + 20z = 25

Solution:

The above equation can be written as

$$x = \frac{1}{20}(17 - y + 2z)$$
$$y = \frac{1}{20}(-18 - 3x + z)$$
$$z = \frac{1}{20}(25 - 2x + 3y)$$

First approximation =

$$x_1 = \frac{1}{20}(17 - y_0 + 2z_0)$$
$$y_1 = \frac{1}{20}(-18 - 3x_0 + z_0)$$
$$z_1 = \frac{1}{20}(25 - 2x_0 + 3y_0)$$

Put $x_0 = y_0 = z_0 = 0 \Rightarrow x_1 = 0.85$, $y_1 = -0.9$, $z_1 = 1.25$

Second approximation

$$x_2 = \frac{1}{20}(17 - y_1 + 2z_1)$$
$$y_2 = \frac{1}{20}(-18 - 3x_1 + z_1)$$
$$z_2 = \frac{1}{20}(25 - 2x_1 + 3y_1)$$

$x_1 = 0.85$, $y_1 = -0.9$, $z_1 = 1.25$ we get

$x_2 = 1.02$ $y_2 = -0.965$ $z_2 = 1.1515$

Third approximation

$$x_3 = \frac{1}{20}(17 - y_2 + 2z_2)$$
$$y_3 = \frac{1}{20}(-18 - 3x_2 + z_2)$$
$$z_3 = \frac{1}{20}(25 - 2x_2 + 3y_2)$$

$x_2 = 1.02,\ \ y_2 = -0.965,\ z_2 = 1.1515$ we get

$$x_3 = 1.0134,\ y_3 = -0.9954,\ z_3 = 1.0032$$

$$x_4 = \frac{1}{20}(17 - y_3 + 2z_3)$$
$$y_4 = \frac{1}{20}(-18 - 3x_3 + z_3)$$
$$z_4 = \frac{1}{20}(25 - 2x_3 + 3y_3)$$

$x_3 = 1.0134,\ \ y_3 = -0.9954,\ z_3 = 1.0032$ we get

$$x_4 = 1.009,\ y_4 = -1.0018,\ z_4 = 0.9993$$

Fifth approximation

$x_5 = \frac{1}{20}(17 - y_4 + 2z_4)$

$y_5 = \frac{1}{20}(-18 - 3x_4 + z_4)$

$z_5 = \frac{1}{20}(25 - 2x_4 + 3y_4)$

$x_4 = 1.009, \quad y_4 = -1.0018, \; z_4 = 0.994$ we get

$x_5 = 1, \; y_5 = -1.0002, \; z_5 = 0.9996$

$\therefore \; x = 1, \quad y = -1, \quad z = 1$

11. Solve by Gauss – seidal method of iteration the equation.

$10x_1 + x_2 + x_3 = 12$

$2x_1 + 10x_2 + x_3 = 13$

$2x_1 + 2x_2 + 10x_3 = 14$

Solution:

From the above equation

$x_1 = \frac{1}{10}(12 - x_2 - x_3)$

$x_2 = \frac{1}{10}(13 - 2x_1 - x_3)$

$x_3 = \frac{1}{10}(14 - 2x_1 - 2x_2)$

Put $x_2 = x_3 = 0$ we get $x_1 = 1.2$, i.e $x_1^{(1)} = 1.2$

Put $x_2^{(1)} = \frac{1}{10}[13 - 2.4 - 0] = \frac{10.6}{10} = 1.06$

$x_1^{(1)} = 1.2 \quad x_2^{(1)} = 1.06$

$x_3^{(1)} = \frac{1}{10}[14 - 2.4 - 2.12] = 0.948$

$$x_1^{(2)} = \frac{1}{10}(12 - 1.06 - 0.948) = 0.992$$

$$x_2^{(2)} = \frac{1}{10}(13 - 2(0.9992) - 0.948) = 1.00536$$

$$x_3^{(2)} = \frac{1}{10}(14 - 2(0.9992) - 2(1.00536)) = 0.999098$$

Thus the iteration process is continued.

i	$x_1^{(i)}$	$x_2^{(i)}$	$x_3^{(i)}$
0	1.2000	0.000	0.000
1	1.2000	1.0600	0.9480
2	0.9992	1.0054	0.9991
3	0.9996	1.001	1.001
4	1.0000	1.0000	1.00
5	1.000	1.000	1.000

The exact values of the roots are

$X_1 = 1$, $x_2 = 1$, $x_3 = 1$.

12. Find the inverse of the matrix $\begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$ **using Gauss Jordan method.**

Solution:

$$\text{Let } A = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$$

$$X = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & 2_{22} & x_{33} \\ x_{31} & x_{32} & x_{33} \end{pmatrix}$$

Ax = I

$$\text{Step 1} \Rightarrow \begin{pmatrix} 2 & 1 & 1 & 1 & 0 & 0 \\ 3 & 2 & 3 & 0 & 1 & 0 \\ 1 & 4 & 9 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{Step 2} \Rightarrow \begin{pmatrix} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{3}{2} & \frac{-3}{2} & 1 & 0 \\ 0 & \frac{7}{2} & \frac{17}{2} & \frac{-1}{2} & 0 & 1 \end{pmatrix} \begin{matrix} R_2 \to R_2 - \frac{3}{2}R_1 \\ R_3 \to R_3 - \frac{1}{2}R_1 \end{matrix}$$

$$\text{Step 3} \Rightarrow \begin{pmatrix} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{3}{2} & \frac{-3}{2} & 1 & 0 \\ 0 & 0 & -2 & 10 & -7 & 1 \end{pmatrix} R_3 \to R_3 - 7R_2$$

$$\text{Step 4} \Rightarrow \begin{pmatrix} 2 & 0 & 0 & -6 & 5 & -1 \\ 0 & \frac{1}{2} & 0 & 6 & \frac{-17}{4} & \frac{3}{4} \\ 0 & 0 & -2 & 10 & -7 & 1 \end{pmatrix} R_1 \to R_1 + \frac{1}{2}R_3$$

$$\text{Step 5} \Rightarrow \begin{pmatrix} 2 & 0 & 0 & -6 & 5 & -1 \\ 0 & \frac{1}{2} & 0 & 6 & \frac{-17}{4} & \frac{3}{4} \\ 0 & 0 & -2 & 10 & -7 & 1 \end{pmatrix} R_1 \to R_1 + 2R_2$$

$$\text{Step 6} \Rightarrow \begin{pmatrix} 1 & 0 & 0 & -3 & \frac{5}{2} & \frac{-1}{2} \\ 0 & 1 & 0 & 12 & \frac{-17}{2} & \frac{3}{2} \\ 0 & 0 & 1 & -5 & \frac{7}{2} & \frac{1}{2} \end{pmatrix} \begin{matrix} R_1 \to R_{\frac{1}{2}} \\ R_2 \to R_2 x_2 \\ R_3 \to R_3(-Y_2) \end{matrix}$$

Hence the inverse of the given matrix

$$\begin{pmatrix} -3 & \frac{5}{2} & \frac{-1}{2} \\ 12 & \frac{-17}{2} & \frac{3}{2} \\ -5 & \frac{-7}{2} & \frac{-1}{2} \end{pmatrix}$$

Question
-------- Method is based on the repeated application of the intermediate value theorem.
The formula for Newton Raphson method is ------------------.
The order of convergence of Newton Raphson method is ----------
Graeffe's root squaring method is useful to find -----------------
The approximate value of the root of f(x) given by the bisection method is ----
In Newton Raphson method, the error at any stage is proportional to the ------- of the error in the previous stage.
The convergence of bisection method is -------.
The order of convergence of Regula falsi method may be assumed to ----------.
------------------ Method is also called method of tangents.
If f (x) contains some functions like exponential, trigonometric, logarithmic etc., then f (x) is called -------------- equation.
A polynomial in x of degree n is called an algebraic equation of degree n if -----
The method of false position is also known as ------------- method.
The Newton Rapson method fails if ---------------.
The bisection method is simple but ------------------.
________Method is also called as Bolzano method or interval having method.
The another name of Bisection method is ___________
The convergence of Bisection is Very__________
In Regula-Falsi method, to reduce the number of iterations we start with ________ interval
The rate of convergence in Newton-Raphson method is of order ________
Newton’s method is useful when the graph of the function crosses the x-axis is nearly ______.
If the initial approximation to the root is not given we can find any two values of x say a and bsuch that f (a) and f(
The Newton – Raphson method is also known as method of __________
If the derivative of f(x) = 0, then _________ method should be used.
The rate of convergence of Newton – Raphson method is _________
If f (a) and f (b) are of opposite signs the actual root lies between _________
The convergence of root in Regula-Falsi method is slower than ________
Regula-Falsi method is known as method of _________
______________method converges faster than Regula-Falsi method.
If f(x) is continuous in the interval (a, b) and if f (a) and f (b) are of opposite signs the equation f(x) = 0 has at leas
x^2 + 3x – 3 = 0 is a polynomial of order
Errors which are already present in the statement of the problem are called____ errors.
Rounding errors arise during_________
The other name for truncation error is _________ error.
Rounding errors arise from the process of _________ the numbers.
Absolute error is denoted by________
Truncation errors are caused by using _________ results.
Truncation errors are caused on replacing an infinite process by ________ one.
If a word length is 4 digits, then rounding off of 15.758 is
The actual root of the equation lies between a and b when f (a) and f (b) are of ____ signs.

Opt 1	**Opt 2**	**Opt 3**	**Opt 4**
Gauss Seidal	Bisection	Regula Falsi	Newton Raphson
$x_{n+1} = f(x_n) / f'(xn)$	$x_{n+1} = xn + f(x_n) / f'(xn)$	$x_{n+1} = xn - f(x_n) / f'(xn)$	$x_{n+1} = xn - f'(x_n) / f(xn)$
4	2	1	0
complex roots	single roots	unequal roots	polynomial roots
$x_0 = a + b$	$x_0 = f(a) + f(b)$	$x_0 = (a + b)/2$	$X_0 = (f(a) + f(b))/2$
cube	square	square root	equal
linear	quadratic	slow	fast
1	1.618	0	0.5
Gauss Seidal	Secant	Bisection	Newton Raphson
Algebraic	transcendental	numerical	polynomial
$f(x) = 0$	$f(x) = 1$	$f(x) < 1$	$f(x) > 1$
Gauss Seidal	Secant	Bisection	Regula falsi
$f'(x) = 0$	$f(x) = 0$	$f(x) = 1$	$f(x) \neq 0$
slowly divergent	fast convergent	slowly convergent	divergent
Bisection	false position	Newton raphson	Horner's
Bozano	Regula falsi	Newtons	Giraffes
slow	fast	moderate	normal
Small	large	equal	none
1	2	3	4
vertical	horizontal	close to zero	none
opposite	same	positive	negative
secant	tangent	iteration	interpolation
Newton – Raphson	Regula-Falsi	iteration	interpolation
quadratic	cubic	4	5
(a, b)	(0, a)	(0, b)	(0, 0)
Gauss – Elimination	Gauss – Jordan	Newton – Raphson	Power method
secant	tangent	chords	elimination
Newton – Raphson	Power method	elimination	interpolation
equation	function	root	polynomial
2	3	1	0
Inherent	Rounding	Truncation	Absolute
Solving	Algorithm	Truncation	Computation
Absolute	Rounding	Inherent	Algorithm
Truncating	Rounding off	Approximating	Solving
Ea	Er	Ep	Ex
Exact	True	Approximate	Real
Approximate	True	Finite	Exact
15.75	15.76	15.758	16
Opposite	same	negative	positive

Opt 5	Opt 6	Answer
		Bisection
		x_{n+1} =xn- f (x_n) /f '(xn)
		2
		polynomial roots
		$x_0 = (a + b)/ 2$
		square
		slow
		1.618
		Newton Raphson
		transcendental
		$f(x) = 0$
		Regula falsi
		$f'(x) = 0$
		slowly convergent
		Bisection
		Bozano
		slow
		Small
		2
		vertical
		opposite
		tangent
		Regula-Falsi
		quadratic
		(a, b)
		Newton – Raphson
		chords
		Newton – Raphson
		root
		2
		Inherent
		Computation
		Algorithm
		Rounding off
		Ea
		Approximate
		Finite
		15.76
		Opposite

UNIT-II

SYLLABUS

Gauss elimination method (with row pivoting) and Gauss-Jordan method – Gauss thomas method for tridiagonal systems. Interative methods: Jacobi and Gauss-seidalinterative methods.

Consider a system of n linear algebraic equations in n unknowns $x_1, x_2, \ldots, x_n$.

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \\ \cdot \quad \cdot \quad \cdots \quad \cdot \quad &\cdot \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n &= b_n \end{aligned} \qquad (1)$$

where a_{ij} $(i = 1,2,\ldots,n \;\&\; j = 1,2,\ldots,n)$ are the known coefficients, $b_i\,(i = 1,2,\ldots,n)$ are the known values and $x_i\,(i = 1,2,\ldots,n)$ are the unknowns to be determined.

Above system of linear equations may be represented at the matrix equations as follow

$$AX = b$$

where

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \cdot & \cdot & \ldots & \cdot \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} \quad and \quad b = \begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ \cdot \\ b_n \end{bmatrix}$$

The system of equations given above is said to be homogeneous if all the $b_i\,(i = 1,2,\ldots,n)$ vanish otherwise it is called as non-homogeneous system of equations.

By finding a solution of a system of equations we mean to obtain the value of $x_1, x_2, \ldots, x_n$ such that they satisfy the given equations and a solution vector of system of equations (1) is a vector X whose components constitute a solution of (1)

There are two types of numerical methods to solve the above system of equations

(I) Direct Methods: direct methods such as Gauss Elimination method, in such methods the amount of computation to get a solution can be specified in advance.

(II) Indirect or Iterative Methods: Such as Gauss-Siedel Methods, in such methods we start from a (possibly crude) approximation and improve it stepwise by repeatedly performing the same cycle of composition with changing data.

Gauss Elimination Method:

Gauss elimination method for solving linear systems is a systematic process of elimination that reduces the system of linear equations to triangular form. In Gauss elimination method, we proceed with the following steps.

Step 1: Elimination of x_1 from the second, third, . . . , n^{th} equations

In the first step of Gauss elimination method we eliminate x_1 from the second, third, . . . , n^{th} equations by subtracting suitable multiple of first equation from second, third, . . . , n^{th} equations.

The first equation is called the pivot equation and the coefficient of x_1 in the first equation i.e., $a_{11} \neq 0$ is called the pivot. Thus first step gives the new system as follows.

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\
a'_{22}x_2 + \ldots + a'_{2n}x_n &= b'_2 \\
. \quad . \quad . \quad . \quad . & \\
a'_{n2}x_2 + \ldots + a'_{nn}x_n &= b'_n
\end{aligned}
$$

Step 2: Elimination of x_2 from the third, . . . , n^{th} equation

In the second step of Gauss elimination method, we take the new second equation (which no longer contains x_1) as the pivot equation and use it to eliminate x_2 from the third, fourth, . . . , n^{th} equation.

In the third step we eliminate x_3 and in the fourth step we eliminate x_4 and so on. After (n-1) steps when the elimination is complete this process gives upper triangular system of the form

$$
\begin{aligned}
c_{11}x_1 + c_{12}x_2 + \ldots + c_{1n}x_n &= d_1 \\
c_{22}x_2 + \ldots + c_{2n}x_n &= d_2 \\
. \quad . \quad . \quad . \quad . & \\
c_{nn}x_n &= d_n
\end{aligned}
$$

Thus, the new system of equations is of upper triangular form that can be solved by the back substitution.

Example 7: Solve the system of equations

$$8x_2 + 2x_3 = -7$$
$$3x_1 + 5x_2 + 2x_3 = 8$$
$$6x_1 + 2x_2 + 8x_3 = 26$$

using Gauss elimination method.

Solution: Given system of equations is

$$8x_2 + 2x_3 = -7 \quad (1)$$

$$3x_1 + 5x_2 + 2x_3 = 8 \quad (2)$$

$$6x_1 + 2x_2 + 8x_3 = 26 \quad (3)$$

Since the coefficient of x_1 in first equation is zero therefore we must rearrange the equations by interchanging first equation to third i.e.,

$$6x_1 + 2x_2 + 8x_3 = 26 \quad (4)$$

$$3x_1 + 5x_2 + 2x_3 = 8 \quad (5)$$

$$8x_2 + 2x_3 = -7 \quad (6)$$

Step 1: Elimination of x_1:

On subtracting $\frac{1}{2}$ times of equation (4) from equation (5) we have

$$6x_1 + 2x_2 + 8x_3 = 26 \quad (7)$$

$$4x_2 - 2x_3 = -5 \quad (8)$$

$$8x_2 + 2x_3 = -7 \quad (9)$$

Step 2: Elimination of x_2:

On subtracting 2 times of equation (8) from equation (9) we have

$$6x_1 + 2x_2 + 8x_3 = 26 \quad (10)$$

$$4x_2 - 2x_3 = -5 \quad (11)$$

$$6x_3 = 3 \quad (12)$$

On solving equation (10), (11) and (12) by back substitution we have

$$x_3 = \frac{1}{2},\ x_2 = -1 \ \ and \ \ x_1 = 4 \ .$$

Thus, the required solution is

$$x_1 = 4,\ x_2 = -1 \ \ and \ \ x_3 = \frac{1}{2}.$$

Example Solve the system of equations

$$10x_1 - 7x_2 + 3x_3 + 5x_4 = 6$$
$$-6x_1 + 8x_2 - x_3 - 4x_4 = 5$$
$$3x_1 + x_2 + 4x_3 + 11x_4 = 2$$
$$5x_1 - 9x_2 - 2x_3 + 4x_4 = 7$$

using Gauss elimination method.

Solution: Given system of equations can be written as

$$x_1 - 0.7x_2 + 0.3x_3 + 0.5x_4 = 0.6 \quad (1)$$

$$-6x_1 + 8x_2 - x_3 - 4x_4 = 5 \quad (2)$$

$$3x_1 + x_2 + 4x_3 + 11x_4 = 2 \quad (3)$$

$$5x_1 - 9x_2 - 2x_3 + 4x_4 = 7 \quad (4)$$

Step 1: Elimination of x_1:

On subtracting (-6) times of equation (1) from equation (2), 3 times of equation (1) from equation (3) and 5 times of equation (1) from eqution (4) we have

$$x_1 - 0.7x_2 + 0.3x_3 + 0.5x_4 = 0.6 \quad (5)$$

$$3.8x_2 + 0.8x_3 - x_4 = 8.6 \quad (6)$$

$$3.1x_2 + 3.1x_3 + 9.5x_4 = 0.2 \quad (7)$$

$$-5.5x_2 - 3.5x_3 + 1.5x_4 = 4 \quad (8)$$

Step 2: Elimination of x_2:

In the above equations (6), (7) and (8) coefficient of x_2 is maximum (numerically) in equation (8) therefore interchanging the equation (6) and (8) After that x_2 is eliminated from equations (7) and (8) we have

$$x_1 - 0.7x_2 + 0.3x_3 + 0.5x_4 = 0.6 \quad (9)$$

$$x_2 + 0.6363x_3 - 0.27275x_4 = -0.72727 \quad (10)$$

$$-1.61818x_3 + 0.03636x_4 = 11.36364 \quad (11)$$

$$1.12727x_3 + 10.34545x_4 = 2.45455 \quad (12)$$

Step 3: Elimination of x_3:

On eliminating x_3 from equation (12) we have

$$x_1 - 0.7x_2 + 0.3x_3 + 0.5x_4 = 0.6 \quad (13)$$

$$x_2 + 0.6363x_3 - 0.27275x_4 = -0.72727 \quad (14)$$

$$x_3 - 0.02247x_4 = -7.02247 \quad (15)$$

$$10.3607947x_4 = 10.37079 \quad (16)$$

On solving equation (13), (14), (15) and (16) by back substitution we have

$x_4 = 1,\ x_3 = -7,\ x_2 = 4 \ \ and \ \ x_1 = 5$.

Thus, the required solution is

$x_1 = 5,\ x_2 = 4,\ x_3 = -7 \ \ and \ \ x_4 = 1$.

Gauss-Jordan Elimination Method:

M. Jordan in 1920 introduced another variant of the Gauss elimination method. In Gauss-Jordan method the coefficient matrix is reduced to a diagonal form rather than a triangular form in the Gauss elimination and we have the solution without further computations. Generally, this method is not used for the solution of a system of equations, because the reduction from the Gauss triangular to diagonal form requires more operations than back substitution does. Therefore this method is disadvantageous for solving system of equations. However it gives a simple method for finding the inverse of a given matrix by operating on the unit matrix I in the same way as the Gauss-Jordan method reducing A to I.

Example 11: Solve the system of equations

$$x_1 + 2x_2 + x_3 = 8$$
$$2x_1 + 3x_2 + 4x_3 = 20$$
$$4x_1 + 3x_2 + 2x_3 = 16$$

using Gauss elimination method.

Solution: Given system of equations is

$$x_1 + 2x_2 + x_3 = 8 \quad (1)$$

$$2x_1 + 3x_2 + 4x_3 = 20 \quad (2)$$

$$4x_1 + 3x_2 + 2x_3 = 16 \quad (3)$$

Step 1: Elimination of x_1:

On eliminating x_1 from equations (2) and (3) we have

$$x_1 + 2x_2 + x_3 = 8 \quad (4)$$

$$-x_2 + 2x_3 = 4 \quad (5)$$

$$-5x_2 - 2x_3 = -16 \quad (6)$$

Step 2: Elimination of x_2:

On eliminating x_2 from equations (4) and (6) we have

$$x_1 + 0.x_2 + 5x_3 = 16 \quad (7)$$

$$-x_2 + 2x_3 = -36 \quad (8)$$

$$-12x_3 = -36 \quad (9)$$

Step 2: Elimination of x_3:

On eliminating x_3 from equations (7) and (8) we have

$$x_1 = 1 \quad (7)$$

$$-x_2 = -2 \quad (8)$$

$$12x_3 = 36 \quad (9)$$

This gives

$$x_1 = 1,\ x_2 = 2 \ \ and \ \ x_3 = 3.$$

Example 12: Find the inverse of the coefficient matrix of the given system of equations

$$x_1 + x_2 + x_3 = 1$$
$$4x_1 + 3x_2 - 1x_3 = 6$$
$$3x_1 + 5x_2 + 3x_3 = 4$$

using Gauss elimination method with partial pivoting and hence solve the system of the equations..

Solution: Given system of equations is

$$AX = b \quad (1)$$

where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad and \quad b = \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix}$$

Using the augmented matrix [A | I], we have

$$[A \mid I] = \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 4 & 3 & -1 & 0 & 1 & 0 \\ 3 & 5 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$\approx \left[\begin{array}{ccc|ccc} 4 & 3 & -1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 3 & 5 & 3 & 0 & 0 & 1 \end{array}\right] \quad [R_1 \leftrightarrow R_2]$$

$$\approx \left[\begin{array}{ccc|ccc} 1 & \frac{3}{4} & -\frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 3 & 5 & 3 & 0 & 0 & 1 \end{array}\right] \quad R_1 \to \frac{1}{4} R_1$$

$$\approx \left[\begin{array}{ccc|ccc} 1 & \frac{3}{4} & -\frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{5}{4} & 1 & -\frac{1}{4} & 0 \\ 0 & \frac{11}{4} & \frac{15}{4} & 0 & -\frac{3}{4} & 1 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$$\approx \left[\begin{array}{ccc|ccc} 1 & \frac{3}{4} & -\frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & \frac{11}{4} & \frac{15}{4} & 0 & -\frac{3}{4} & 1 \\ 0 & \frac{1}{4} & \frac{5}{4} & 1 & -\frac{1}{4} & 0 \end{array}\right] \quad R_2 \leftrightarrow R_3$$

$$\approx\left[\begin{array}{ccc|ccc} 1 & \frac{3}{4} & -\frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & 1 & \frac{15}{11} & 0 & -\frac{3}{11} & \frac{4}{11} \\ 0 & \frac{1}{4} & \frac{5}{4} & 1 & -\frac{1}{4} & 0 \end{array}\right] \qquad R_2 \to \frac{4}{11}R_2$$

$$\approx\left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{14}{11} & 0 & \frac{5}{11} & -\frac{3}{11} \\ 0 & 1 & \frac{15}{11} & 0 & -\frac{3}{11} & \frac{4}{11} \\ 0 & 0 & \frac{10}{11} & 1 & -\frac{2}{11} & -\frac{1}{11} \end{array}\right] \qquad \begin{array}{l} R_1 \to R_1 - \frac{3}{4}R_2 \\ R_3 \to R_3 - \frac{1}{4}R_2 \end{array}$$

$$\approx\left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{14}{11} & 0 & \frac{5}{11} & -\frac{3}{11} \\ 0 & 1 & \frac{15}{11} & 0 & -\frac{3}{11} & \frac{4}{11} \\ 0 & 0 & 1 & \frac{11}{10} & -\frac{1}{5} & -\frac{1}{10} \end{array}\right] \qquad R_3 \to \frac{11}{10}R_3$$

$$\approx\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{7}{5} & \frac{1}{5} & -\frac{2}{5} \\ 0 & 1 & 0 & -\frac{3}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{11}{10} & -\frac{1}{5} & -\frac{1}{10} \end{array}\right] \qquad \begin{array}{l} R_1 \to R_1 + \frac{14}{11}R_3 \\ R_2 \to R_2 - \frac{15}{11}R_3 \end{array}$$

Thus the inverse of the coefficient matrix A is

$$A^{-1} = \begin{bmatrix} \frac{7}{5} & \frac{1}{5} & -\frac{2}{5} \\ -\frac{3}{2} & 0 & \frac{1}{2} \\ \frac{11}{10} & -\frac{1}{5} & -\frac{1}{10} \end{bmatrix}$$

Therefore the solution of the system of equation (1) is

$$X = A^{-1}b$$

$$\Rightarrow \quad X = \begin{bmatrix} \frac{7}{5} & \frac{1}{5} & -\frac{2}{5} \\ -\frac{3}{2} & 0 & \frac{1}{2} \\ \frac{11}{10} & -\frac{1}{5} & -\frac{1}{10} \end{bmatrix} \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}$$

Thus,

$$x_1 = 1, \; x_2 = \frac{1}{2} \; and \; x_3 = -\frac{1}{2}.$$

Jacobi-Iterative Method or Gauss-Jacobi Iterative Method:

Let us consider the system of simultaneous linear equation as

In the matrix form the solution of system of equations can be written as

$$\widehat{X} = HX + C$$

where H is called iteration matrix.

Thus, the (n+1)th approximation of iteration formula can be written as

$$X^{(n+1)} = HX^{(n)} + C.$$

This method is also called the method of simultaneous displacements.

Example 6: Solve the system of equations

$$\begin{aligned} 27x_1 + 6x_2 - x_3 &= 85 \\ 6x_1 + 15x_2 + 2x_3 &= 72 \\ x_1 + x_2 + 54x_3 &= 110 \end{aligned}$$

by Jacobi iterative method.

Solution: Given system of equations is

$$\begin{aligned} 27x_1 + 6x_2 - x_3 &= 85 \\ 6x_1 + 15x_2 + 2x_3 &= 72 \\ x_1 + x_2 + 54x_3 &= 110 \end{aligned} \qquad (1)$$

Rearranging the equations for the unknown with the largest coefficient in terms of the remaining unknowns,

$$\begin{aligned} x_1 &= \frac{1}{27}(85 - 6x_2 + x_3) \\ x_2 &= \frac{1}{15}(72 - 6x_1 - 2x_3) \\ x_3 &= \frac{1}{54}(110 - x_1 - x_2) \end{aligned} \qquad (2)$$

Starting with the approximations

$$x_1 = 0, \; x_2 = 0 \; and \; x_3 = 0$$

we have, first approximation

$$x_1^{(1)} = \frac{85}{27} = 3.15, \; x_2^{(1)} = \frac{72}{15} = 4.8 \; and \; x_3^{(1)} = \frac{110}{54} = 2.04$$

Second approximations to the solution,

$$x_1^{(2)} = \frac{1}{27}\left(85 - 6x_2^{(1)} + x_3^{(1)}\right)$$
$$x_2^{(2)} = \frac{1}{15}\left(72 - 6x_1^{(1)} - 2x_3^{(1)}\right) \qquad (3)$$
$$x_3^{(2)} = \frac{1}{54}\left(110 - x_1^{(1)} - x_2^{(1)}\right)$$

on putting the values of $x_1^{(1)},\ x_2^{(1)}\ \ and\ \ x_3^{(1)}$ in (3) we have

$$x_1^{(2)} = 2.16,\ x_2^{(2)} = 3.27\ \ and\ \ x_3^{(2)} = 1.89$$

Third approximations to the solution,

$$x_1^{(3)} = \frac{1}{27}\left(85 - 6x_2^{(2)} + x_3^{(2)}\right)$$
$$x_2^{(3)} = \frac{1}{15}\left(72 - 6x_1^{(2)} - 2x_3^{(2)}\right) \qquad (4)$$
$$x_3^{(3)} = \frac{1}{54}\left(110 - x_1^{(2)} - x_2^{(2)}\right)$$

on putting the values of $x_1^{(2)},\ x_2^{(2)}\ \ and\ \ x_3^{(2)}$ in (4) we have

$$x_1^{(3)} = 2.426,\ x_2^{(2)} = 3.572\ \ and\ \ x_3^{(2)} = 1.926$$

Fourth approximations to the solution,

$$x_1^{(4)} = \frac{1}{27}\left(85 - 6x_2^{(3)} + x_3^{(3)}\right)$$
$$x_2^{(4)} = \frac{1}{15}\left(72 - 6x_1^{(3)} - 2x_3^{(3)}\right) \qquad (5)$$
$$x_3^{(4)} = \frac{1}{54}\left(110 - x_1^{(3)} - x_2^{(3)}\right)$$

on putting the values of $x_1^{(3)},\ x_2^{(3)}\ \ and\ \ x_3^{(3)}$ from (5) we have

$$x_1^{(4)} = 2.4257,\ x_2^{(4)} = 3.5728\ \ and\ \ x_3^{(4)} = 1.9259$$

Since the values of $x_1^{(4)}, x_2^{(4)}\ and\ x_3^{(4)}$ are sufficiently close to $x_1^{(3)},\ x_2^{(3)}\ \ and\ \ x_3^{(3)}$ respectively. Hence the values $x_1^{(4)} = 2.4257,\ x_2^{(4)} = 3.5728\ \ and\ \ x_3^{(4)} = 1.9259$ can be considered as the solution of the given system.

Gauss-Seidel Iterative Method:

Gauss-Seidel iterative method is of great practical importance. This method is a modification to Jacobi iteration method.

Let us consider the system of simultaneous linear equation as

$$\begin{aligned} &a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ . \ . \ . + a_{1n}x_n = b_1 \\ &a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ . \ . \ . + a_{2n}x_n = b_2 \\ &- \quad - \quad - \quad - \quad - \\ &a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \ . \ . \ . + a_{nn}x_n = b_n \end{aligned} \qquad (1)$$

Let the diagonal coefficients a_{ii} in (1) do not vanish. If this condition is not satisfied then rearrange the equation to satisfy this condition.

Now rearrange the equations as

$$\begin{aligned} &x_1 = \frac{b_1}{a_{11}} - \frac{a_{12}}{a_{11}}x_2 - \frac{a_{13}}{a_{11}}x_3 - \ . \ . \ . - \frac{a_{1n}}{a_{11}}x_n \\ &x_2 = \frac{b_2}{a_{22}} - \frac{a_{21}}{a_{22}}x_1 - \frac{a_{23}}{a_{22}}x_3 - \ . \ . \ . - \frac{a_{2n}}{a_{22}}x_n \\ &- \quad - \quad - \quad - \quad - \\ &x_n = \frac{b_n}{a_{nn}} - \frac{a_{n1}}{a_{nn}}x_1 - \frac{a_{n2}}{a_{nn}}x_2 - \ . \ . \ . - \frac{a_{n(n-1)}}{a_{nn}}x_{(n-1)} \end{aligned} \qquad (2)$$

Let the first approximations to the unknowns $x_1, x_2, x_3, \ldots, x_n$ be $x_1^{(1)}, x_2^{(1)}, x_3^{(1)}, \ldots, x_n^{(1)}$. Then the second approximation system of next approximations is given by

$$x_1^{(2)} = \frac{b_1}{a_{11}} - \frac{a_{12}}{a_{11}} x_2^{(1)} - \frac{a_{13}}{a_{11}} x_3^{(1)} - \ . \ . \ . - \frac{a_{1n}}{a_{11}} x_n^{(1)}$$
$$x_2^{(2)} = \frac{b_2}{a_{22}} - \frac{a_{21}}{a_{22}} x_1^{(2)} - \frac{a_{23}}{a_{22}} x_3^{(1)} - \ . \ . \ . - \frac{a_{2n}}{a_{22}} x_n^{(1)}$$
$$x_3^{(2)} = \frac{b_2}{a_{33}} - \frac{a_{31}}{a_{33}} x_1^{(2)} - \frac{a_{32}}{a_{33}} x_2^{(2)} - \ . \ . \ . - \frac{a_{3n}}{a_{33}} x_n^{(1)} \qquad (3)$$
$$- \quad - \quad - \quad - \quad -$$
$$x_n^{(2)} = \frac{b_n}{a_{nn}} - \frac{a_{n1}}{a_{nn}} x_1^{(2)} - \frac{a_{n2}}{a_{nn}} x_2^{(2)} - \ . \ . \ . - \frac{a_{n(n-1)}}{a_{nn}} x_{(n-1)}^{(2)}$$

Continuing in this way let $x_1^{(n)}, x_2^{(n)}, x_3^{(n)}, \ldots, x_n^{(n)}$ be the nth approximations, then the system of (n+1)th approximations is given by

$$x_1^{(n+1)} = \frac{b_1}{a_{11}} - \frac{a_{12}}{a_{11}} x_2^{(n)} - \frac{a_{13}}{a_{11}} x_3^{(n)} - \ . \ . \ . - \frac{a_{1n}}{a_{11}} x_n^{(n)}$$
$$x_2^{(n+1)} = \frac{b_2}{a_{22}} - \frac{a_{21}}{a_{22}} x_1^{(n+1)} - \frac{a_{23}}{a_{22}} x_3^{(n)} - \ . \ . \ . - \frac{a_{2n}}{a_{22}} x_n^{(n)}$$
$$x_3^{(n+1)} = \frac{b_2}{a_{33}} - \frac{a_{31}}{a_{33}} x_1^{(n+1)} - \frac{a_{32}}{a_{33}} x_2^{(n+1)} - \ . \ . \ . - \frac{a_{3n}}{a_{33}} x_n^{(n)} \qquad (4)$$
$$- \quad - \quad - \quad - \quad -$$
$$x_n^{(n+1)} = \frac{b_n}{a_{nn}} - \frac{a_{n1}}{a_{nn}} x_1^{(n+1)} - \frac{a_{n2}}{a_{nn}} x_2^{(n+1)} - \ . \ . \ . - \frac{a_{n(n-1)}}{a_{nn}} x_{(n-1)}^{(n+1)}$$

This arrangement may also be written as

$$a_{11}x_1^{(n+1)} = -\sum_{i=2}^{n} a_{1i}x_i^{(n)} + b_1$$
$$a_{21}x_1^{(n+1)} + a_{22}x_2^{(n+1)} = -\sum_{i=3}^{n} a_{2i}x_i^{(n)} + b_2 \qquad (5)$$
$$- \quad - \quad - \quad - \quad -$$
$$a_{n1}x_1^{(n+1)} + a_{n2}x_2^{(n+1)} + \ . \ . \ . + a_{nn}x_n^{(n+1)} = b_n$$

In matrix notation, the system of equations (3) can be written as

$$(D+L)X^{(n+1)} = -UX^{(n)} + b$$

or
$$X^{(n+1)} = -(D+L)^{-1}UX^{(n)} + (D+L)^{-1}b$$
$$= HX^{(n)} + C, \ n = 0, 1, 2, \ldots \qquad (6)$$

where $H = -(D+L)^{-1}U$ *and* $C = (D+L)^{-1}b$.

This solution can also be written as

$$\begin{aligned} X^{(n+1)} &= X^{(n)} - \left[I + (D+L)^{-1}U \right] X^{(n)} + (D+L)^{-1}b \\ &= X^{(n)} - (D+L)^{-1}(D+L+U)X^{(n)} + (D+L)^{-1}b \\ &= X^{(n)} - (D+L)^{-1}AX^{(n)} + (D+L)^{-1}b \\ &= X^{(n)} + (D+L)^{-1}\left(b - AX^{(n)}\right). \end{aligned}$$

$$\Rightarrow \quad X^{(n+1)} - X^{(n)} = (D+L)^{-1}\left(b - AX^{(n)}\right)$$

$$\Rightarrow \quad (D+L)\left(X^{(n+1)} - X^{(n)}\right) = \left(b - AX^{(n)}\right)$$

or we may write it as

$$(D+L)V^{(n)} = r^{(n)} \qquad (7)$$

where $V^{(n)} = X^{(n+1)} - X^{(n)}$ *and* $r^{(n)} = b - AX^{(n)}$

Solve the equation (7) for $V^{(n)}$ by forward substitution. The solution is then found from

$$X^{(n+1)} = X^{(n)} + V^{(n)}.$$

This gives the final solution of the Gauss-Seidel method.

We proceed in this way until we get a result of desired accuracy. Gauss-Seidel method is also called the method of successive displacements.

Example 9: Solve the system of equations

$$\begin{aligned} 27x_1 + 6x_2 - x_3 &= 85 \\ 6x_1 + 15x_2 + 2x_3 &= 72 \\ x_1 + x_2 + 54x_3 &= 110 \end{aligned}$$

by Gauss-Seidel iterative method.

Solution: Given system of equations is

$$\begin{aligned} 27x_1 + 6x_2 - x_3 &= 85 \\ 6x_1 + 15x_2 + 2x_3 &= 72 \\ x_1 + x_2 + 54x_3 &= 110 \end{aligned} \qquad (1)$$

Rearranging the equations for the unknown with the largest coefficient in terms of the remaining unknowns,

$$x_1 = \frac{1}{27}(85 - 6x_2 + x_3)$$

$$x_2 = \frac{1}{15}(72 - 6x_1 - 2x_3) \qquad (2)$$

$$x_3 = \frac{1}{54}(110 - x_1 - x_2)$$

let $x_1^{(n)}, x_2^{(n)}$ and $x_3^{(n)}$ be the nth approximations, then the (n+1)th approximations is given by

$$x_1^{(n+1)} = \frac{1}{27}\left(85 - 6x_2^{(n)} + x_3^{(n)}\right) \qquad (3)$$

$$x_2^{(n+1)} = \frac{1}{15}\left(72 - 6x_1^{(n+1)} - 2x_3^{(n)}\right) \qquad (4)$$

$$x_3^{(n+1)} = \frac{1}{54}\left(110 - x_1^{(n+1)} - x_2^{(n+1)}\right) \qquad (5)$$

Starting with the initial approximations

$$x_1^{(0)} = 0,\ x_2^{(0)} = 0 \ \ and\ \ x_3^{(0)} = 0$$

first approximation to x_1 *i.e.* $x_1^{(1)}$, using equation (3) we have

$$x_1^{(1)} = \frac{1}{27}\left(85 - 6x_2^{(0)} + x_3^{(0)}\right)$$

$$x_1^{(1)} = \frac{85}{27} = 3.15$$

first approximation to x_2 *i.e.* $x_2^{(1)}$, using equation (4) we have

$$x_2^{(1)} = \frac{1}{15}\left(72 - 6x_1^{(1)} - 2x_3^{(0)}\right)$$

$$\Rightarrow \quad x_2^{(1)} = \frac{1}{15}(72 - 6\times 3.15 - 2\times 0) = 3.54$$

first approximation to x_3 *i.e.* $x_3^{(1)}$, using equation (5) we have

$$x_3^{(1)} = \frac{1}{54}\left(110 - x_1^{(1)} - x_2^{(1)}\right)$$

$$\Rightarrow \quad x_3^{(1)} = \frac{1}{54}(110 - 3.15 - 3.54) = 1.91$$

Thus first approximation to the solution is

$$x_1^{(1)} = 3.15,\ x_2^{(1)} = 3.54 \ \ and \ \ x_3^{(1)} = 1.91$$

second approximation to x_1 *i.e.* $x_1^{(2)}$, using equation (3) we have

$$x_1^{(2)} = \frac{1}{27}\left(85 - 6x_2^{(1)} + x_3^{(1)}\right)$$

$$\Rightarrow \quad x_1^{(1)} = \frac{1}{27}\left(85 - 6\times 3.54 + 1.91\right) = 2.43$$

second approximation to x_2 *i.e.* $x_2^{(2)}$, using equation (4) we have

$$x_2^{(2)} = \frac{1}{15}\left(72 - 6x_1^{(2)} - 2x_3^{(1)}\right)$$

$$\Rightarrow \quad x_2^{(2)} = \frac{1}{15}\left(72 - 6\times 2.43 - 2\times 1.91\right) = 3.57$$

second approximation to x_3 *i.e.* $x_3^{(2)}$, using equation (5) we have

$$x_3^{(2)} = \frac{1}{54}\left(110 - x_1^{(2)} - x_2^{(2)}\right)$$

$$\Rightarrow \quad x_3^{(2)} = \frac{1}{54}\left(110 - 2.43 - 3.57\right) = 1.926$$

Thus second approximation to the solution is

$$x_1^{(2)} = 2.43,\ x_2^{(2)} = 3.57 \ \ and \ \ x_3^{(2)} = 1.926$$

Similarly third approximation to the solution is

$$x_1^{(3)} = 2.426,\ x_2^{(3)} = 3.572 \ \ and \ \ x_3^{(3)} = 1.926$$

Since the values of $x_1^{(2)},\ x_2^{(2)}\ and\ x_3^{(2)}$ are sufficiently close to $x_1^{(3)},\ x_2^{(3)}\ and\ x_3^{(3)}$ respectively. Hence the values $x_1^{(3)} = 2.426,\ x_2^{(3)} = 3.572\ and\ x_3^{(3)} = 1.926$ can be considered as the solution of the given system.

Method of Factorization (Triangularization Method):

This method is also known as decomposition method. This method is based on the fact that a square matrix A can be factored into the product of a lower triangular matrix L and an upper triangular matrix U, if all the principal minors of A are non-singular, i.e. if

$$|a_{11}| \neq 0, \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0, \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{22} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0, \text{ etc.}$$

Thus, the matrix A can be expressed as

$$A = LU \qquad (1)$$

where

$$L = \begin{bmatrix} l_{11} & 0 & 0 & \dots & 0 \\ l_{21} & l_{22} & 0 & \dots & 0 \\ l_{31} & l_{32} & l_{33} & \dots & 0 \\ . & . & . & \dots & . \\ l_{n1} & l_{n2} & l_{n3} & \dots & l_{nn} \end{bmatrix} \quad and \quad U = \begin{bmatrix} u_{11} & u_{12} & u_{13} & \dots & u_{1n} \\ 0 & u_{22} & u_{23} & \dots & u_{2n} \\ 0 & 0 & u_{33} & \dots & u_{3n} \\ . & . & . & \dots & . \\ 0 & 0 & 0 & \dots & u_{nn} \end{bmatrix}$$

Using the matrix multiplication rule to multiply the matrices L and U and comparing the elements of the resulting matrix with those of A we obtain

$$l_{i1}u_{1j} + l_{i2}u_{2j} + \dots + l_{in}u_{nj} = a_{ij} \quad (i = 1, 2, \dots, n \text{ and } j = 1, 2, \dots, n)$$

where

$$l_{ij} = 0 \text{ if } i < j \text{ and } u_{ij} = 0 \text{ if } i > j$$

this system of equations involves $n^2 + n$ unknowns. Thus there are n parameters family of solutions. To produce a unique solution it is convenient to choose either

$$u_{ii} = 1 \text{ or } l_{ii} = 1 \quad i = 1, 2, \dots, n$$

Now,

If we take $l_{ii} = 1$, in the factorization method then the factorization method is called Doolittle's method.

Now if $l_{ii} = 1$, then we have

$$L = \begin{bmatrix} 1 & 0 & 0 \dots & 0 \\ l_{21} & 1 & 0 \dots & 0 \\ l_{31} & l_{32} & 1 \dots & 0 \\ . & . & . \dots & . \\ l_{n1} & l_{n2} & l_{n3} \dots & 1 \end{bmatrix} \quad and \quad U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \dots & u_{1n} \\ 0 & u_{22} & u_{23} \dots & u_{2n} \\ 0 & 0 & u_{33} \dots & u_{3n} \\ . & . & . \dots & . \\ 0 & 0 & 0 \dots & u_{nn} \end{bmatrix}$$

thus, for the system of equations

AX = b (2)

We have

LUX = b (3)

Putting UX = y in equation (3), we have

Ly = b (4)

On solving equation (4) by forward substitution, we find the vector y now solv the system of equations

UX = b

by backward substitution we get the values

$x_1, x_2, \dots, x_n$.

We have

UX = y

and Ly = b

$\Rightarrow \quad y = L^{-1}b \quad and \quad x = U^{-1}y$

Thus the inverse of A can also be determined as

$A^{-1} = U^{-1}L^{-1}$.

Example 3: Solve the system of equations

$$2x+3y+z=9$$
$$x+2y+3z=6$$
$$3x+y+2z=8$$

using factorization method (Doolittle's method).

Solution (Doolittle's method): We have system of equations

AX = b (1)

where

$$A=\begin{bmatrix}2 & 3 & 1\\ 1 & 2 & 3\\ 3 & 1 & 2\end{bmatrix},\ X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}\ and\ \ b=\begin{bmatrix}9\\ 6\\ 8\end{bmatrix}$$

Now let

A = L U (2)

where

$$L=\begin{bmatrix}1 & 0 & 0\\ l_{21} & 1 & 0\\ l_{31} & l_{32} & 1\end{bmatrix}\ and\ \ U=\begin{bmatrix}u_{11} & u_{12} & u_{13}\\ 0 & u_{22} & u_{23}\\ 0 & 0 & u_{33}\end{bmatrix}$$

Thus from equation (2) we have

L U = A

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$$

$$u_{11} = 2,\ u_{12} = 3,\ u_{13} = 1$$

$$\Rightarrow \quad l_{21}u_{11} = 1 \Rightarrow l_{21} = \frac{1}{2} \ and \ l_{31}u_{11} = 3 \Rightarrow l_{31} = \frac{3}{2}$$

$$l_{21}u_{12} + u_{22} = 2 \Rightarrow u_{22} = \frac{1}{2} \ and \ l_{21}u_{13} + u_{23} = 3 \Rightarrow u_{23} = \frac{5}{2}$$

$$l_{31}u_{12} + l_{32}u_{22} = 1 \Rightarrow l_{32} = -7 \ and \ l_{31}u_{13} + l_{32}u_{23} + u_{33} = 2 \Rightarrow u_{33} = 18$$

Thus, we have

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{3}{2} & -7 & 1 \end{bmatrix} \ and \ U = \begin{bmatrix} 2 & 3 & 1 \\ 0 & \frac{1}{2} & \frac{5}{2} \\ 0 & 0 & 18 \end{bmatrix}$$

Now using equation (1) and (2) we have

LUX = b

Let

UX=Y (3)

where $Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

$\Rightarrow$ LY = b

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{3}{2} & -7 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$$

$$\Rightarrow \quad \begin{aligned} &y_1 = 9 \\ &\frac{1}{2}y_1 + y_2 = 6 \\ &\frac{3}{2}y_1 - 7y_2 + y_3 = 8 \end{aligned}$$

On solving using forward substitution we have

$$y_1 = 9,\ y_2 = \frac{3}{2}\ \ and\ \ y_3 = 5$$

Now using the equation (3) we have

$$\begin{bmatrix} 2 & 3 & 1 \\ 0 & \frac{1}{2} & \frac{5}{2} \\ 0 & 0 & 18 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ \frac{3}{2} \\ 5 \end{bmatrix}$$

$$\Rightarrow \quad \begin{aligned} &2x + 3y + z = 9 \\ &\frac{1}{2}y + \frac{5}{2}z = \frac{3}{2} \\ &18z = 5 \end{aligned}$$

On solving using backward substitution we have

$$x = \frac{35}{18},\ y = \frac{29}{18}\ \ and\ \ z = \frac{5}{18}$$

Thus, the solution of the given system of equations is

$$x = \frac{35}{18},\ y = \frac{29}{18}\ \ and\ \ z = \frac{5}{18}.$$

If we take $u_{ii} = 1$, in the factorization method then the factorization method is called the Crout's method.

For the matrix A where

$$A = LU \qquad (1)$$

if $l_{ii} = 1$, then we have

$$L = \begin{bmatrix} l_{11} & 0 & 0 \dots & 0 \\ l_{21} & l_{22} & 0 \dots & 0 \\ l_{31} & l_{32} & l_{33} \dots & 0 \\ . & . & . \dots & . \\ l_{n1} & l_{n2} & l_{n3} \dots & l_{nn} \end{bmatrix} \quad and \quad U = \begin{bmatrix} 1 & u_{12} & u_{13} \dots & u_{1n} \\ 0 & 1 & u_{23} \dots & u_{2n} \\ 0 & 0 & 1 \dots & u_{3n} \\ . & . & . \dots & . \\ 0 & 0 & 0 \dots & 1 \end{bmatrix}$$

thus, for the system of equations

AX = b (2)

We have

LUX = b (3)

Putting UX = y in equation (3), we have

Ly = b (4)

On solving equation (4) by forward substitution, we find the vector y now solve the system of equations

UX = b

by backward substitution we get the values

$x_1, x_2, \dots, x_n$.

We have

UX = y

and Ly = b

$\Rightarrow \quad y = L^{-1}b \quad and \quad x = U^{-1}y$

Thus the inverse of A can also be determined as

$A^{-1} = U^{-1}L^{-1}$.

Example 4: Solve the system of equations

$$x + y + z = 1$$
$$4x + 3y - z = 6$$
$$3x + 5y + 3z = 4$$

using factorization method (Crout's method).

Solution (Crout's method): We have system of equations

AX = b (1)

where

$$A=\begin{bmatrix}1 & 1 & 1\\ 4 & 3 & -1\\ 3 & 5 & 3\end{bmatrix},\ X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}\ \ and\ \ b=\begin{bmatrix}1\\ 6\\ 4\end{bmatrix}$$

Now let

A = L U (2)

where

$$L=\begin{bmatrix}l_{11} & 0 & 0\\ l_{21} & l_{22} & 0\\ l_{31} & l_{32} & l_{33}\end{bmatrix}\ \ and\ \ U=\begin{bmatrix}1 & u_{12} & u_{13}\\ 0 & 1 & u_{23}\\ 0 & 0 & 1\end{bmatrix}$$

Thus from equation (2) we have

L U = A

$$\Rightarrow \begin{bmatrix}l_{11} & 0 & 0\\ l_{21} & l_{22} & 0\\ l_{31} & l_{32} & l_{33}\end{bmatrix}\begin{bmatrix}1 & u_{12} & u_{13}\\ 0 & 1 & u_{23}\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 1 & 1\\ 4 & 3 & -1\\ 3 & 5 & 3\end{bmatrix}$$

$$\Rightarrow \begin{array}{l} l_{11}=1,\ l_{21}=4,\ l_{31}=3\\ l_{11}u_{12}=1\Rightarrow\ u_{12}=1\ \ and\ \ l_{11}u_{13}=1\Rightarrow\ u_{13}=1\\ l_{21}u_{12}+l_{22}=3\Rightarrow\ l_{22}=-1\ \ and\ \ l_{21}u_{13}+l_{22}u_{23}=-1\Rightarrow\ u_{23}=5\\ l_{31}u_{12}+l_{32}=5\Rightarrow\ l_{32}=2\ \ and\ \ l_{31}u_{13}+l_{32}u_{23}+l_{33}=3\Rightarrow\ l_{33}=-10 \end{array}$$

Thus, we have

$$L=\begin{bmatrix}1 & 0 & 0\\ 4 & -1 & 0\\ 3 & 2 & -10\end{bmatrix}\ \ and\ \ U=\begin{bmatrix}1 & 1 & 1\\ 0 & 1 & 5\\ 0 & 0 & 1\end{bmatrix}$$

Now using equation (1) and (2) we have

LUX = b

Let

UX=Y (3)

where $Y=\begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix}$

$\Rightarrow$ LY = b

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 4 & -1 & 0 \\ 3 & 2 & -10 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix}$$

$$\Rightarrow \begin{aligned} & y_1 = 1 \\ & 4y_1 - y_2 = 6 \\ & 3y_1 + 2y_2 - 10y_3 = 4 \end{aligned}$$

On solving using forward substitution we have

$$y_1 = 1,\ y_2 = -2 \ \ and \ \ y_3 = -\frac{1}{2}$$

Now using the equation (3) we have

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ -\frac{1}{2} \end{bmatrix}$$

$$\Rightarrow \begin{aligned} & x + y + z = 1 \\ & y + 5z = -2 \\ & z = -\frac{1}{2} \end{aligned}$$

On solving using backward substitution we have

$$x = 1,\ y = \frac{1}{2} \ \ and \ \ z = -\frac{1}{2}$$

Thus, the solution of the given system of equations is

$$x = 1,\ y = \frac{1}{2} \ \ and \ \ z = -\frac{1}{2}.$$

UNIT II

Iterative method is a ------------ method

------------- is also a self-correction method.

The condition for convergence of Gauss Seidal method is that the ------ should be diagonally dominant

In ----------- method, the coefficient matrix is transformed into diagonal matrix

------------- Method takes less time to solve a system of equations

The iterative process continues till ------------- is secured.

In Gauss elimination method, the solution is getting by means of ----------- from which the unknowns are found by back substitution.

The --------------- is reduced to an upper triangular matrix or a diagonal matrix in direct methods.

The augment matrix is the combination of ------------------.

The given system of equations can be taken as in the form of -----------

Which is the condition to apply Gauss Seidal method to solve a system of equations?

Crout's method and triangularisation method are ------ method.

The solution of simultaneous linear algebraic equations are found by using-----

The matrix is ____ if the numerical value of the leading diagonal element in each row is greater than or equal to the

If the Eigen values of A are -6, 2, 4 then ______ is dominant.

The Gauss – Jordan method is the modification of ________ method.

$x^2 + 5x + 4 = 0$ is a ________ equation.

a + b logx + c sinx + d = 0 is a _________ equation.

In Gauss – Jordan method, the augmented matrix is reduced into _____matrix

The 1st equation in Gauss – Jordan method, is called __________ equation.

The element a_{11} in Gauss – Jordan method is called _________ element.

The system of simultaneous linear equation in n unknowns AX = B if A is diagonally dominant then the system is sa

The convergence of Gauss – Seidal method is roughly _____ that of Gauss – Jacobi method

Jacobi's method is used only when the matrix is ____

Gauss Seidal method always --------- for a special type of systems.

Condition for convergence of Gauss Seidal method is ---------------.

Modified form of Gauss Jacobi method is -------------------- method.

In Gauss elimination method by means of elementary row operations, from which the unknowns are found by ---------------- method

In iterative methods, the solution to a system of linear equations will exist if the absolute value of the lar

In ------------- iterative method, the current values of the unknowns at each stage of iteration are used in p

The direct method fails if any one of the pivot elements become ----.

In Gauss elimination method the given matrix is transformed into --------------.

If the coefficient matrix is not diagonally dominant, then by ----------- that diagonally dominant coefficient matrix is formed.

Gauss Jordan method is a -----------------.

Gauss Jacobi method is a -----------------.

The modification of Gauss – Jordan method is called ---------------------.
Gauss Seidal method always converges for ----------- of systems
In solving the system of linear equations, the system can be written as ---
In solving the system of linear equations, the augment matrix is --------
In the direct methods of solving a system of linear equations, at first the given system is written as ------------- form.
All the row operations in the direct methods can be carried out on the basis of --
The direct method fails if -----------.
The elimination of the unknowns is done not only in the equations below, but also in the equations above the leading diagonal is called ------------

In Gauss Jordan method, we get the solution ---------------------

If the coefficient matrix is diagonally dominant, then ---------- method converges quickly.
Which is the condition to apply Jocobi's method to solve a system of equations
Iterative method is a ------------ method
As soon as a new value for a variable is found by iteration it is used immediately in the equations is called --------------.
------------- is also a self-correction method.
The condition for convergence of Gauss Seidal method is that the ------ should be diagonally dominant
In ----------- method, the coefficient matrix is transformed into diagonal matrix
We get the approximate solution from the ---------------.
The iterative process continues till ------------- is secured.

In Gauss elimination method, the solution is getting by means of ----------- from which the unknowns are found by back substitution.

The method of iteration is applicable only if all equation must contain one coefficient of different unknowns as ---------- than other coefficients.
The --------------- is reduced to an upper triangular matrix or a diagonal matrix in direct methods.

The augment matrix is the combination of ------------------.

The given system of equations can be taken as in the form of -----------
The sufficient condition of iterative methods will be satisfied if the large coefficients are along the ---------------- of the coefficient matrix.
Which is the condition to apply Gauss Seidal method to solve a system of equations.

In the absence of any better estimates, the -------------of the function are taken as x = 0, y = 0, z = 0.

The solution of simultaneous linear algebraic equations are found by using-

Direct method	InDirect method	both 1st & 2nd
Iteration method	Direct method	Interpolation
Constant matrix	unknown matrix	Coefficient matrix
Gauss elimination	Gauss jordan	Gauss jacobi
Direct method	Indirect method	Regula falsi
convergency	divergency	oscillation
Elementary operations	Elementary column operations	Elementary diagonal operations
Coefficient matrix	Constant matrix	unknown matrix
Coefficient matrix and constant matrix	Unknown matrix and constant matrix	Coefficient matrix and Unknown matrix
A = B	BX= A	AX= B
1st row is dominant	1st column is dominant	diagonally dominant
Direct	Indirect	Iterative
Direct method	Indirect method	both 1st & 2nd
orthogonal	symmetric	diagonally dominant
2	-6	4
Gauss –Elimination	Gauss – Jacobi	Gauss – Seidal
algebraic	transcendental	wave
algebraic	transcendental	wave
upper triangular	lower triangular	diagonal
pivotal	dominant	reduced
Eigen value	Eigen vector	pivot
dominant	diagonal	scalar
twice	thrice	once
symmetric	skew-symmetric	singular
Converges	diverges	oscillates
Coefficient matrix is diagonally dominant	pivot element is Zero	Coefficient matrix is not diagonally dominant
Gauss Jordan	Gauss Siedal	Gauss Jacobbi
Forward substitution	Backward substitution	random
less than	greater than or equal to	equal to
Gauss Siedal	Gauss Jacobi	Gauss Jordan
Zero	one	two
Unit matrix	diagonal matrix	Upper triangular matrix
Interchanging rows	Interchanging Columns	adding zeros
Direct method	InDirect method	iterative method
Direct method	InDirect method	iterative method

Gauss Jordan	Gauss Siedal	Gauss Jacobbi
Only the special type	all types	quadratic types
BX = B	AX = A	AX = B
(A, A)	(B, B)	(A, X)
An augment matrix	a triangular matrix	constant matrix
all elements	pivot element	negative element
1st row elements 0	1st column elements 0	Either 1st or 2nd
Gauss elimination	Gauss jordan	Gauss jacobi
without using back substitution method	By using back substitution method	by using forward substitution method
Gauss elimination	Gauss jordan	Direct
1st row is dominant	1st column is dominant	diagonally dominant
Direct method	InDirect method	Interpolation
Iteration method	Direct method	Interpolation
Iteration method	Direct method	Interpolation
Constant matrix	unknown matrix	Coefficient matrix
Gauss elimination	Gauss jordan	Gauss jacobi
Direct method	InDirect method	fast method
convergency	divergency	oscillation
Elementary operations	Elementary column operations	Elementary diagonal operations
smaller	larger	equal
Coefficient matrix	Constant matrix	unknown matrix
Coefficient matrix and constant matrix	Unknown matrix and constant matrix	Coefficient matrix and Unknown matrix
A = B	BX= A	AX= B
Rows	Coloumns	Leading Diagonal
1st row is dominant	1st column is dominant	diagonally dominant
initialapproximations	roots	points
Direct method	InDirect method	fast method

either 1st &2nd
none

Unit matrix

Gauss seidal
Bisection
none

Elementary row
operations

Augment matrix

Coefficient matrix,
constant matrix and
Unknown matrix

AB = X
last row dominant
Interpolation
Bisection
singular
-2
interpolation
heat
heat
scalar
normal
root
singular
4 times
non-singular
equal

pivot element is non Zero

Gauss Elimination

Gauss Elimination

not equal
Gauss Elimination
negative
lower triangular matrix

Interchangingrow and Columns

convergent
convergent

gauss elemination
first type
AB = X
(A, B)

Coefficient matrix

positiveelement

2 nd row is dominant

Gauss siedal

Without using forward
substitution method

Gauss siedal

2 nd row is dominant
extrapolation

extrapolation

extrapolation

extrapolation

Gauss seidal
Bisection
point

Elementary row
operations

non zero

Augment matrix
Coefficient matrix,
constant matrix and
Unknown matrix
AB = X

elements

Leading Diagonal

final value

Bisection

InDirect method
Iteration method

Coefficient matrix
Gauss jordan
Direct method
convergency

Elementary row
operations

Augment matrix
Coefficient matrix
and constant matrix

AX= B
diagonally dominant

Direct
InDirect method

diagonally dominant

Gauss –Elimination
algebraic
transcendental

diagonal
pivotal
pivot
diagonal
twice
symmetric
Converges
Coefficient matrix is
diagonally dominant
Gauss Siedal
Backward substitution
greater than or equal to
Gauss Siedal
Zero
Upper triangular matrix
Interchangingrow and Columns
Direct method
InDirect method
Gauss Siedal
Only the special type
AX = B
(A, B)
An augment matrix
pivot element
Either 1st or 2nd
Gauss jordan
By using back
substitution method
Gauss siedal
diagonally dominant
InDirect method
Iteration method
Iteration method
Coefficient matrix
Gauss jordan
InDirect method
convergency
Elementary row
operations
larger
Augment matrix
Coefficient matrix
and constant matrix

UNIT-III

SYLLABUS

Gauss elimination method (with row pivoting) and Gauss-Jordan method – Gauss thomas method for tridiagonal systems. Interative methods: Jacobi and Gauss-seidalinterative methods.

First differences of the function.

Let y = f(x) be any function given by the values y_0, y_1, ……y_n. Which it takes for the equidistant values x_0, x_1, ……x_n of the independent variable x then $y_1 - y_0$, $y_2 – y_1$, …….$y_n – y_{n-1}$ are called the first differences of the function y. Denoted by Δy_0, Δy_1, …..

Shift operator:

Let y = f(x) be function of x and x, $x+h_1$ x + 2h … etc. be the consecutive values of x, then the operator E is defined as E[f(x)] = f(x+h) ∵ E is called Shift Operator.

Formula for Newton forward and Newton Backward differences:

The Newton's forward interpolation formula is

$$y(x_0+nh) = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + ...$$

The Newton's backward interpolation formula is

$$y(x_0+nh) = y_0 + n\nabla y_0 + \frac{n(n+1)}{2!}\nabla^2 y_0 + ...$$

Formula for Lagrange's interpolation .

Let Y = f(x) be a function which assumes the values $f(x_0)$, $f(x_1)$ ….. $f(x_n)$ corresponding to the values x: x_1, x_1 …..x_n.

$$Y = f(x) = \frac{(x-x_1)(x-x_2)...(x-x_n)}{(x_0-x_1)(x_0-x_2)...(x_0-x_n)} f(x_0) + \frac{(x-x_0)(x-x_2)...(x-x_n)}{(x_1-x_0)(x_1-x_2)...(x_1-x_n)} f(x_1) + ...$$

Formula for inverse Lagrange's interpolation:

$$x = \frac{(y-y_1)(y-y_2)\ldots(y-y_n)}{(y_0-y_1)(y_0-y_2)\ldots(y_0-y_n)}x_0 + \frac{(y-y_0)(y-y_2)\ldots(y-y_n)}{(y_1-y_0)(y_1-y_2)\ldots(y_1-y_n)}x_1 + \ldots\ldots\ldots$$

OperatorΔ, ∇, E, δ, μ.

(i) $\Delta y_x = y_{x+h} - y_x$

(ii) $\nabla y_x = y_x - y_{x-h}$

(iii) $\delta y_x = y_{x+\frac{1}{2}h} - y_{x-\frac{1}{2}h}$

$\mu = Y_2 \left(y_{x+\frac{1}{2}h} + y_{x-\frac{1}{2}h} \right)$

Relation between the operator δ and E:

$$\delta f(x) = f\left(x+\tfrac{1}{2}h\right) - f\left(x-\tfrac{1}{2}h\right)$$

$$= E^{\frac{1}{2}} f(x) - E^{-\frac{1}{2}} f(x)$$

$$= \left(E^{\frac{1}{2}} - E^{-\frac{1}{2}}\right) f(x)$$

$$\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}} \qquad = E^{-\frac{1}{2}}(E-1)$$

$$= E^{-\frac{1}{2}}\Delta$$

Relation between the operator μ and E:

$$\mu\,[f(x)] = \tfrac{1}{2}\left[f(x+\tfrac{1}{2}h) + f(x-\tfrac{1}{2}h) \right]$$

$$= \tfrac{1}{2}\left[E^{\frac{1}{2}} + E^{-\frac{1}{2}} \right] f(x)$$

$$\mu = \frac{1}{2}\left[E^{\frac{1}{2}} + E^{-\frac{1}{2}} \right]$$

Error formula in interpolation:

$$f(t) = f(t) - \phi(t) - \left(f(x) - \phi(x) \right) \frac{(t - x_0)(t - x_1)....(t - x_n)}{(x - x_0)(x - x_1)...(x - x_n)}$$

The above function f(t) is continuous in $[x_0, x_n]$.

Numerical Examples:

01.Prove that if m and n are positive integers then

$\Delta^m \Delta^n f(x) = \Delta^{m+n} f(x)$.

Proof:

$\Delta^m \Delta^n f(x) = (\Delta \times .... \text{ m times}) (\Delta \times \Delta \times..... \times \text{ n times}) f(x)$

$= \Delta.\Delta ...... (m + n) \text{ times } f(x)$

$= \Delta^{m+n} f(x)$.

02. Find Δ log x.

Solution:

$\Delta \log x = \log (x + h) - \log x$

$$= \log\left(\frac{x+h}{x}\right)$$

$\Delta \log x = \log (1 + h/x)$

03. Find $\Delta \tan^{-1} x$

Solution:-

$\Delta \tan^{-1} x = \tan^{-1}(x+h) - \tan^{-1}(x)$

$$= \tan^{-1}\left[\frac{\not{x} + h - \not{x}}{1 + (x+h)x}\right]$$

$$= \tan^{-1}\left(\frac{h}{x^2 + hx + 1}\right).$$

04. It n is a positive integer then

$y_n = y_0 + nc_1 \Delta y_0 + nc_2 \Delta^2 y_0 + \ldots\ldots. + \Delta^n y_0.$

<u>Proof:</u>

From the let

$y_1 = Ey_0 = (1 + \Delta) y_0$

$= y_0 + \Delta y_0$

$y_2 = E^2 y_0 = (1 + \Delta)^2 y_0$

$= (1 + 2c_1 \Delta + \Delta^2) y_0$

$= y_0 + 2c_1 \Delta y_0 + \Delta^2 y_0$

Similarly $y_n = E^n y_0 = (1 + \Delta)^n y_0$

$= (1 + nc_1\Delta + \ldots\ldots.\Delta^n) y_0$

$= y_0 + nc_1 \Delta y_0 + \ldots..\Delta^n y_0$

Hence proved.

05. Prove that $E \Delta \equiv \Delta E$

<u>Proof:-</u>

$E\Delta f(x) = E[f(x+h) - f(x)]$

$= E\,[f(x + h) - E\,[f(x)]$

$= f(x + 2h) - f(x + h)$

06. Prove that $f(4) = f(3) + \Delta f(2) + \Delta^2 f(1) + \Delta^3 f(1)$

Solution:-

$f(4) - f(3) = \Delta f(3)$

$= \Delta\,[f(2) + \Delta f(2)] \quad \because f(3) - f(2) = \Delta f(2)$

$= \Delta f(2) + \Delta^2 f(2)$

$= \Delta f(2) + \Delta^2\,[f(1) + \Delta f(1)]$

$= \Delta f(2) + \Delta^2 f(1) + \Delta^3 f(1).$

$f(4) = f(3) + \Delta f(2) + \Delta^2 f(1) + \Delta^3 f(1).$

07. Express $f(x) = x^4 - 5x^3 + 3x + 4$ in terms of factorial polynomials.

Solution:

Synthetic division method:-

Given $f(x) = x^4 - 5x^3 + 3x + 4$

By Synthetic division

$$
\begin{array}{c|ccccl}
1 & 1 & -5 & 0 & 3 & 4(=E) \\
 & 0 & 1 & -4 & -4 & \\
\hline
2 & 1 & -4 & -4 & -1 & =(-D) \\
 & 0 & 2 & -4 & & \\
\hline
3 & 1 & -2 & -8 & (=C) & \\
 & 0 & 3 & & & \\
\hline
 & 1(=A) & 1(=B) & & &
\end{array}
$$

$f(x) = x^{(4)} + x^{(3)} - 8x^{(2)} - x^{(1)} + 4.$

08. Estimate y_2 from the following table.

X	1	2	3	4	5
Y_x	7	?	13	21	37

Solution:-

Here we are given four entries

Viz. y_1, y_3, y_4 and y_5. Therefore the function y_x can be represented by a third degree polynomial $\Delta^3 y_x$ = Constant and $\Delta^4 y_x = 0$ in particular

$\Delta^4 y_1 = 0 \Rightarrow (E - 1)^4 y_1 = 0; (E^4 - 4E^3 + 6E^2 - 4E + 1) y_1 = 0$

$y_5 - 4y_4 + 6y_3 - 4y_2 + 7 = 0$; $38 - 4y_2 = 0$; $y_2 = 9.5$

09. Obtain a function whose first differences is

$6x^2 + 16x + 11$

Solution:-

Expressing the function in factorial notation, we get

$6x^2 + 16x + 11 = 6x^{(2)} + 16x^{(1)} + 11$

$\Delta f(x) = 6x^{(2)} + 16x^{(1)} + 11$

Integrating we get

$$f(x) = \frac{6x^{(3)}}{3} + \frac{16x^{(2)}}{2} + \frac{11x^{(1)}}{1} + K$$
$$= 2x^{(3)} + 8x^{(2)} + 11x^{(1)} + K$$

which is the required function.

10. Find Δ^{10} (1 – ax) (1 – bx^2) (1 – cx^3) (1 – dx^4).

Solution:-

Let f(x) = (1 – ax) (1 – bx^2) (1 – cx^3) (1 – dx^4).

f(x) is a polynomial of degree 10 and the coefficient of x^{10} is a, b, c, d.

Δ^{10} f(x) = Δ^{10} (abcd x^{10})

= abcd $\Delta^{10} x^{10}$ = 10! abcd.

11. Find Δ^n (1/x).

Solution:-

Now $\Delta\left(\frac{1}{x}\right) = \frac{1}{x+1} - \frac{1}{x}$

$$= \frac{-1}{x(x+1)}$$

$$\Delta^2\left(\frac{1}{x}\right) = \frac{(-1)^2}{x(x+1)\ (x+2)}$$

and so on

Preceding like this

$$\Delta^n\left(\frac{1}{x}\right) = \frac{(-1)^n}{x(x+1)(x+2)....(x+n)}$$

12. Evaluate $\left(\frac{\Delta^2}{E}\right)x^3$

Solution:-

Let h be the interval of differencing

$$\left(\frac{\Delta^2}{E}\right)x^3 = (\Delta^2 E^{-1})x^3$$

$$= (E-1)^2 E^{-1}x^3$$
$$= (E^2 - 2E + 1)E^{-1}x^3$$
$$= (E - 2 + E^{-1})x^3 \Rightarrow Ex^3 - 2x^3 + E^{-1}x^3$$
$$= (x+h)^3 - 2x^3 + (x-h)^3$$
$$= 6xh.$$

13. Given $u_0 = 1$, $u_1 = 11$, $u_2 = 21$, $u_3 = 28$ and $u_4 = 29$ find $\Delta^4 y_0$.

Solution:-

$\Delta^4 y_0 = (E^4 - 4C_1 E^3 + 4C_2 E^2 - 4C_3 E + 1)\, y_0$

$= E^4 y_0 - 4E^3 y_0 + 6E^2 y_0 - 4E y_0 + y_0$

$= y_4 - 4y_3 + 6y_2 - 4y_1 + y_0$

$= 29 - 112 + 126 - 44 + 1$

$= 0.$

14. Write the relation between E and Δ.

Solution:

$\Delta f(x) = f(x) - f(x-h)$

$= f(x) - E^{-1} f(x)$

$= (1 - E^{-1})\, f(x)$

$$\Delta = \frac{E-1}{E}$$

15. Prove that (1+ Δ) (1 - Δ) = 1

Solution:-

$(1 + \Delta)(1 - \Delta) f(x) = E E^{-1} f(x)$

$= E f(x - h)$

$= f(x)$

$(1+ \Delta)(1 - \Delta) = 1$

16. Prove that ΔV = Δ - V

Proof:-

$\Delta\nabla f(x) = (E - 1)(1 - E^{-1}) f(x)$

$= (E - 1)[f(x) - f(x-h)]$

$= E[f(x)] f(x) - E[f(x - h)] + f(x - h)$

$= f(x+h) - f(x) - f(x) + f(x - h)$

$= [Ef(x) - f(x)] - [f(x) - f(x - h)]$

$= (E - 1) f(x) - (1 - E^{-1}) f(x)$

$= [(E - 1) - (1 - E^{-1})] f(x)$

$= (\Delta - \nabla) f(x).$

17. Prove that ∇f(x) = (1 – E^{-1}) f(x).

Proof:-

∇ f(x) = f(x) – f(x – h)

E^{-1} [∇f(x)] = E^{-1} [f(x + h) – f(x)]

= f(x) – f(x – h) ∇

∇ = E^{-1} Δ

18. Prove that $\nabla^2_{0^3} = 6,\ \nabla^3_{0^3} = 6$

Solution:-

$$\because \Delta^n_{0^r} = 0 \quad \because n > r$$
$$\Delta^n\ 0^n = n!$$
$$\Delta 0^r = 1^r = 1$$

$$\nabla^2_{0^3} = 2^3 - 2.1^3 = 6$$
$$\nabla^3_{0^3} = 3^3 - 3.2^3 + 3.i^3 = 6$$

19. Calculate (i) $\nabla^3_{0^6}$ **(ii)** $\nabla^5_{0^6}$

Solution:-

(i) $\nabla^3_{0^6}$ =3^6 – 3.2^6 + 3.1^6

= 729 – 192 + 3 = 540

(ii) $\nabla^5_{0^6}$ $=5^6 - 5.4^6 + 10.3^6 - 10.2^6 + 5.1^6$

$=15625 - 20480 + 7290 - 640 + 5$

$=1800.$

20. Prove that $E^{1/2} = \mu + \frac{1}{2}\delta,\ E^{-1/2} = \mu - \frac{1}{2}\delta$.

Solution:-

$$\mu + \tfrac{1}{2}\delta = \tfrac{1}{2}\left(E^{1/2} + E^{-1/2}\right) + \tfrac{1}{2}\left(E^{1/2} - E^{-1/2}\right)$$
$$= \tfrac{1}{2}\left[2E^{\frac{1}{2}}\right] = E^{1/2}$$
$$\mu - \tfrac{1}{2}\delta = \tfrac{1}{2}\left(E^{1/2} + E^{-1/2}\right) - \tfrac{1}{2}\left(E^{1/2} - E^{-1/2}\right)$$
$$= E^{-1/2}$$

21. Prove that $\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1 + \delta^2/4}$

Proof:-

$$\tfrac{1}{2}\delta^2 + \delta\sqrt{1 + \delta^2/4} = \tfrac{1}{2}\left(E^{1/2} - E^{-1/2}\right)^2 + \delta\sqrt{1 + \frac{\left(E^{1/2} - E^{-1/2}\right)^2}{4}}$$
$$= \tfrac{1}{2}\left(E^{1/2} - E^{-1/2}\right)^2 + \left(E^{1/2} - E^{-1/2}\right)\sqrt{\frac{4 + E + E^{-1-2}}{4}}$$
$$= \tfrac{1}{2}\left(E + E^{-1} - 2\right) + \left(E^{1/2} - E^{-1/2}\right)\sqrt{\frac{\left(E^{1/2} - E^{-1/2}\right)^2}{4}}$$
$$= \tfrac{1}{2}\left(E + E^{-1} - 2\right) + \left(\frac{E^{1/2} - E^{-1/2}}{2}\right)\left(E^{1/2}E^{-1/2}\right)$$
$$= E - 1 = \Delta.$$

23. Prove that (E + 1) δ = 2 (E – 1) μ.

Solution:-

$$(E+1)\delta = (E+1)\left(E^{1/2} - E^{-1/2}\right)$$

$$= \left(E^{1/2}.E^{1/2} + E^{1/2}.E^{-1/2}\right)\left(E^{1/2} - E^{-1/2}\right)$$

$$= E^{1/2}\left(E^{1/2} - E^{-1/2}\right)\left(E^{1/2} - E^{-1/2}\right)$$

$$= E^{1/2}\left(E^{1/2} - E^{-1/2}\right)\left(E^{1/2} + E^{-1/2}\right)$$

$$= (E-1) - 2.\left(\frac{E^{1/2} + E^{-1/2}}{2}\right)$$

$$=2(E-1)\mu.$$

24. Find the missing y_x values from the first differences provided.

y_x	0	a	b	C	d	e
Δy_x	0	1	2	4	7	11

Solution:-

By def:

a – 0 = 1 a = 1

b – a = 2, b – 1 = 2 b = 3

c – b = 4, c – 3 = 4 c = 7

d – c = 7, d – 7 = 7 d = 14

e – d = 11, e – 14 = 11 e = 25

25. Evaluate $\Delta^n (ax^n + bx^{n-1})$

Solution:-

$(n (ax^n + bx^{n-1}) = (n (ax^n) + (n (bx^{n-1})$

$= a\ n! + b(0)$

$= an!$

26. Find Δf(x) if $x^2 + 2x + 2 = f(x)$ and the interval of differencing as unity.

Solution:-

$\Delta f(x) = f(x + 1) – f(x)$

$= (x + 1)^2 + 2(x + 1) + 2 – [x^2 + 2x + 2]$

$$= \cancel{x^2} + \cancel{2x} + 1 + 2x + 2 + \cancel{2} - \cancel{x^2} - \cancel{2x} - \cancel{2}$$
$$= 2x + 3$$

27. Find the second degree polynomial fitting the following data.

x	**1**	**2**	**4**
y	**4**	**5**	**13**

<u>Solution:-</u>

$x_0 = 1, x_1 = 2, x_2 = 4, y_0 = 4, \quad y_1 = 5, \; y_2 = 13$

By Lagrange's formula

$$f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_0)} y_2$$

$$= \frac{(x-2)(x-4)}{3} 4 + \frac{(x-1)(x-4)}{-2} 5 + \frac{(x-1)(x-2)}{6} 13$$

$$= \frac{1}{6}\left[8x^2 - 48x + 64 - 15x^2 + 75x - 60 + 13x^2 - 39x + 26\right]$$

$$= \frac{1}{6}\left[6x^2 - 12x + 30\right]$$

$$= x^2 - 2x + 5$$

28. Prove that $\left(\frac{\Delta^2}{E}\right) e^x . \frac{Ee^x}{\Delta^2 e^x} = e^x$

Solution:-

$$(\Delta^2 E^{-1}) e^x . \frac{Ee^x}{\Delta^2 e^x} = \Delta^2 e^{x-h} \frac{Ee^x}{\Delta^2 e^x}$$

$$= \Delta^2 e^x . e^{-h} \frac{Ee^x}{\Delta^2 e^x} = e^{-h} Ee^x$$

$$= e^{-h} - x^{x+h} = e^x$$

29. Show that $\underset{bcd}{\overset{3}{\Delta}} \left(\frac{1}{a}\right) = -\frac{1}{abcd}$

Solution:

If f(x) = 1/x, f(a) = 1/a

f(a, b) = $\underset{b}{\Delta}\left(\frac{1}{a}\right) = \frac{\frac{1}{b} - \frac{1}{a}}{b-a} = -\frac{1}{ab}$

f(a, b, c) $= \frac{f(b,c) - f(a,b)}{c-a} = \frac{-\frac{1}{bc} + \frac{1}{ab}}{c-a}$

$= \frac{1}{abc}$

$$f(a,b,c,d) = \frac{f(b,c,d) - f(a,b,c)}{d-a}$$

$$= \frac{\frac{1}{\text{bcd}} - \frac{1}{abc}}{\text{d-a}} = -\frac{1}{abcd}$$

30. A function f(x) is given by the following table. Find f(0.2) by a suitable formula.

x	0	1	2	3	4	5	6
F(x)	176	185	194	203	212	220	229

Solution:

The difference table is follows:-

x	y = f(x)	Δ	Δ2	(3	(4	(5	(6

0	176	(y0					
1	185	9	(2y0				
2	194	9	0	(3 y0			
3	**203**	**9**	**0**	**0**	**$\Delta^4 y_0$**		
4	**212**	**9**	**0**	**0**	**0**	**$\Delta^5 y_0$**	
5	**220**	**8**	**-1**	**-1**	**-1**	**-1**	**$\Delta^6 y_0$**
6	**229**	**9**	**1**	**2**	**3**	**4**	**5**

Here $x_0 = 0$, h = 1, $y_0 = 176 = f(x)$

We have to find the value of f (0.2). By Newton's forward interpolation formula we have

$$f(x_0 + nh) = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + ...$$
$$f(0.2) = ?$$

$x_0 + nh = 0.2$

$0 + n = 0.2 \Rightarrow \quad n = 0.2$

$$f(0.2) = 176 + (0.2)\ 9 + \frac{(0.2)(0.2-1)}{2}\ 0$$
$$= 176 + 1.8$$
$$= 177.8$$

31. From the given table compute the value of sin 38.

x	**0**	**10**	**20**	**30**	**40**
Sin x	**0**	**0.17365**	**0.34202**	**0.5**	**0.64276**

Solution:-

As we have to determine the value of y = sin x near the lower end, we apply Newton's backward interpolation formula.

The difference table is as given below.

x^0	$y(x) = \sin x^0$	Δ_y	Δ^2_y	Δ^3_y	Δ^4_y
0	0				
10	0.17365	0.17365			
20	0.34202	0.16837	- 0.00528		
30	0.5000	0.15798	- 0.01039	- 0.00511	
40	0.64279	0.14279	- 0.01519	- 0.0048	0.00031
X_0	Y_0	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$

Here $x_0 = 40$, h = 0.64279, h = 10

Newton's backward differences formula

$$y(x_0+nh) = y_0 + n\nabla y_0 + \frac{n(n+1)}{2!}\nabla^2 y_0 + ...$$

$$y(38) =$$

$$x_0 + nh = 38 \qquad 40 + n(10) = 38 \qquad \boxed{n = -0.2}$$

$$y\ (38) = 0.64279 + (-0.2)\ (0.14279) + \frac{(-0.2)\ (-0.2+1)}{2!}\ (-0.01519)$$

$$+\frac{(-0.2)\ (-0.2+1)\ (-0.2+2)}{3!}\ (-0.0048) + \textit{ neglible term}$$

$$= 0.64279 - 0.028558 + 0.0012152 + 0.0002304$$

$$= 0.61566$$

32. If $U_x = ax^2 + bx + C$, then show that

$$U_{2n} - nC_1 2U_{2n-1} + nC_2 2U_{2n-2} .... + (-2)^n u_n = (-1)^n (1-2an)$$

<u>Solution:-</u>

Given that

$U_x = ax^2 + bx + C$

$U_n = an^2 + bn + C$, Uni > a polynomial of

degree 2 in n

$\Delta^3 u_n = \Delta^4 u_n = 0$

Let the interval of differencing be equal to 1. Now

$U_n \quad = an^2 + bn + C$

$\Delta U_n \quad = \quad a(n+1)^2 + b(n+1) + C - an^2 - bn - C$

$\quad = \quad 2an + a + b$

Δ^2U_n = $\Delta[\Delta U_n)$

= $2a (n + 1) + a + b – 2an – a – b$

= $2a$

LHS :-

$U_{2n} – nC_1 2U_{2n-1} + nC_2 U_{2n-2} - ....$

= $[E^n U_n – nC_1 2E^{n-1} U_n + nC_2 E^{n-2} .....] U_n$

= $[E^n – nC_1 2E^{n-1} U_n + nC_2 E^{n-2} ....] U_n$

= $(E – 2)^n U_n$

= $(E – 1 -1)^n u_n$

= $(\Delta - 1)^n U_n$ $\because$ $E – 1 = \Delta$

= $(-1)^n (1 - \Delta)^n U_n$

= $(-1)^n [1 – nC_1 \Delta + nC_2 \Delta^2 + ...]U_n$

= $(-1)^n [U_n – n \Delta U_n + \frac{n(n-1)}{2!} \Delta^2 U_n + ...]$

= $(-1)^n \left[an^2 + bn + C - n(2an + a + b) + \frac{n^2 - n}{2} 2a \right]$

= $(-1)^n (C – 2an)$

= RHS.

UNIT IV

Simpson's Rule.

Evaluate $\int_0^1 e^{-x^2} dx$ by dividing the range of initiation in to 4 equal parts using (i) Trapezoidal rule (ii) Simpson's rule.

Solution:

Here the length of the interval is h = $\frac{1-0}{u}$ = 0.25. The values of the function y = e^{-x2} for each point of sub divisions are given below.

X	0	0.25	0.5	0.75	1
e^{-x2}	1	0.9394	0.7784	0.5694	0.3628
	Y_0	Y_1	Y_2	Y_3	Y_4

(i) By Trapezoidal rule

$$\int_0^1 e^{-x^2} dx = \frac{h}{2}\left[(y_0 + y_4) + 2(y_1 + y_2 + y_3)\right]$$
$$= \frac{0.25}{2}\left[1.3678 + 2(2.8761)\right]$$
$$= (0.125)(5.943)$$
$$= 0.7428$$

(ii) By Simpson's rule:

$$\int_0^1 e^{-x^2} dx = \frac{h}{3}\left[(y_0 + y_4) + 2y_2 + 4(y_1 + y_3)\right]$$
$$= \frac{0.25}{2}\left[1.3678 + 1.5576 + 6.0352\right]$$
$$= 0.7467$$

14. The velocity v of a partial ad distances from a point on its path is given by the table.

F	0	10	20	30	40	50	60	Feet
V	47	58	64	65	61	52	38	Feet/fer

Estimate the time taken to travel 60feet by using Simpson's one third value. Compare the result with Simpson's 3/8 rule.

Solution:

We know that the rate of charge of displacement is velocity

$$\frac{ds}{dt} = V$$
$$ds = vdt$$
$$dt = \frac{1}{v}ds \qquad \rightarrow (1)$$

Here we find to find the time taken to travel 60 feet. Therefore interstate (1) from 0 to 60

We get

$$\int_0^{60} dt = \int_0^{60} \frac{1}{v} ds$$

The time taken to travel 60 feet is

$$t = \int_0^{60} \frac{1}{v} ds = \int_0^{60} y dx$$

The given table can write as given below:

X(5)	0	10	20	30	40	50	60
$Y = 1/V$	0.2127	0.017	0.0156	0.01538	0.0164	0.01923	0.0263

	Y_0	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6

Simpson's one third we have

$$\int_0^{60} ydx = \frac{h}{3}\left[(y_0+y_6)+2(y_2+y_4)+4(y_1+y_3+y_5)\right]$$

$$=\frac{10}{3}\left[(0.02127+0.0263)+2(0.01563+0.0164)\right.$$

$$\left.+4(0.01724+0.01538+0.01923)\right]$$

$$=\frac{10}{3}\left[0.04758+0.020740+0.06406\right]$$

$$=1.06\,\text{sec}$$

Hence time taken to travel 60 feet is 1.063 feets

By Simpson's 3/8 rule

$$\int_0^{60} ydx = {}^{3h}\!/_8\left[(y_0+y_6)+3(y_1+y_2+y_4+y_5)+2(y_3)\right]$$

$$=\frac{3(10)}{8}\left[(0.02127+0.02630)+3(0.01723+0.0156)\right.$$

$$\left.+(0.01640+0.01923)+2(0.01538)\right]$$

$$=3.75\left[0.04757+0.20547+0.03076\right]$$

$$=1.064\,\text{sec}$$

15. Evaluate $\int_{0.2}^{1.4}\left(\sin x - \ln x + e^x\right)dx$ by Simpson's 1/3 rule.

Solution:

Let us divide the interval at integration in to twelve equal parts by taking h = 0.1. Now the table of values of the given function y = sinx – lnx + e^x at each point of subdivision is as given below.

X	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Y	3.0291	2.8493	2.7975	2.8213	2.8915	3.014	3.348	3.559	3.559

X	1.1	1.2	1.3	1.4
Y	3.8007	4.0698	4.3705	4.7041

Simpson's rule

$$\int_{0.2}^{1.4} y dx = \frac{h}{3}\left[(y_0 + y_{12}) + 2(y_2 + y_4 + y_6 + y_8 + y_{10}) + 4(y_1 + y_3 + y_5 + y_7 + y_9 + y_{11})\right]$$

$$= \frac{0.1}{3}\left[7.73369 + 2(16.49077) + 4(20.20418)\right]$$

$$= 4.05106$$

33. In an examination the number of candidates who obtained marks between certain limits were as follows:-

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of Students	31	42	51	35	31

Find the number at candidate whose scores lie between 45 and 50.

Solution:-

First we construct a cumulative frequency table for the given table.

$$\begin{array}{llllll} Upper\ limits & 40 & 50 & 60 & 70 & 80 \\ C.F. & 31 & 73 & 124 & 159 & 190 \end{array}$$

The difference table is

x Marks	y C.F.	Δ_y	Δ^2_y	Δ^3_y	Δ^4_y
40	31				
50	73	42			
60	124	51	9		
70	159	35	-16	- 25	
80	190	31	- 4	12	37

We have $x_0 = 40$, $x = 45$, $h = 10$

$$U = \frac{x - x_0}{h} = \frac{45 - 40}{10} = 0.5$$

$y_0 = 73$, $\Delta y_0 = 42$, $\Delta^2 y_0 = 9$, $\Delta^3 y_0 = -25$, $\Delta^4 y_0 = 37$

From Newton's forward interpolation formula

$$f(x) = y_0 + U\Delta y_0 + \frac{U(U-1)}{2!}\Delta^2 y_0 + \frac{U(U-1)\ (U-2)}{3!}\Delta^3 y_0 + \frac{U(U-1)\ (U-2)\ (U-3)}{4!}\Delta^4 y_0 + \ldots\ldots$$

$$f(45) = 31 + (0.5)\ 42 + \frac{(0.5)\ (-0.5)}{2!}\ (9)\ + \frac{(0.5)\ (0.5-1)\ (0.5-2)}{6}\ (-25) + \frac{(0.5)\ (-0.5)\ (-1.5)\ (-2.5)}{24}\ (37)$$

= 47.8673 = 48 approximately.

The number of students who obtained marks less than 45 = 48, and the number of students who scored marks between 45 and 50 = 73 – 48 = 25.

X	0	1	2	5
f(x)	2	3	12	147

34. Find the form of the function f(x) under suitable assumption from the following

Solution:-

The divided differences table is given below.

X	f(x)	Δf(x)	Δ^2f(x)	Δ^3f(x)
0	2			
1	3	$\frac{3-2}{1-0} = 1$		

2	12	$\frac{12-3}{2-1}=9$	$\frac{9-1}{2-0}=4$	
5	147	$\frac{147-12}{5-2}=45$	$\frac{45-9}{5-1}=9$	$\frac{9-4}{5-0}=1$

We have $x_0 = 0$, $f(x_0 = 2$, $f(x_0, x_1) = 1$, $f(x_0, x_1, x_2) = 4$

$f(x_0, x_1, x_2, x_3) = 1$

The Newton's divided difference interpolation formula is

$$f(x) = (fx_0) + (x-x_0)\,f(x_0,x_1) + (x-x_0)\,(x-x_1)\,f(x_0,x_1,x_2)$$
$$+\ (x-x_0)\,(x-x_1)\,(x-x_2)\,f(x_0,x_1,x_2,x_3)$$
$$= 2 + (x-0)\,1 + (x-0)\,(x-1)\,4 + (x-0)\,(x-1)\,(x-2)\,1$$
$$= x^3 + x^2 - x + 2$$

35. Using Lagrange's interpolation formula, find the value of y corresponding to x = 10 from the following table.

x	**5**	**6**	**9**	**11**
f(x)	**12**	**13**	**14**	**16**

Solution:-

We have $x_0 = 5$, $x_1 = 6$, $x_2 = 9$, $x_3 = 11$

$Y_0 = 12$, $y_1 = 13$, $y_2 = 14$, $y_3 = 16$

Using Lagrange's interpolation formula, we have

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

Substitute

$$f(10) = \frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)}(12) + \frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)}(13)$$

$$+ \frac{(10-5)(10-6)(10-11)}{(9-6)(9-6)(9-11)} 14 + \frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)} \times 16$$

$$= 2 - \frac{13}{3} + \frac{35}{3} + \frac{16}{3} = \frac{42}{3}$$

36. Find the value of x when y = 85, using Lagrange's formula from the following table.

x	**2**	**5**	**8**	**14**
y	**94.8**	**87.9**	**81.3**	**68.7**

Solution:-

$x_0 = 2, x_1 = 5, x_2 = 8, x_3 = 14$

$y_0 = 94.8, y_1 = 87.9, y_2 = 81.3, y_3 = 68.7$

$\because$ y = 85

We know that the Lagrange's inverse formula is

$$x = \frac{(y-y_1)(y-y_2)(y-y_3)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)} x_0 + \frac{(y-y_0)(y-y_2)(y-y_3)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)} x_1$$

$$+ \frac{(y-y_0)(y-y_1)(y-y_3)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)} x_2 + \frac{(y-y_0)(y-y_1)(y-y_2)}{(y_3-y_0)(y_3-y_1)(y_3-y_2)} x_3$$

Substituting the above values we get,

$$x = \frac{(85-87.9)\,(85-81.3)\,(85-68.7)}{(94.8-87.9)\,(94.8-81.3)\,(94.8-68.7)} \times 2 + \frac{(85-94.8)\,(85-81.3)\,(85-68.7)}{(87.9-94.8)\,(87.9-81.3)\,(87.9-68.7)} .5$$

$$+ \frac{(85-94.8)\,(85-87.9)\,(85-68.7)}{(81.3-94.8)\,(81.3-87.9)\,(81.3-68.7)} (8) + \frac{(85-94.8)\,(85-87.9)\,(85-68.7)}{(68.7-94.8)\,(68.7-87.9)\,(68.7-81.3)} 14$$

x = 6.5928.

37. Find the first term of the series whose second and subsequent terms are 8, 3, 0, -1, 0 ……..

Solution:-

Given f(2) = 8, f(3) = 3, f(4) = 0, f(4) = -1, f(5) = 0

We are to find f(1)

We construct the difference table with the given values.

X	f(x)	Δf(x)	Δ^2 f(x)	Δ^3 f(x)	Δ^4 f(x)
2	8				

3	3	-5			
4	0	-3	2		
5	-1	-1	2	0	
6	0	1	2	0	0

We have $\Delta^3 f(x) = \Delta^4 f(x) = 0$

Using the displacement operator

$f(1) = E^{-1} (f(2))$

$= (1 + \Delta)^{-1} f(2)$

$= (1 - \Delta + \Delta^2 - \Delta^3 + \ldots.) f(2)$

$= f(2) - \Delta f(2) + \Delta^2 f(2) - \Delta^3 f(2) + \ldots\ldots..$

$= 8 - (- 5) + 2$

$= 15$

$f(1) = 15$

38. Find f(x) as a polynomial in x for the following data by Newton's divided difference formula

X :	-4	-1	0	2	5
F(x) :	1245	33	5	9	1335

Solution:

X	F(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
-4	1245				
		-404			
-1	33		94		
		-28		-14	
0	5		10		3
		2		13	
2	9		88		
		442			
5	1335				

By Newton's divided difference interpolation formula

f(x)= 1245 + (x+4) (-404) + (x+4) (x+1) 94 + (x+4)

(x+1) x(-14) + (x+4) (x+1) x(x-2)3

= $3x^4 + x^3 - 14x+5$

39. The following table gives same relation between steam pressure and temperature. find the pressure at temperature 372.1^0

T	361^0	367^0	378^0	387^0	399^0
P	154.9	167.9	191.0	212.5	244.2

Solution:

T	P	Δp	$\Delta^2 p$	$\Delta^3 P$	$\Delta^4 p$
361	154.9				
		2.016666			
367	167.0		0.0097147		
		2.18181818		0.000024	
378	191.0		0.0103535		0.00000073
		2.388889		0.000052	
387	212.5		0.01203703		
		2.641667			
399	244.2				

By Newton's divided difference formula

$P(T=372.1^0) = 154.9 + (11.1)(2.016666) + (11.1)$

$(5.1)(0.009914) + (11.1)(5.1)(-5.9)(0.000024)$

$+ (11.1)(5.1)(-5.9)(-14.9)(0.00000073)$

$= 177.8394819$

40. From the data given below, find the number of students whose weight is between 60 to 70

Weight	0-40	40-60	60-80	80-100	100-120
No of Students	250	120	100	70	50

Solution:

x Weight	Y No of Students	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
Below 40	250				
		120			
Below 60	370		-20		
		100		-10	
Below 80	470		-30		20
		70		10	
Below 100	540		-20		
		50			
Below 120	590				

$$u = \frac{x - x_0}{h} = \frac{70 - 40}{20} = 1.5$$

$$y(70) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + ...$$

$$=250+(1.5)(120)+\frac{(1.5)(1.5)}{2}(-20)+\frac{(0.5)(0.5)(0.5)}{6}(-10)$$

$$+\frac{(1.5)(0.5)(-0.5)(-1.5)}{24}$$

=424

$\therefore$ Number of students whose weight is between

60 and 70

= y(70) – y(60) = 424 – 370 = 54

Questions

The x values of Interpolating polynomial of newton -Gregory has_________

The value of E is __________

We use the central difference formula such as ___________

-------------- Formula can be used for unequal intervals.

The difference value $\nabla y_1 - \nabla y_0$ in a Newton forward differenc table is denoted by

By putting n = 3 in Newton cote's formula we get ----------- rule.

The process of computing the value of a function outside the range is called -----

The process of computing the value of a function inside the range is called ------

The difference value $y_2 - y_1$ in a Newton's forward difference table is denoted by

_________ Formula can be used for interpolating the value of f(x) near the end of the tabular values.

The technique of estimating the value of a function for any intermediate value is

The $(n+1)^{th}$ and higher differences of a polynomial of the nth degree are --------

Numerical evaluation of a definite integral is called --------

The values of the independent variable are not given at equidistance intervals, we use --------------- formula.

To find the unknown values of y for some x which lies at the --------- of the table, we use Newton's Backward formula.

To find the unknown values of y for some x which lies at the --------- of the table, we use Newton's Forward formula.

To find the unknown value of x for some y, which lies at the unequal intervals we use ------------------- formula.

If the values of the variable y are given, then the method of finding the unknown variable x is called ----------------

In Newton's backward difference formula, the value of n is calculated by -------.

In Newton's forward difference formula, the value x can be written as -----------.

In Newton's backward difference formula, the value x can be written as ---------

------------- Interpolation formula can be used for equal and unequal intervals.

The fourth differences of a polynomial of degree four are ----------.

If the values $x_0 = 0$, $y_0 = 0$ and h = 1 are given for Newton's forward method, then the value of x is -------

The second difference $\Delta^2 y_0$ is equal to

The second difference $\Delta^3 y_0$ is equal to

The differences of constant functions are -----

Δy_2 = -----

$y_n = y_0 + n\ \Delta y_0 + n\,(n-1) / 2!\ \Delta^2 y_0 + n\,(n-1)\,(n-2)/3!\ \Delta^3 y_0 + \ldots\ldots$ is known as

In Newton's forward interpolation formula, the first two terms will give the -----

In Newton's forward interpolation formula, the three terms will give the ------

The difference $\Delta^3 f(x)$ is called -------differences f(x).

n th difference of a polynomial of n th degree are constant and all higher order difference are

In divided difference the value of any difference is ----- of the order of their argument

Central difference equivalent to shift operator is

The differences Δy are called -------differences f(x).

The value (delta +1)is ____________

Opt 1	**Opt 2**	**Opt 3**
even space	equal space	odd space
delta -1	1-delta	delta+1
lagrange's	Newton	Euler
Newton's forward	Newton's backward	Lagrange
$\nabla^2 y_0$	$\nabla^2 y_1$	∇y_1
Simpson's 1/3 rule	Simpson's 3/8 rule	Trapezoidal rule
interpolation	extrapolation	triangularisation
interpolation	extrapolation	triangularisation
Δy_0	∇y_1	Δy_2
Newton's forward	Newton's backward	Lagrange
interpolation	extrapolation	forward method
zero	one	two
integration	differentiation	interpolation
Newton's forward	Newton's backward	Lagrange
beginning	end	center
beginning	end	center
Newton's forward	Newton's backward	Lagrange
Newton's forward	Newton's backward	interpolation
$n = (x - x_n) / h$	$n = (x_n - x) / h$	$n = (x - x_0) / h$
$x_0 - nh$	$x_n - nh$	$x_n + nh$
$x_0 - nh$	$x_n - nh$	$x_n + nh$
Newton's forward	Newton's backward	Lagrange
zero	one	two
0	1	n
$y_2 + 2y_1 - y_0$	$y_2 - 2y_1 - y_0$	$y_2 - 2y_1 + y_0$
$y_3 - 3y_2 + 3y_1 - y_0$	$y_3 + 3y_2 + 3y_1 - y_0$	$y_3 + 3y_2 + 3y_1 + y_0$
Not equal to zero	zero	one
$y_2 - y_3$	$y_1 - y_2$	$y_0 - y_2$
Newton's formula for equal intervals	Bessel's formula	Newton's formula for unequal intervals
extrapolation	linear interpolation	parabolic interpolation
extrapolation	linear interpolation	parabolic interpolation
first	fourth	second
constant	variable	zero

Independent	dependent	Inverse
$E^{½} + E^{-½}$	$E^{½} - E^{-½}$	$E^{½} . E^{-½}$
first	fourth	second
E	h	h2

Opt 4	Opt 5	Opt 6	Answers
unequal			equal space
delta+2			delta+1
bessel's			bessel's
stirling			Lagrange
Δy_0			$\nabla^2 y_0$
Simpson’s rule			Simpson’s 3/8 rule
integration			extrapolation
integration			interpolation
$\nabla^2 y_0$			∇y_1
stirling			Newton’s backward
backward method			interpolation
three			zero
triangularisation			integration
stirling			Lagrange
outside			end
outside			beginning
inverse interpolation			Lagrange
inverse interpolation			inverse interpolation
$n = (x_0 – x) / h$			$n = (x – x_n) / h$
$x_0 + nh$			$x_0 + nh$
$x_0 + nh$			$x_n + nh$
none			Lagrange
three			zero
X			n
$y_2 + 2y_1 + y_0$			$y_2 - 2y_1 + y_0$
$y_3 + 3y_2 + 3y_1 + y3$			$y_3 – 3y_2 + 3y_1 – y_0$
two			zero
$y_3 – y_2$			$y_3 – y_2$
Newton’s formula for Equal and unequal intervals			Newton’s formula for equal intervals
interpolation			linear interpolation
interpolation			parabolic interpolation
third			third
negative			zero

direct	Independent
E	$E^{½} . E^{-½}$
third	first
h4	E

UNIT-IV

SYLLABUS

Numerical differentiation: First derivatives and second order derivates – Richardson extrapolation. Numerical integration: traphezoid rule – simpson's rule (only method) – newton – Cotes open formulas.

Numerical differentiation:

Numerical differentiation is the process of calculating the derivatives of a given function by means of a table given values of that function. i.e., if y (x_i, y_i) are the given set of values, then the process of computing the values of $\frac{dy}{dx}, \frac{d^2y}{dx^2}$......... is called numerical differentiation.

Formula of forward difference formula to compute the derivatives:

$$f^1(x_0) = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 ....\right]$$
$$f^{11}(x_0) = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 ....\right]$$
$$f^{111}(x_0) = \frac{1}{h^3}\left[\Delta^3 y_0 - \frac{3}{2}\Delta^4 y_0 + ...\right]$$

Formula of backward difference formula to computer the derivatives:

$$f^1(x_0) = \frac{1}{h}\left[\nabla y_0 + \frac{1}{2}\nabla^2 y_0 + \frac{1}{3}\nabla^4 y_0 ....\right]$$
$$f^{11}(x_0) = \frac{1}{h^2}\left[\nabla^2 y_0 + \nabla^3 y_0 + \frac{11}{12}\nabla^4 y_0 + ...\right]$$
$$f^{111}(x_0) = \frac{1}{h^3}\left[\nabla^3 y_0 + \frac{3}{2}\nabla^4 y_0 + ...\right]$$

Numerical Integration:

The term numerical integration is the numerical evaluation of a definite integral.

$A = \int_a^b f(x)dx$ Where a and b are given constants and f(x) is a function.

Formula for Trapezoidal Rule:

$$\int_{x_0}^{x_0+nh} y(x)dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + ... + y_{n-1})\right]$$

Formula for Simpson's 1/3 rule:

$$\int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{3}\Big[y_0 + y_n + 4(\text{sum of odd ordinates})$$
$$+2(\text{sum of even ordinates})\Big]$$

Formula for Simpson's 3/8 rule:

$$\int_{x_0}^{x_0+nh} f(x)dx = \frac{3h}{8}\Big[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + ...y_{n-6})$$
$$+2(y_3 + y_6 + ...y_{n-3})\Big]$$

Truncation error in the Trapezoidal rule:

The total error

$$E = \frac{-h^3}{12}\left[y_1^{11} + y_2^{11} + ... + y_n^{11}\right]$$
$$E < \frac{nh^3}{12} y^{11}(\varepsilon)$$

Error in the trapezoidal rule is of the order h^2.

Truncation error in Simpson rules:

The error in the interval (r_1, r_3)

$$= \left(\frac{4}{15} - \frac{5}{18}\right) h^5 y_1^{iv}$$

$$= \left(\frac{24-25}{90}\right) h^5 y^{iv}$$

$$= \frac{-h^5}{90} y_1^{iv}$$

$\therefore$ The total error: $-\frac{nh^5}{90} y^{iv}(\varepsilon)$

Error in the Simpson 1/3 rule is of the order h^4.

Formula for Romberg method:

$$I = I_2 + \frac{(I_2 - I_1)}{3}$$

$I_1 \rightarrow$ dividing h into two parts $\left(h/2\right)$

$I_2 \rightarrow$ dividing h into four parts

$I_3 \rightarrow$ dividing h into eight parts

05. Using Simpson rule find $\int_0^4 e^x dx$ given that $e^0 = 1$, $e^1 = 2.72$, $e^2 = 7.39$, $e^3 = 20.09$,

$e^4 = 54.6$.

Solution:

By Simpson rule we have

$$\int_0^4 e^x dx = \frac{h}{u}\left[(y_0 + y_4) + 2y_2 + 4(y_1 + y_3)\right]$$

$$= \frac{1}{3}\left[(54.6+1) + 2(7.39) + 4(2.72 + 20.04)\right]$$

$$=53.8733$$

06. Write the polynomial to calculate the value of x when?

X	3	5	7	9
Y	6	24	58	108

Solution:

X	Y	Δy	Δ²y	Δ³y
3	6			
		18		
5	24		16	
		34		0
3	58		16	
		50		
9	108			

$$x_0 + nx = x$$

$$3 + n2 = x; \quad 2n = x - 3; \quad n = \frac{x-3}{2}$$

$$y(x_0+nx)=y_0+n\Delta y_0+\frac{n(n-1)}{2!}\Delta^2 y_0+.....$$

$$y(x)=6+\left(\frac{x-3}{2}\right)18+\frac{\left(\frac{x-3}{2}\right)\left(\frac{x-3}{2}-1\right)}{2}(16)$$

$$y(x)=2x^2-3x+9$$

07. Evaluating $\int_0^1 \frac{dx}{1+x^2}$ by a numerical iteration method, Find this value?

Solution:

$$\int_0^1 \frac{dx}{1+x^2}=\left[\tan^{-1}(x)\right]_0^1$$

$$=\tan^{-1}(1)-\tan^{-1}(0)$$

$$=\tan^{-1}(1)$$

$$=\pi/2$$

09. Find the value of log $2^{1/3}$ from $\int_0^1 \frac{x^2}{1+x^3}dx$ using Simpson's 1/3 rule with h = 0.25.

Solution:

Given h = 0.25

X	0	0.25	0.5	0.75	1
Y	0	0.06154	0.222	0.395	0.5

By Simpson's 1/3 rule

$$\int_0^1 \frac{x^2}{1+x^3}dx=\frac{h}{3}\left[(y_0+y_4)+2y_2+4(y_1+y_3)\right]$$

$$=0.23\log 3$$

We know that

$$\int_0^1 \frac{x^2}{1+x^3}dx = \left[\frac{1}{3}\log\left(1+x^3\right) \right]_0^1$$

$$= \frac{1}{3}\log e^2$$

$$\log\left(2^{1/3}\right) = \int_0^1 \frac{x^2}{1+x^3}dx = 0.231083$$

10. Find the first second and 3rd derivatives of the function tabulated below at the point x=1.5?

X	1.5	2.0	2.5	3.0	3.5	4.0
F(x)	3.325	7.0	13.625	24.0	38.87	59.0

Solution:

The table of difference is as follows:

X	F(x)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1.5	3.375				
		3.625			
2.0	7.0		3.0		
		6.625		0.75	
2.5	13.625		3.25		0
		10.375		0.75	
3.0	24.0		4.50		0
		14.875		0.75	
3.5	38.875		5.25		

4.0	59.0	20.125			

Here we have to find the derivatives at the point x = 1.5 which is the starting value of the table. Therefore newton's forward differences formula for derivatives at $x = x_0$, we have

$$f^1(x_0) = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 ....\right]$$

$$f_0 = 1.5,\ h = 0.5$$

$$f^1(1.5) = \frac{1}{0.5}\left[3.625 - \frac{1}{2}(3) + \frac{1}{3}(0.75)\right]$$

$$=4.75$$

$$f^{11}(x_0) = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 ....\right]$$

$$=\frac{1}{(0.5)^2}[3.0 - 0.75]$$

$$=9.0$$

$$f^{111}(x_0) = \frac{1}{h^3}\left[\Delta^3 y_0 - \frac{3}{2}\Delta^4 y_0\right]$$

$$=\frac{1}{(0.5)^3}[0.75]$$

$$=6.0$$

11. The population of certain town is shown in the following table.

Year	**1931**	**1941**	**1951**	**1961**	**1971**
Population (in thousands)	**40.6**	**60.8**	**79.9**	**103.6**	**132.7**

Find the rate of growth of the population in the year 1961.

Solution:

The table of difference is as follows:

X	Y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1931	40.6				
		20.2			
1941	60.8		-1.1		
		19.1		5.7	
1951	79.9		4.6		-4.9
		23.7		0.8	
1961	103.6		5.4		
		29.7			
1971	132.7				

Here h = 10, x_0 = 1971

We know that

$$y^1(x_0+nh) = \frac{1}{h}\left[\nabla y_0 + \frac{2n+1}{2}\nabla^2 y_0 + \frac{3r^2+6r+2}{6}\nabla^3 y_0 + \frac{2r^3+9r^2+11r+3}{12}\nabla^4 y_0\right]$$

$y^1(1961) = ?$

$x_0 + nh = 1961$

$1971 + n10 = 1961$

$n10 = -10 \Rightarrow \boxed{n=-1}$

$$y^1(1961) = \frac{1}{10}\left[29.1 + \left(-\frac{1}{2}\right)5.4 + \left(-\frac{1}{6}\right)(0.8) + \left(-\frac{1}{12}\right)(-4.9)\right]$$

$$= \frac{1}{10}[29.1 - 2.7 - 0.1334 + 0.4083]$$

$$=2.6775$$

$$= \frac{1}{3}[4\times 0.7899 - 0.7943] = 0.7884$$

we get

$$I\left(h_1, \frac{h}{2}, \frac{h}{4}\right) = \pm I(0.6, 0.3, 0.15)$$

$$= \frac{1}{3}\left[4\times I(0.15)0.3 - I(0.3,\ 0.6)\right]$$

$$= \frac{1}{3}[4\times 0.7884 - 0.7886]$$

$$=0.7883$$

The table of these values

0.8113		
	0.7886	
0.7948		0.7883
	0.7884	
0.7899		

$$I = \int_0^2 \frac{dx}{1+x} = 0.7883$$

17. Evaluate to integral $I = \int_1^2 \int_1^2 \frac{dxdy}{x+y}$ **Using the trapezoidal rule with h = k = 0.5 and h = k = 0.25**

Solution:

When h = k = 0.5

y / x	$Y_0=1$	$Y_1=1.5$	$Y_2=2$
$X_0=1$	$f_{00}=0.5$	$f_{01} = 0.4$	$f_{02}=0.33$
$X_1=1.5$	$f_{20}=0.33$	$f_{11}=0.33$	$f_{22}=0.25$
$X_2 = 2$		$f_{21}=0.285$	

$$I=\frac{hk}{u}\left[f_{00}+f_{02}+f_{21}+f_{22}+2\left(f_{01}+f_{10}+f_{21}\right)+4\left(f_{11}\right)\right]$$

$$=\frac{1}{16}\left[\left(0.5+0.33+0.25+0.33\right)+2\left(0.4+0.4+0.285\right)+0.285+4\left(0.33\right)\right]$$

$$=0.3418$$

When

y/x	**Y_0=1**	**Y_1=1.25**	**Y_2=1.5**	**Y_3=1.25**	**Y_4=2.0**
X_0=1	f(1,1) f(1.25,1)	f(1,1.25)	f(1,1.5)	f(1,1.75)	f(1,2.0) f(1.25,2)
X_1=1.25		f(1.25,1.25)	f(1.25,1.5)	f(1.25,1.75)	
X_2=1.5	f(1.5,1) f(2,1)	f(1.5,1.25) f(2,1.25)	f(1.5,1.5) f(2,1.5)	f(1.5,1.75) f(2,1.75)	f(1.5,2) f(2,2)
X_3=1.75		f(1.75,1.25)	f(1.75,1.5)	f(1.75,1.75)	
X_4=2.0					

$$I = \frac{1}{65}\{f(1,1) + f(1,2) + f(2,1) + f(2,2) + 2f(1,1.25) + f(1,1.5)$$
$$+f(1,1.75) + f(1.25,1) + f(1.5,1) + f(1.75,1) + f(2,1.25)$$
$$+f(2,1.5) + f(2,1.75) + f(1.25,2) + f(1.5,2) + f(1.75,2)\}$$
$$+4\{f(1.25,1.25) + f(1.25,1.5) + f(1.25,1.25) + f(1.5,1.25)$$
$$+f(1.5,1.25) + f(1.5,1.5) + f(1.5,1.75) +$$
$$f(1.75,1.25) + f(1.25,1.5) + f(1.25,1.25)\}$$
$$= 0.3401$$

18. Using Trapezoidal and Simpson's rule evaluate $I = \int_4^{4.4}\int_2^{2.6} \frac{dydx}{xy}$

Solution:

Taking h = 0.3 along x direction and k =0.2 along y direction we can construct the following table where

$$f(x,y) = \frac{1}{xy}$$

y/x	**2.0**	**2.3**	**2.6**
$X_0=4.0$	$f(x_0,y_0)=0.125$	$f(x_0,y_1)=0.108$	$f(x_0,y_2)=0.0961$
$X_1=4.2$	$f(x_1,y_0)=0.119$	$f(x_1,y_1)=0.1035$	$f(x_1,y_1)=0.0916$

$X_2=4.4$	$f(x_2,y_0)=0.1236$	$f(x_2,y_1)=0.0988$	$f(x_2,y_2)=0.0874$

$$I=\frac{0.2\times0.3}{4}\left[0.1250+0.0961+0.1136+0.0874\right.$$
$$\left.+2\left(0.1190+0.1087+0.0916+0.0988\right)+4\left(0.1035\right)\right]$$
$$=0.0250$$

using Simpson's rule

$$I=\frac{hk}{9}\left[f_{00}+f_{20}+f_{22}+4\left(f_{10}+f_{01}+f_{11}+f_{21}\right)+16f_{11}\right]$$
$$=\frac{0.2\times0.3}{9}\left[0.1250+0.0961+0.1136+0.0874+4(0.1087\right.$$
$$\left.+0.1190+0.0916+0.988)+16\left(0.1035\right)\right]$$
$$=0.0250$$

19. If D, E, δ and μ be the operators with usual meaning and if hD = u where h13 the interval at differencing. Prove that the following relations between the operator's.

(i) E= e^u

(ii) $\delta=2\sinh\left(u/2\right)$

(iii) $\mu=\cosh\left(u/2\right)$

(iv) (E+1) δ=2(E-1)μ

Solution:

(i) E = e^{hD}

$=E^u$ (hD=u)

(ii) 2 sin h $u/2$

$$= 2\left[\frac{e^{u/2} - e^{-u/2}}{2}\right]$$
$$= \left(E^u\right)^{1/2} - \left(E^u\right)^{-1/2}$$
$$= \delta(\text{by dt})$$

(iii) cos $u/2$

$$= 1/2\left(E^{u/2} + E^{-u/2}\right)$$
$$= \frac{\left(E^u\right)^{1/2} + \left(E^u\right)^{-1/2}}{2}$$
$$= \frac{E^{1/2} + E^{-1/2}}{2}$$
$$= \mu$$

(iv)

$$(E+1)\delta = (E+1)\left(E^{1/2} - E^{-1/2}\right)$$
$$=E^{1/2}\left(E^{1/2} + E^{-1/2}\right)\left(E^{1/2} - E^{-1/2}\right)$$
$$=(E\text{-}1)2\left(\frac{E^{1/2} + E^{-1/2}}{2}\right)$$
$$=2(E\text{-}1)\mu$$

20. **Given that** $\sqrt{12500} = 111.8034, \ \sqrt{125110} = 111.8481$

find the value of $\sqrt{12516}$ **.**

The difference table

X	$y=\sqrt{x}$	Δy	$\Delta^2 y$
12500	111.8034		
		0.0471	
12510	111.8481		0
		0.0447	
12520	111.8928		0
		0.0447	
12530	111.9375		

We have x_0 = 12500, h = 10, x = 12516

$$u=\frac{x-x_0}{h}=\frac{12516-12510}{10}=1.6$$

$$f(x)=y_0+u\Delta y_0+\frac{u(u-1)}{21}\Delta^2 y_0+.....$$

$$f(12516)=111.8034+1.6\times 0.0447$$

$$=111.8034+0.07152$$

$$=111.87492$$

$$\sqrt{12516}=111.87492$$

21.Use Newton's forward interpolation and find value of sin 52 from the following data. Estimate the error.

X	**45**	**50**	**55**	**60**
Y=sinx	**0.7071**	**0.7660**	**0.8192**	**0.8660**

Solution: The difference table

X	Sin x	Δy	$\Delta^2 y$	$\Delta^3 y$
45	0.7071			
		0.0589		
50	0.7660		-0.0057	
		0.0532		-0.0007
55	0.8192		-0.0064	
		0.0462		
60	0.8660			

We have $x_0 = 45$, $x_1 = 52$, $y_0 = 0.7071$, $\Delta y_0 = 0.0589$,

$\Delta^2 y_0 = -0.0057$, $\Delta^3 y_0 = -0.0007$

$$u = \frac{x - x_0}{h} = \frac{52^\circ - 45^\circ}{5^\circ} = 1.4$$

Newton's formula

$$y = u_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + ...$$

$$f(52) = 0.7071 + 1.4(0.0589) + \frac{(1.4)(1.4-1)}{2}(-0.0057) + \frac{(1.4)(1.4-1)(1.4-2)}{3!}(-0.0007)$$

=0.7071+0.8246-0.001596+0.000392

$$\sin 52^\circ = 0.7880032$$

$$Error = \frac{u(u-1)(u-2)....(u-n)}{(n+1)!}\Delta^{n+1} y_0$$

using n = 2 we get

$$= \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

$$= \frac{(1.4)(1.4-1)(1.4-2)}{6}(-0.0007) = 0.0000392$$

22.The following table gives the values of the probability integral $y = \frac{2}{\sqrt{\pi}}\int_0^n e^{-x^2}dx$ corresponding to certain values of x. For what value of x is this integral equation of to ½?

X	0.46	0.47	0.48	0.49
$y = \frac{2}{\sqrt{\pi}}\int_0^n e^{-x^2}dx$	0.4846	0.4937	0.5027	0.5116

Solution:

Here x_0=0.46, x_1=0.47, y_0 = 0.487 y = ½

From Laurens's inverse interpolation formula

$$x=\frac{(y-y_1)(y-y_2)(y-y_3)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)}x_0+\frac{(y-y_0)(y-y_2)(y-y_3)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)}x_1$$
$$+\frac{(y-y_0)(y-y_1)(y-y_3)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)}x_2+\frac{(y-y_0)(y-y_1)(y-y_2)}{(y_3-y_0)(y_3-y_1)(y_3-y_2)}x_3$$

$$=\frac{(0.5-0.49)(0.5-0.5274)(0.5-0.51)}{(0.48-0.49)(0.48-0.52)(0.48-0.51)}(0.46)+$$
$$\frac{(0.5-0.48)(0.5-0.502)(0.5-0.511)}{(0.49-0.48)(0.49-0.502)(0.49-0.511)}(0.47)+$$
$$\frac{(0.5-0.48)(0.5-0.49)(0.5-0.511)}{(0.52-0.48)(0.52-0.49)(0.52-0.511)}(0.48)+$$
$$\frac{(0.5-0.48)(0.5-0.49)(0.5-0.502)}{(0.51-0.48)(0.51-0.49)(0.51-0.502)}(0.49)$$
$$=-0.0207787+0.157737+0.369928-0.0299495$$
$$=0.476937$$

Questions

If the given integral is approximated by the sum of 'n' trapezoids, then the rule is called as ----------------.

The order of error in Trapezoidal rule is -------------.

The general quadratic formula for equidistant ordinates is ________________

h/2[(sum of the first and last ordinates)+2(sum of the remaining ordinates)] is_____________

Use trapezoidal rule for y(x) _______________

Simpson's rule is exact for a ----------------- even though it was derived for a Quadratic.

What is the order of the error in Simpson's formula?

Simpson's 1/3 is findind y(x) upto ____________

In simpson's 1/3, the number of intervels must be ___________

In simpson's 1/3, the number of ordinates must be _______________

Simpson's one-third rule on numerical integration is called a --------- formula.

In simphson's 3/8 rule, we calculate the polynomial of degree ________

The number of interval is multiple of three the use_________

The number of interval is multiple of six _____________

The error in Simpson's 1/3 is ------------.

Modulus of E is ________

The order of error is h^2 for_____________

h^4 is the error of ________

The value of integral e^x is evaluated from 0 to 0.4 by the following formula. Which method will give the least error

Using Simpson's rule the area in square meters included between the chain line, irregular boundary and the first anc

By putting n = 1 in Newton cote's formula we get ----------- rule.

$I = (3h / 8) \{ (y_0 + y_n) + 3 (y_1 + y_2 + y_4 + y_5 + \ldots..)+2(y_3 + y_6 + y_9 + \ldots..) \}$ is known as ------------------.

$I = (h / 3) \{ (y_0 + y_n) + 2 (y_2+ y_4 + y_6 + y_8 + \ldots..)+ 4(y_1 + y_3 + y_5 + \ldots..) \}$ is known as ------------------.

The differentiation of logx is ________

∫ f(x) dx of (a, b) is ____________

h/3[(sum of first and last ordinates)+2(sum of even ordinates)+4(sum of odd ordinates)] is the formula for _______

In simpson 1/3 rule, the integral value is h/3[y0+4(y1)+y2] ______________

Differentiation of sinx is ________________

Integration of cosx _____________

If y(x) is linear then use ___________________

The differentiation of secx is ____________

The notation h is __________

While evaluating the definite integral by Trapezoidal rule, the accuracy can be increased by taking---------

Numerical integration when applied to a function of a single variable, it is known as--------

Opt 1	**Opt 2**	**Opt 3**
Newton's method	Trapezoidal rule	simpson's rule
h	h^3	h^2
raphson	Newton-cote's	interpolation
simphson's 3/8	simphson's 1/3	trapezoidal
linear	second degree	third degree
cubic	less than cubic	linear
Four	three	two
linear	second degree	degree n
any integer	odd	even
any integer	odd	prime
closed	open	semi closed
degree n	linear	second degree
simpson's 1/3	trapezoidal	simpson's 3/8
simpson's 1/3	simphson's 3/8	weddle
h	h^3	h^2
<M(b-a)h4/180	0	>M(b-a)h4/180
lagrange's	trapezoidal	weddle
simphson's 3/8	simphson's 1/3	trapezoidal
Trapezoidal rule with h = 0.2	Trapezoidal rule with h = 0.1	Simpson's 1/3 rule with h = 0.1.
7.33.28 sq-m	744.18 sq-m	880.48 sq-m.
Simpson’s 1/3 rule	Simpson’s 3/8 rule	Trapezoidal rule
Simpson’s 1/3 rule	Simpson’s 3/8 rule	Trapezoidal rule
Simpson’s 1/3 rule	Simpson’s 3/8 rule	Trapezoidal rule
1/x	e(x)	sinx
F(a)		F(a+b)
trapezoidal	simphson's 1/3	simphson's 3/8
for n=1	for n=2	for n=3
cosx	tanx	sinx
cosx	tanx	sinx
simphson's 3/8	simphson's 1/3	trapezoidal
secx tanx	cotx	cosecx
differece of ordinates	sum of ordinates	number of ordinates
Large number of sub-intervals	even number of sub-intervals	multipleof6
maxima	minima	quadrature

Opt 4	Opt 5	Opt 6	Answers
power			Trapezoidal rule
h^4			h^2
divide difference			Newton-cote's
taylor series			trapezoidal
degree n			linear
quadratic			linear
one			Four
third degree			second degree
prime			even
even			odd
semi opened			closed
third degree			third degree
taylor series			simpson's 3/8
trapezoidal			weddle
h^4			h^4
M(b-a)h4/180			<M(b-a)h4/180
simpson's 1/3			trapezoidal
taylor series			simphson's 1/3
weddle			Simpson's 1/3 rule with h = 0.1.
820.38 sq-m			820.38 sq-m
Simpson's rule			Trapezoidal rule
Simpson's rule			Simpson's 3/8 rule
Simpson's rule			Simpson's 1/3 rule
cosx			1/x
F(b)-F(a)			F(b)-F(a)
taylor series			simphson's 1/3
for n=4			for n=2
logx			cosx
logx			sinx
taylor series			trapezoidal
tanx			secx tanx
product of ordinates			differece of ordinates
has multiple of 3			Large number of sub-intervals
quadrant			quadrature

UNIT-V

SYLLABUS

Extrapolation method : Romberg integration- Cosine quadrature. Ordinary differential equations: Euler's method modified Euler's methods – Heun method and mid-point method – Runge-kutta second methods – Heun method without iteration – mid-point method and Ralston's method – classical 4^{th} order Runge-Kutta method.

Formula for Taylor series:

$$Y_{n+1} = y_n + \frac{h}{1!} y_n^1 + \frac{h^2}{2!} y_n^{II} + \frac{h^3}{3!} y_n^{III} + ...$$

Formula for Euler's method or Euler's algorithm:

$Y_{n+1} = y_n + h\, f(x_n, y_n)$, $n = 0,1,2,.....$

Formula for improved Euler's method?:

$Y_{n+1} = y_n + \frac{1}{2} h\, [f(x_n, y_n) + f(x_n + h, y_n + h\, f(x_n, y_n))]$

Formula for modified Eulers method:

$Y_{n+1} = y_n + h[f(x_n + h/2, y_n + h/2\, f(x_n, y_n))$

Formula for fourth order Runge-kutta method:

$K_1 = h\, f(x,y)$

$K_2 = h\, f(x + h/2, y + k_1/2)$

$K_3 = h\, f(x + h/2, y + k_2/2)$

$K_4 = h\, f(x + h, y + k_3)$

$$\Delta y = \frac{1}{6}(K_1 + 2k_2 + 2K_3 + k_4)$$
$$y(x+h) = y(x) + \Delta y$$

Runge-kutta method for simultaneous first order differential equations:

To solve numerically the simultaneous equations

$\frac{dy}{dx} = f_{1(x,y,z)}, and \ \frac{dz}{dx} = f_2(x, y, z) given$ the initial conditions $y(x_0) = y_0$,
$z(r_0) = Z_o$

we starting from (x_0, Y_0, z_0) the increments Δy and ΔZ in y and z respectively are given by formulae

K_1=hf, (x_0,y_0,z_0) $l_1 = hf_2$ (x_o, y_o, z_o)

$$K_2 = hf_1(x_0+\frac{h}{2},y_0+\frac{k_1}{2}+Z_0+\frac{l_1}{2}) \quad l_2 = hf_2(x_0+\frac{h}{2},y_0+\frac{k_1}{2},z_0+\frac{l_1}{2})$$
$$K_3 = hf_1(x_0+\frac{h}{2},y_0+\frac{k_2}{2}+Z_0+\frac{l_2}{2}) \quad l_3 = hf_2(x_0+\frac{h}{2},y_0+\frac{k_2}{2},z_0+\frac{l_2}{2}) \text{ where } h = \Delta x$$
$$K_4 = hf_1(x_0+h,y_0+k_3,Z_0+l_3) \quad l_4 = hf_2(x_0+h,y_0+k_3,z_0+l_3)$$
$$\Delta y = \frac{1}{6}(k_1+2k_2+2k_3+k_4) \ \Delta z = \frac{1}{6}(l_1+2l_2+2l_3+l_4)$$

$y_1=y_0+\Delta y$ and $z_1=z_0+\Delta z$

having got (x_1,y_1,z_1) we get (x_2,y_2,z2) by repeating the above algorithm once again starting from (x_1,y_1,z_1)

Runge-kutta method for second order differential equation (or R-K-method of order from to solve $y^{||} = f(x,y, y^1)$, given $y(x_0)= y_0$ and $y^1(x_0) =y_0^1$?

To solve $y^{II} =f(x,y,y^1)$, given $y(x_0) =y_0$ $y^1(x_0) = y^1{}_0$

Now, set y^1=Z and y" = z^1

Hence, differential equation reduce to $\frac{dy}{dx} = y^1 = z$ and

$\frac{dz}{dx} = z^1 = y'' = f(x, y, y'') = f(x, y, z)$

$\therefore \frac{dy}{dx} = z$ and $\frac{dz}{dy} = f(x, y, z)$ are simultaneous equation Where $f_1(x, y, z) = z$, $f_2(x, y, z) = f(x, y, z)$ given

Also y (0) and z (0) are given

Starting from these equations, we can use the R – K method for simultaneous equation and solve the problem.

Milne's predictor formula:

$$Y_{n+1, P} = y_{n-3} + \frac{4h}{3} (2y_{n-2}^1 - y^1_{n-1} + 2y^1_n)$$

Milne's corrector formula:

$$Y_{n+1, C} = y_{n-1} + \frac{h}{3} (y^1_{n-1} + 4y^1_n + y^1_{n+1})$$

Adam – Bashforth predictor formula:

$$Y_{n+1, P} = y_n + \frac{h}{24} [55y^1_n - 59y^1_n + 37y^1_{n-2} - 9 y^1_{n-3}]$$

Adam – Bashforth corrector formula:

$$Y_{n+1, C} = y_n + \frac{h}{24} [9y^1_{n+1} + 19y^1_n - 5y^1_{n-1} + y^1_{n-2}]$$

Relation between Runge – kutta method of second order and modified Euler's method:

In second order Runge – kutta method,

$\Delta y_0 = k_2 = hf\left(x_0 + \frac{h}{2},\ y_0 + \frac{k_1}{2}\right)$

$\Delta y_0 = hf\left(x_0 + \frac{h}{2},\ y_0 + \frac{1}{2} h\, f(x_0, y_0)\right)$

$\therefore y_1 = y_0 + \Delta y_0 + y_0 + hf\left(x_0 + \frac{h}{2},\ y_0 + \frac{h}{2}\, f(x_0, y_0)\right)$

This is exactly the modified Euler method

So, the Runge – kutta method of second order is nothing but the modified Euler method.

Numerical Examples:

01. Using Taylor series method, find correct to four decimal places, the values of y (0.1), given $\frac{dy}{dx} = x^2 + y^2$ and y (0) = 1

Solution:

We have $y^1 = x^2 + y^2$

$Y^{ii} = 2x + 2yy'$

$Y^{iii} = 2 + 2yy'' + 2'^2$

$Y^{iv} = 2yy^{iii} + 2y^i y^{ii} + 4y^i y^{ii}$

$= 2yy^{iii} + 6y^i y^{ii}$

$x_0 = 0,\ y_0 = 1,\ h = 0.1$

$x_1 = 0.1$, $y_1 = y(0.1) = ?$

$Y_0^1 = x_0^2 + y_0^2 = 0 + 1 = 1$

$Y_0^{ii} = 2x_0 + 2y_0y_0^1 = 2$

$Y_0^{iii} = 2 + 2(1)(2) + 2(1)^2 = 8$

$Y_0^{iv} = 2 \times 1 \times 8 + 6(1)(2) = 28$

By Taylor series method

$Y_1 = y_0 + \frac{h}{1!}y_0^1 + \frac{h^2}{2!}y_0^2 + \frac{h^3}{3!}y_0^3 + \ldots\ldots$

$Y(0.1) = y_1 = 1 +$

$\frac{0.1}{1}(1) + \frac{(0.1)^2}{2}(2) + \frac{(0.1)^3}{6}(8) + \frac{(0.1)^4}{24}(28) + \ldots.$

$= 1 + 0.1 + 0.01 + 0.0013333 + 0.000116666$

$= 1.11144999$

$= 1.11145$

02. Using Taylor series method, find y (1.1) correct to four decimal places given $\frac{dy}{dx} = xy^{1/3}$ and y (1) = 1

Solution:

Take $x_0 = 1$, $y_0 = 1$, $h = 0.1$

$Y^1 = xy^{1/3}$

$Y^{ii} = \frac{1}{3}xy^{-2/3}y^1 + y^{2/3}$

$= \frac{1}{3} x^2 y^{-1/3} + y^{1/3}$

$y^{iii} = \frac{x^2}{3}\left(\frac{-1}{3}\right) y^{-\frac{4}{3}} y^1 + \frac{2x}{3} y^{-\frac{1}{3}} + \frac{1}{3} y^{-\frac{2}{3}} y^1$

$y_0^1 = 1\,(1)^{1/3} = 1$

By Taylor series $Y_1 = y\,(1.1) = 1+0.1 + \frac{(0.2)^2}{2}\left(\frac{4}{3}\right) + \frac{(0.1)^3}{6}\left(\frac{8}{9}\right) + \ldots\ldots$

$= 1+0.1 + 0.00666 + 0.000148 + \ldots\ldots$

$= 1.10681$

03. Using Taylor series method, find y (0.1) given $\frac{dy}{dx} = x^2 - y$, y (0) = 1 (correct to 4 decimal places)

Solution:

$X_0 = 0$, $y_0 = 1$, $h = 0.1$, $x_1 = 0.1$

$Y^1 = x^2 - y$

$Y^{ii} = 2x - y^1$

$Y^{iii} = 2 - y^{ii}$

$Y^{iv} = - y^{iii}$

$Y_0^1 = x_0^2 - y_0 = 0 - 1 = -1$

$Y_0^{11} = 2x_0 - y_0^1 = 0 - (-1) = 1$

$Y_0^{iii} = 2 - 1 = 1$

$Y_0^{iv} = -1$

$\therefore y\,(o.1) = 1+0.1\,(-1)+$

$$\frac{0.01}{2}(1)+\frac{(0.001)}{6}(1)+\frac{(0.0001)}{24}(-1,+.....)$$

=0.905125

04. Given y^1 = - y and y (0) = 1, determine the value of y at x = (0.01) (0.01) (0.04) by Euler method

Solution:

$Y^1 = -y$, $x_0 = 0$, $y_0 = 1$, $x_1 = 0.01$, $x_2 = 0.02$, $x_3 = 0.03$, $x_4 = 0.04$

We have to find y_1, y_2, y_3, y_4 takes h = 0.01

By Euler algorithm, $y_{n+1} = y_n + hy_n^1 = y_n + hf(x_n, y_n)$

$Y_1 = y_0 + h f(x_0, y_0) = 1 + (0.01)(-1) = 0.99$

$Y_2 = y_1 + hy_1^1 = 0.99 + (0.01)(-y_1)$

= 0.99 + (0.01) (-0.99)

=0.9801

$y_3 = y_2 + hf(x_2, y_2) = 0.9801 + (0.01)(-0.9801)$

= 0.9703

$y_4 = y_3 + h f(x_3, y_3) = 0.9703 + (0.01)(-0.9703) = 0.9606$

05. Compute y at x = 0.25 by modified Euler method given $y^1 = 2xy$, y (0) = 1

Solution:

Here $f(x, y) = 2xy$, $x_0 = 0$, $y_0 = 1$

Take h = 0.25, $x_1 = 0.25$

By modified Euler method

$Y_1 = y_0 + h$ [f $\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2} \text{ f } (x_0, y_0) \right)$

f (x_0, y_0) = f (0, 1) = 2 (0) (1) = 0

$\therefore y_1$ = 1 +0.25 [6 (0.125, 1)]

= 1+0.25 [2 × 0.125, 1]

=1 = 0.25 [2 × 0.125 ×1]

= 1.0625

06. Solve $\frac{dy}{dx}$=-2x – y, y (0) = -1 by Taylor series method to find y (0.1) compare it with exact solution?

Solution:

Here $x_0 = 0$, $y_0 = -1$, h = 0.1

$Y^1 = -2x - y$

$Y^{ii} = -2 - y^1$

$Y^{iii} = - y^{ii}$

$Y^{iv} = - y^{iii}$

$Y_0^1 = -2x_1 - y_0 = 1$

$Y_0^{11} = -2 - 1 = -3$

$Y_0^{iii} = 3$

$Y_0^{iv} = -3$

$\therefore y_1 = 1 + \frac{0.1}{1!} \times 1 + \frac{(0.1)^2}{2!} \times (-3) + \frac{(0.1)^3}{3!} \times 3 + \frac{(0.1)^4}{4!} \times (-3) + ...$

= 1+ 0.1 -0.015 +0.0005 – 0.0000125

= -0.91451

07. Solve $\frac{dy}{dx}$=x (1+x^3y), y (0) = 3 by Euler's method for y (0.1)

Solution:

$X_0 = 0$, $y_0 = 3$, h = 0.1, $x_1 = 0.1$

By Euler's algorithm is $y_1 = y_0 + hf(x_0, y_0)$

= 3 +0.1 f (0, 3) = 3 + 0.1(0)

= 3

08. Solve $\frac{dy}{dx}$= 2x +3y, y(0) = 1 by Euler's method for y (0.1), y (o.2)

Solution:

$X_0 = 0$, $y_0 = 1$, $x_1 = 0.1$

By Euler algorithm, $y_1 = 1+0.1\,[2 \times 0 + 3\times 1] = 1.3$

$Y_2 = y_1 + hf(x_1, y_1)$

= 1.3 + 0.1f [0.1, 1.3]

= 1.3 + 0.1 [2 × 0.1 + 3 × 1.3] = 1.71

09. Obtain the values of y at x = 0.1 using Runge – kutta method of fourth order for the differential equation y^1 = - y, given y (0) = 1

Solution:

Here f (x, y) = - y, $x_0 = 0$, $y_0 = 1$, $x_1 = 0.1$

$K_1 = hf(x_0, y_0) = 0.1 f(0, 1) = -0.1$

$K_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}) = (0.1) f(0.05, 0.95) = -0.095$

$K_3 = hf\left(x_0 + \frac{h}{2}, y_0 \frac{K_2}{2}\right) = (0.1) f(0.05, 0.9525) = -0.09525$

$K_4 = hf(x_0+h, y_0 + K_3) = (0.1) f(0.1, 0.90475) = -0.090475$

$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$y_1 = y_0 + \Delta y = 0.9048375$

10. Compute y (0.3) given $\frac{dy}{dx} + y + xy^2 = 0$, y (0) = 1 by taking h = 0.1 using R.K method of fourth order?

Solution:

$Y^1 = -(xy^2 + y) = f(x, y)$, $x_0 = 0$, $y_0 = 1$, $h = 0.1$ $x_1 = 0.1$

$K_1 = h f(x_0, y_0) = 0.1 [-(x_0 y_0^2 + y_0)] = -0.1$

$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = -0.1 [(0.05)(0.95)^2 + 0.95] = -0.0995$

$K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = (0.1) f(o.1, 0.9005) = -0.0982$

$\therefore y_1 = 1 + \frac{1}{6}[-0.1 + 2(-0.0995) + 2(-0.0995) - 0.0982]$

= 0.9006

11. What are the values of k_1 and l_1 to solve y^{11} +xy^1 + y = 0; y (0) =1, y^1 (0) =0 by Runge kutta method of fourth order

$y^{11} = -xy^1 - y$, $x_0 = 0$, $y_0 = 1$

Setting $y^1 = z$, the equation becomes $y^{11} = z^1 = -xz - y$

$\therefore \frac{dy}{dx} = z = 6, (x, y, z), \frac{dz}{dx} = -xz - y = f_2 (x, y, z)$

given $y_0 = 1$, $z_0 = y_0^1 = 0$

By algorithm, $k_1 = hf_1 (x_0, y_0, z_0) = 0.1\ f_1 (0, 1, 0) = 0$

$L_1 = hf_2 (x_0, y_0, z_0) = 0.1\ f_2 (0, 1, 0) = -1 (0.1) = -0.1$

12. What are the values of k_1 and l_1 solve y^{11} +$2xy^1$-4y=0 ,y(0)=0.2,y^1(0)=0.5.

Solution:

$Let \frac{dy}{dx} = z$ then $\frac{d^2}{dx^2} = \frac{dy}{dx}$ the given differential equation becomes

$\frac{dz}{dx} = -2xz + 4y$ now $\frac{dy}{dx} = z$ and $\frac{dz}{dx} = 2xz + 4y$

$x_0 = 0, y_0 = 0.2\ h = 0.2$ $f_1(x,y,z)=z$, $f_2(x_1,x_2,x_3)=-2x2+4yK_1=hf_1(x_0,y_0,z_0)=0.1\times0.5=0.05$,

$l_1 = ht_2(x_0,y_0,z_0)=0.1[-2\times0\times0.5+4\times.5]=0.8$

13. What are the values of k_1 and l_1 to solve $y^{11} - x^2y^1 - 2xy = 1$ y (0) = 1, y^1 (0) = 0

Solution:

Let $\frac{dy}{dx}$=z

$\therefore$ The given differential equation becomes $\frac{d^2y}{dx^2} = x^2y^1 + 2xy + 1$

$\frac{dz}{dx}$ =x^2z +2xy +1, x_0 =0, y_0= 1, z_0 =0, f_1 (x, y, z) = z

f_2 (x, y, z) = x^2z +2xy +1, h = 0.1

x_1 =$hf_1(x_0, y_0, z_0)$= 0.1 f (0, 1, 0) = 0.1 × 0 = 0

l_1 =hf_2 (x_0, y_0, z_0) = 0.1

14. What are the values of k_1, k_2, l_1 and l_2 from the system of equations, $\frac{dy}{dx}$= x +z, $\frac{dz}{dx}$= x – y given y (0) =2, z (0) = 1 using Runge – Kutta method of fourth order.

Solution:

f_1 (x, y, z) = x + z; f_2 (x, y, z) = x –y

X_0 = 0, y_0 =2, z_0 =1, h = 0.1

Now

K_1 = hf_1 (x_0, y_0, z_0)

= (0.2) f_1 (0, 2, 1)

= (0.1) (0+1)

= 0.1

$K_2 = hf_1\left(x_0+\frac{h}{2},\ y_0+\frac{k_1}{2}, z_0+\frac{l_1}{2}\right)$

= 0.1 f_1 (0.05, 2.05, 0.8)

= 0.085

l_1 = (0.1) f_2 (0, 2, 1)

= (0.1) (0 -2^2)

= - 0.4

$l_2 = hf_2\left(x_0+\frac{h}{2},\ y_0+\frac{k_1}{2}, z_0+\frac{l_1}{2}\right)$

= (0.1) f_2 (0.05, 2.05, 0.8)

= - 0.41525

15. Solve by Euler's method $\frac{dy}{dx}$ = x^2 + y, y (0) = 1 of x = 0.02, 0.04

Solution:

Here $x_0 = 0$, $y_0 =1$, f (x, y) = x^2 + y, h = 0.2

By Euler's algorithm, $y_1 = y_0 + h f (x_0, y_0)$

i.e. $y_1 =1 + 0.02 (x_0^2 + y_0) = 1.02$

$y_2 = y_1 + hf (x_1, y_1)$

= 1.02 +0.02 [$(0.02)^2$ + 1.02]

=1.04041

16. Solve $\frac{dy}{dx}$=x + y, given y (1) = 0 and get y (1.1) by Taylor series method?

Solution:

Here $x_0 = 1$, $y_0 =0$, h = 0.1

$Y^1 =x + y$

$Y^{ii} = 1+y^1$

$Y^{iii} = y^{ii}$

$Y^{iv} = y^{iii}$

$Y_0^1 = x_0 + y_0 = 1 + 0 = 1$

$Y_0^{11} = 1 + y_0^1 = 2$

$Y_0^{iii} = 2$

$Y_0^{iv} =$

By Taylor series, we have

$$Y_1 = y_0 + \frac{h}{1!} y_0^1 + \frac{h^2}{2!} y_0^{11} + \frac{h^3}{3!} y_0^{111} + ...$$

$Y_1 = y\ (1.1) = 0 +$

$$\frac{0.1}{1}(1) + \frac{(0.1)^2}{2} 2 + \frac{(0.1)^3}{6} \times 2 + \frac{(0.1)^4}{24} \times 2 + ...$$

$= 0.11033847$

17. Using Taylor method, compute y (0.2) correct to 4 decimal places given $\frac{dy}{dx}$ = 1 – 2xy and y (0) = 0

Solution:

Here $x_0 = 0$, $y_0 = 0$, h =0.2

$Y^1 = 1 - 2xy$

$Y^{11} = -2\ (xy^1 + y)$

$Y^{iii} = -2\ [xy^{11} + 2y^1]$

$Y^{iv} = -2\ [xy^{iii} + 3y^{11}]$

$Y^{v} = -2\ (xy^{iv} + 4y^{iii}]$

$Y_0^1 = 1 - 2.0.0 = 1$

$Y_0^{11} = 0$

$Y_0^{111} = -4$

$Y_0^{iv} = 0$

$Y_0^{v} = 32$

By Taylor series,

Y_1 =y (0.2) = 0 +

$$\frac{0.2}{1}(1)+\frac{(0.2)^2}{2}(0)+\frac{(0.2)^3}{6}(-4)+0+\frac{(0.2)^5}{120}(32)+....$$

= 0.1948

18. Solve dy/dx = x+y, given y(1) =0, and get y(1.1), y(1.2) by Taylor series method. Compare your result with the analysis.

Solution:

Here x_0= 1, y_o =0x =0.1

Y^1=x + y y_0^I= x0+y0=1+0 = 1

$y^{II} = 1+y^1$ $y_0^{II}=1xy_0^1 = 2$

$y^{III} = y^{II}$ $y_0^{III} = y_0^{II} = 2$

$y^{IV} = y^{III}$ y_0^{iv} = 2 etc

By Taylor series, are have

$$y_1 = y_0 + \frac{h}{1!}y_0^{\ 1} + \frac{h^2}{Z!}y_0^{\ II} + \frac{h^3}{3!}y_0^{\ III} + ...$$

$$\therefore y_1 = y(1.1) = 0 + \frac{0.1}{1}(1) + \frac{(0.1)^2}{2}(2) + \frac{(0.1)^3}{6} + \frac{(0.1)^4}{24}(2)$$

$$+\frac{0.1^5}{120}.2+....---(2)$$

= 0.1+0.01+0.00033+0.00000833+0.000000166+….

Y(1.1) = 0.11033847

Now, take x_0 = 0.1103847

Now, take x_0 = 1.1 h = 0.1,

$$y_2 = y_1 + \frac{h}{1!}y_1^1 + \frac{h^2}{2!}y_I^{II} + \frac{h^3}{3!}y_I^{III} + \frac{h^4}{4!}y_1^{IV} + ...---(3)$$

we calculate $y_1^I, y_1^{II}, y_1^{III}, ...,$

$x_1 = 1.1, y_1 = 0.11033847$

$y_1^I = x_1 + y_1 = 1.1 + 0.11033847 = 1.21033847$

$y_1^{II} = 1 + y_1^I = 2.21033847$

$y_1^{III} = y_1^{II} = y_1^{IV} = y_1^{V} = ... = 2.21033847$

using in (3),

y_2 = y(1.2) = 0.11033847 + 0.1 / 1 (1.21033847)

$$+\frac{(0.1)^2}{2}(2.21033847)+\frac{(0.1)^3}{6}(2.21033847)+\frac{(0.1)^h}{2h}(2.21033847)+....$$

= 0.11033847+2.21033847(0.005+0.0016666+….)

= 0.2461077

The exact solution $\frac{dy}{dx} = x + y$ is y = -x-1+2e^{x-1}

Y (1.1) = -1 1 -1 +2e $^{0.1}$

= 0.11034

y (1.2) = -1.2 – 1+ze$^{0.2}$ = 0.2428

y (1.1) = 0.11033847

y (1.2) = 0.2461077

Exact values: y(1.1) = 0.110341876

Y(1.2) = 0.24280552

19.Using Taylor method compute y (0.2) and y(0.4) correct to 4 decimal places given

$\frac{dy}{dx} = 1 - 2xy$ and y(0) =0

Soln

We know y^1=1-2xy	Here x_0=0 y_0 =0, h=0.2
y^{II} = -2(xy^1+y)	$y_0{}^1$=1-zx_0y_0 = 1
y^{III} =-2(xy^{II}+2y^I)	$y_0{}^{II}$ = 0
y^{IV} = -2(xy^{III} +Jy^{II})	$y_o{}^{III}$ =-4
y^V = -2(xy^{iv}+4y^{III})	$y^{iv}{}_o$ =0
	$y_0{}^{II}$ =J 32

by Taylor series

$$y_I = y_0 + \frac{h}{I!} y_0^{\,1} + \frac{h^2}{2!} y_o^{\,II} + \frac{h^3}{3!} y_0^{\,III} + \ldots.------(!)$$

$$y_1 = y(0.2) = 0 + \frac{(0.2)}{1} 1 + \frac{(0.2)^2}{2}(0) + \frac{(0.2)^3}{6} + \frac{(0.2)^4{}_0}{24} + \frac{(0.2)^5}{120}(32) + \ldots.$$

$y^I = Z - x$ | $Z^1 = x + y$

$y^{II} = Z^1 - 1$ | $Z^{II} = 1 + y^1$

$y^{III} = Z^{II}$etc $Z^{III} = y^{II}$ etc

By Taylor series, for $y_1$and z we have

$$Y_1 = y\,(0.1) = y_0 + hy_0^{\,1} + \frac{h^2}{2!} y_0^{II} + \frac{h^3}{3!} y_0^{\,III} + \ldots\ldots---(1)$$

$$\text{And } Z_1 = Z\,(0.1) = Z_0 + hZ_0^{\,1} + \frac{h^2}{2} Z_0^{\,II} + \frac{h^3}{6} Z_0^{\,III} + \ldots..(2)$$

$Y_0 = 1$ | $z_0 = 1$

$Y_0^1 = Z_0 - x_0 = 1 - 0 = 1$ | $z_0^1 = x_0 + y_0 + 0 = 1 = 1$

$Y_0^{II} = Z_0^1 - 1 = 1 - 1 = 0$ | $z_0^{II} = 1 + y_o^1 = 1 + 1 = 2$

$Y_0^{III} = z_0^{II} = 2$ $z_0^{III} = y_0^{II} = 0$

Substituting in (1) and (2), we get $z_0^{IV} = y_0^{III} = 2$

$$Y_1 = y\,(0.1) = 1 + (0.1) + \frac{(0.01)}{2}(0) + \frac{(0.001)}{6} 2 + \ldots..$$

$= 1 + 0.1 + 0.000333 + \ldots. = 1.1007$ (correct to 4 decimals)

$z_1 = z\,(0.1) = 1 + (0.1) +$

$$\frac{(0.01)}{2}2+\frac{(0.001)}{6}(0)+\frac{0.0001}{24}\times 2+....$$

=1+0.1+0.01 +0.0000083+….

=1.1100 (correct to 4 decimal places)

$\therefore$ y (0.1) = 1.1003 and z (0.1) = 1.1100

20. Solve $\frac{dy}{dx}$ = z - x, $\frac{dz}{dx}$ = y + x with y (0) = 1, z (0) = 1, by taking h = 0.1, to get y (0.1) and z (0.1). Here y and z are dependent variables and x is independent.

Solution:

$Y^1 = z - x$	and $z^1 = x + y$
Take $x_0 = 0$, $y_0 = 1$	take $x_0 = 0$, $z_0 = 1$ and h = 0.1
$Y_1 = y(0.1) = ?$	$Z_1 = z(0.1) = ?$

Using in (6)

$$Y_1 = y(0.1) = 0 + \frac{0.1}{2}[1+0.9] = \frac{0.19}{2} = 0.095$$

$$Y_2 = y_1 + \frac{1}{2}h\,[f(x_1, y_1) + f(x_2, y_1 + hf(x_1, y_1)] \rightarrow (7)$$

$F(x_1, y_1) = 1 - y_1 = 1 - 0.095 = 0.905$

$F(x_2, y_1 + h f(x_1, y_1) = f(0.2, 0.095 + (0.1)(0.905)) = 0.8145$

Using in (7) we get $y_2 = y\ (0.2) = 0.095 + \frac{0.1}{2}[0.905 + 0.8145]$

Y (0.2) = 0.18098

$Y_3 = y_2 + \frac{1}{2} h\ [f\ (x_2, y_2) + 6\ (x_3, x_2 + h\ f\ (x_2, y_2))] \rightarrow (8)$

Using in (8)

$Y_3 = y\ (0.3) = 0.18098 + \frac{0.1}{2}(0.81902 + 1 - 0.26288)$

Y (0.3) = 0.258787

The values are tabulated

X	Modified Euler	Improved Euler	Exact solution
0.1	0.095	0.095	0.09516
0.2	0.18098	0.18098	0.18127
0.3	0.258787	0.258787	0.25918

Modified Euler and improved Euler methods give the same values come A to sin decimal places.

21. Evaluate the values of y (0.1) and y (0.2) given $y^{II} - x\ (y^1)^2 + y^2 = 0$; y (0) =1, y^1 (0) =0 by using Taylor series method?

Solution:

$Y^{II} - x\ (y^1)^2 + y^2 = 0$

Put $y^1 = z \rightarrow (1)$

Hence the eqn reduces to $z^1 - xz^2 + y^2 = 0$

$\therefore z^1 = xz^2 - y^2 \rightarrow (2)$

By initial condition, $y_0 = y(0) = 1$, $z_0 = y_0^1 = 0 \rightarrow (3)$

$Y_1 = 0.2 - 0.00533333 + 0.000085333$

$= 0.194752003$

Now again starting with x = 0.2 as the starting value so, use again eqn (1)

Now $y_0 = 0.2$, $y_0 = 0.194752003$, $h = 0.2$

$Y_0^1 = 1 - 2x_0y_0 = 1 - 2(0.2)(0.194752003) = 0.9220992$

$Y_0^{II} = -2(x_0y_0^1 + y_0) = -2[(0.2)(0.9220992) + 0.194752003]$

$= -0.758343686$

$y_0^{III} = -2[x_0y_0^{II} + 2y_0^1]$

$= -2[(0.2)(-0.758343686) + 2(0.9220992)]$

$= -3.38505933$

$y_0^{IV} = -2[(0.2)(-3.38505933) + 3(-0.758343686)]$

$= 5.90408585$

Using eqn (1), again

$Y_2 = y(0.4) = 0.194752003 + (0.2)(0.9220992)$

$$\frac{(0.2)^2}{2}(-0.758343686) + \frac{(0.2)^3}{6}(-3.38505933) + \frac{(0.2)^4}{24}(5.90408585) = 0.359883723$$

22. Using improved Euler method find y at x =0.1 and y at x =0.2 give $\frac{dy}{dx} = y - \frac{2x}{y}$,

y (0) = 1

Solution:

By improved Euler method,

$Y_{n+1} = y_n + \frac{1}{2} h [f(x_n, y_n) + f(x_n + h, y_n + hf(x_n, y_n))] \rightarrow (1)$

$\therefore y_1 = y_0 + \frac{1}{2} h [f(x_0, y_0 + h f(x_0, y_0))] \rightarrow (2)$

$f(x_0, y_0) = y_0 - \frac{2x_0}{y_0} = 1 - 0 = 1$

$f(x_1, y_0 + h f(x_0, y_0)) = f(0.1, 1.1) = 1.1 - \frac{2 \times (0.1)}{1.1} = 0.91818$

$y(0.1) = y_1 = 1 + \frac{0.1}{2} [1+0.91818] = 1.095909$

$y_2 = y(0.2) = y_1 + \frac{1}{2} h [f(x_1, y_1) + f(x_2, x_1 + hf(x_1, y_1))] \rightarrow (3)$

$f(x_1, y_1) = y_1 - \frac{2x_1}{y_1} = 1.095909 - \frac{2 \times 0.1}{1.095909}$

$= 0.913412$

$f(x_2, y_1 + h f(x_1, y_1) = f(0.2, 1.095909 + (0.1)(0.9134121)$

$= f(0.2, 1.18732) = 1.18732 - \frac{2 \times 0.2}{1.18732} = 0.8594268$

Using in (3), $y_2 = 1.095909 + \frac{0.1}{2} [0.913412 + 0.850427]$

$= 1.\ 1841009$

X	0	0.1	0.2
Y	1	1.095907	1.1841009

23. Apply the fourth order Runge – kutta method, to find y (0.2) given that $y^1 = x + y$,

y (0) = 1

Solution:

Since h is not mentioned in the question we take h = 0.1

$Y^1 = x + y$; y (0) = 1

$\therefore f(x, y) = x+y,\ x_0 = 0,\ y_0 = 1$

$x_1 = 0.1,\ x_2 = 0.2$

By fourth order Runge – kutta method, for the first interative

$K_1 = h\, f(x_0, y_0) = (0.1)(x_0 + y_0) = (10.1)(0+1) = 0.1$

$K_2 = h\, f(x_0 + \frac{1}{2}h,\ y_0 + \frac{1}{2}k_1)$

$= (0.1)\, f(0.05, 1.05) = (1.0)(0.05 + 1.05) = 0.11$

$k_3 = h\, f(x_0 + \frac{1}{2}h,\ y_0 + \frac{1}{2}k_2) = (0.1)\, f(0.05, 1.055)$

$= (0.1)(0.05 + 1.055) = 0.1105$

$k_4 = hf(x_0 + h,\ y_0 + k_3)$

$= (0.1)\, f(0.1, 11\ 05) = (0.1)(0.1 + 1.1105)$

$= 0.12105$

$\therefore \Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$= \frac{1}{6}[0.1 + 0.22 + 0.2210 + 0.12105) = 0.110341667$

$y(0.1) = y_1 = y_0 + \Delta y = 1.110341667 \approx 1.110342$

Now starting from (x_1, y_1) we get (x_2, y_2) again

Apply Runge kutta algorithm replacing (x_0, y_0) by (x_1, y_1)

$K_1 = h\,f(x_1, y_1) = (0.1)(x_1+y_1) = (0.1)(0.1 + .110342) = 0.1210342$

$K_2 = h\,f(x_1+\frac{h}{2}, y_1 + \frac{1}{2}k_1) = (0.1)\,f(0.15, 1.170859)$

$= (0.1)(0.15 + 1.170859) = 0.1320859$

$k_3 = hf(x_1 + \frac{h}{2}, y_1 + \frac{1}{2}k_2) = (0.1)\,f(0.15, 1.1763848$

$= (0.1)(0.15 + 1.1763848) = 0.13262848$

$k_4 = hf(x_1 + h, y_1 + k_3) = (0.1)\,f(0.2, 1.24298048)$

$= 0.144298048$

$Y(0.2) = y(0.1) + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= 1.110342 + \frac{1}{6}(0.794781008$

$Y(0.2) = 1.2428055$.Correct to four decimals places, $y(0.2) = 1.2428$

24. Using the Runge – kutta method, tabulate the solution of the system $\frac{dy}{dx} = x + z$, $\frac{dz}{dx} =$ $x - y$, $y = 0$, $z = 1$ when $x = 0$ at intervals of $h = 0.1$ from $x = 0.0$ to $x = 0.2$.

Solution:

Given f (x, y, z) = x + z, g (x, y, z) = x – y, x_0 =0, y_0 =0, z_0 =1 and h = 0.1

To compute y (0.1) and z (0.1)

$K_1 = hf(x_0, y_0, z_0)$ $= h(x_0 + z_0)$ $= (0.1)(0+1) = 0.1$	$L_1 = hg(x_0, y_0, z_0)$ $= h(x_0 - y_0)$ $= (0.1)(0-0) = 0$
$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right)$ $=h\left[\left(x_0 + \frac{h}{2}\right)+\left(z_0 + \frac{l_1}{2}\right)\right]$ $= (0.1)\left[\left(0+\frac{0.1}{2}\right)+\left(1+\frac{0}{2}\right)\right]$ $=0.105$	$L_2 = hg\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right)$ $=h\left[\left(x_0 + \frac{h}{2}\right)-\left(y_0 + \frac{k_2}{2}\right)\right]$ $= (0.1)\left[\left(0+\frac{0.1}{2}\right)-\left(0+\frac{0.1}{2}\right)\right]$
$K_3 = hf\left[x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right]$ $= h\left[\left(x_0 + \frac{h}{2}\right)+\left(z_0 + \frac{l_2}{2}\right)\right]$ $= (0.1)\left[\left(0+\frac{0.1}{2}\right)+\left(1+\frac{0}{2}\right)\right]$ $= 0.105$	$L_3 = hg\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right)$ $= 4\left[\left(x_0 + \frac{h}{2}\right)-\left(y_0 + \frac{kl2}{2}\right)\right]$ $= (0.1)\left[\left(0+\frac{0.1}{2}\right)-\left(0+\frac{0.105}{2}\right)\right]$ $= -0.00026$

$K_4 = hf(x_0 + h, y_0 + k_3, z_0 + l_3)$ $= h[x_0 + h) + (z_0 + l_3)$ $= (0.1)[(0+0.1) + (1 - 0.00026)$ $= 0.1099$	$L_4 = hg(x_0 + h, y_0 + k_3, z_0 + l_3)$ $= h[x_0 + h) - (y_0 + k_3)$ $= (0.1)[(0 + 0.1) - (0 + 0.105)]$ $= -0.0005$
$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$ $= \frac{1}{6}[0.1+2(0.105) + 2(0.105) + 0.1099)]$ $= 0.1050$	$\Delta z = \frac{1}{6}[l_1 + 2l_2 + 2l_3 + l_4]$ $= \frac{1}{6}[0+0+2(-0.00026) - 0.0005]$ $= 0.00017$
$Y_1 = y_0 + \Delta y$ $= 0 + 0.1050$ $y(0.1) = 0.1050$	$Z_1 = z_0 + \Delta z$ $= 1 - 0.00017$ $z(0.1) = 0.9998$

To compute y (0.2) and z (0.2)

Here $x_1 = 0.1$, $y_1 = 0.1050$, $z_1 = 0.9998$

$K_1 = hf(x_1, y_1, z_1)$ $= h(x_1 + z_1)$ $= (0.1)(0.1 + 0.9998)$ $= 0.1099$	$L_1 = hg(x_1, y_1, z_1)$ $= h(x_1 - y_1)$ $= (0.1)(0.1 - 0.1050)$ $= -0.0005$

$K_2 = hf\left(x_1+\frac{h}{2}, y_1+\frac{k_1}{2}, z_1+\frac{l_1}{2}\right)$ $= h\left[\left(x_1+\frac{h}{2}\right)+\left(z_1+\frac{l_1}{2}\right)\right]$ $= (0.1)\left[\left(0.1+\frac{0.1}{2}\right)+\left(0.9998+\frac{0.0005}{2}\right)\right]$ $= 0.1149$	$L_2=hg\left(x_1+\frac{h}{2}, y_1+\frac{k_1}{2}, z_1+\frac{l_1}{2}\right)$ $=h\left[\left(x_1+\frac{h}{2}\right)-\left(y_1+\frac{k_1}{2}\right)\right]$ $=(0.1)\left[\left(0.1+\frac{0.1}{2}\right)\left(0.105+\frac{0.1099}{2}\right)\right]$ $= -0.0099$
$K_3 = hf\left(x_1+\frac{h}{2}, y_1+\frac{k_2}{2}, z_1+\frac{l_2}{2}\right)$ $= h\left(\left(x_1+\frac{h}{2}\right)+\left(z_1+\frac{l_2}{2}\right)\right)$ $= (0.1)\left[\left(0.1+\frac{0.1}{2}\right)+\left(0.9998+\frac{0.00099}{2}\right)\right]$ $= 0.1149$	$L_3 = hg\left(x_1+\frac{h}{2}, y_1+\frac{k_2}{2}, z_1+\frac{l_2}{2}\right)$ $= h\left[\left(x_1+\frac{h}{2}\right)-\left(y_1+\frac{k_2}{2}\right)\right]$ $=(0.1)\left[\left(0.1+\frac{0.1}{2}\right)-\left(0.1050+\frac{0.1149}{2}\right)\right]$ $= -0.00125$
$K_4 = hf (x_1 + h, y_1+k_3, z_1+l_3)$ $= h [(x_1 +h) + z_1+l_3)]$ $= (0.1) [(0.1+0.1) + (0.9998 – 0.00125)]$ $= 0.1198$	$L_4 = hg [x_1+h, y_1+k_3, z_1 + l_3]$ $= h [(x_1+h) – (y_1 + k_3)]$ $= (0.1) [(0.1 +0.1) – (0.1050 + 0.1149)]$ $= -0.00199$
$\Delta y = \frac{1}{6}[k_1 +2k_2 + 2k_3 +k_4]$ $= \frac{1}{6}$ [0.1099 + 2 (0.1149) + 2 (0.1149) + 0.1198)]	$\Delta z = \frac{1}{6}[l_1 + 2l_2 + 2l_3 +l_4]$ $= \frac{1}{6}$[-0.0005 +2(-0.00049)+2 (-0.00125) – 0.001199]

$= \frac{1}{6}[0.1099+0.2298 + 0.2298 + 0.1198]$ $= 0.1149$	$=\frac{1}{6}[-0.0005 -0.00198 -0.00199]$ $= \frac{1}{6}[-0.0005 – 0.00198 – 0.00199$ $= -0.00116$
$Y_2 = y_1 + \Delta y$ $= 0.1050 +0.1149$ $= 0.2199$ $y (0.2) = 0.2199$	$Z_2 = z_1 + \Delta z$ $= 0.9998 – 0.00116$ $= 0.9986$ $z (0.1) = 0.9986$

	X=0	X = 0.1	X = 0.2
Y	0	0.1050	0.2199
X	1	0.9998	0.9986

25. Solve $\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^2 + y^2 = 0$ **using Runge – kutta method for x = 0.2 correct to 4 decimal places. Initial condition are x = 0, y =1, $y^1 = 0$**

Solution:

Given:

$$\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^2 + y^2 = 0 \rightarrow (1)$$

Put $\frac{dy}{dx}$=z $\rightarrow$ (2)

$\therefore \frac{d^2y}{dx} = \frac{d^2}{dx} \rightarrow$ (3)

Substituting (2) and (3) in (1), we get

$\frac{dz}{dx} = xz^2 - y^2$

Let $\frac{dz}{dx} = xz^2 - y^2 = g\ (x, y, z)$

$\frac{dz}{dx} = xz^2 - y^2 = g\ (x, y, z)$

Also we are give that $y_0 = 0$, $y_0 = 1$, $y_0^1 = 0$ (or) $z_0 = 0$, $h = 0.2$

Now

$K_1 = hf\ (x_0, y_0, z_0)$ $\quad l_1 = hg\ (x_0, y_0, z_0)$

$= hz_0 = 0$ $\quad = h\ (x_0z_0^2 - y_0^2)$

$= (0.2)\ (0 - 1) = -\ 0.2$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right)$

$= h\left[z_0 + \frac{l_1}{2}\right] = (0.2)\left(0 - \frac{0.2}{2}\right) = -\ 0.02$

$l_2 = hg\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 = \frac{l_1}{2}\right)$

$= h\left[\left(x_0 + \frac{h}{2}\right)\left(z_0 + \frac{l_1}{2}\right)^2 - \left(y_0 + \frac{k_1}{2}\right)^2\right]$

$l_2 = (0.2)\left[\left(0+\frac{0.2}{2}\right)\left(0-\frac{0.2}{2}\right)^2-\left(1+\frac{0}{2}\right)^2\right]$

= -0.1998

$k_3 = hf\left(x_0+\frac{h}{2}, y_0+\frac{k_2}{2}, z_0+\frac{l_2}{2}\right)$

$= h\left[z_0+\frac{l_1}{2}\right] = (0.2)\left(0-\frac{0.1998}{2}\right)$

= - 0.01998

$l_3 = hg\left(x_0+\frac{h}{2}, y_0+\frac{k_2}{2}, z_0+\frac{l_2}{2}\right)$

$= h\left[\left(x_0+\frac{h}{2}\right)\left(z_0+\frac{l_2}{2}\right)^2-\left(y_0+\frac{k_2}{2}\right)^2\right]$

$= (0.2)\left[\left(0+\frac{0.2}{2}\right)\left(0-\frac{0.01998}{2}\right)^2-\left(1-\frac{0.02}{2}\right)^2\right]$

= - 0.1958

$k_4 = hf\,(x_0 + h,\ y_0 + k_3,\ z_0 + h_2)$

$= h\,(z_0 + l_3) = (0.2)\,(0 - 0.1958)$

= - 0.0392

$l_4 = hg\,(x_0 + h,\ y_0 + k_3,\ z_0 + l_3)$

$= h\,[(x_0 + h)\,(z_0 + l_3)^2 - (y_0 + k_3)^2]$

$= (0.2)\,[(0.2)\,(0 - 0.1958)^2 - (1 - 0.01998)^2]$

= - 0.1906

$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= \frac{1}{6}[0 + 2(-0.02) + 2(-0.01998) - 0.0392]$

$= -0.0199$

$\therefore y(0.2) = y_1 = y_0 + \Delta y$

$= 1 - 0.0199$

$= 0.9801$

$\therefore y(0.2) = 0.9801$

26. The differential equation $\frac{dy}{dx} = y - x^2$ is satisfied by y (0) = 1, y (0.2) = 1.12186, y (0.4) = 1.46820, y (0.6) = 1.7379 compute the value of y (0.8) by Milne's predictor corrector formula?

Solution:

Given $\frac{dy}{dx} = y^1 = y - x^2$ and h = 0.2

$X_0 = 0$ $y_0 = 1$

$X_1 = 0.2$ $y_1 = 1.12186$

$X_2 = 0.4$ $y_2 = 1.46820$

$X_3 = 0.6$ $y_3 = 1.7379$

$X_4 = 0.8$ $y_4 = ?$

By Milne's predictor formula, we have

$Y_{n+1, P} = y_{n-3} + \frac{4h}{3} [zy_{n-2}^1 - y^1_{n-1} + 2y^1_n] \rightarrow (1)$

To get y_n, put n = 3 in (1) we get

$Y_{n, P} = y_0 + \frac{4h}{3} [2y^1_1 - y^1_2 + 2y^1_3] \rightarrow (2)$

Now $y^1_1 = (y - x)_1{}^2 = y_1 - x_1^2$

$= 1.12186 - (0.2)^2 = 1.08186 \rightarrow (3)$

$y_2^1 = (y - x^2)_2 = y_2 - x_1^2$

$= 1.\ 46820 - (0.4)^2 = 1.3082 \rightarrow (4)$

$y_3^1 = (y - x^2)_3 = y_3 - x_3^2$

$= 1.7379 - (0.6)^2 = 1.3779 \rightarrow (5)$

Substituting (3), (4) and (5) and (2), we get

$Y_{h, g} = 1 + \frac{4(0.2)}{3} [2(1.08186) - 1.3082 + 2 (1.3779)]$

$= 1+0.266 [2.1637 - 1.3082 + 2.7558]$

$= 1.9630187$

$\therefore y (0.8) = 1.9630187$ (by predictor formula)

By Milne's corrector formula we have

$Y_{n+1, C} = y_{n-1} + \frac{h}{3} (y^1_{n-1} + 4y^1_n + y^1_{n+1})$

To get y_h, put n = 3, we get

$Y_{h,C} = y_2 + \frac{h}{3}(y_2^1 + hy_3^1 + y_n^1) \rightarrow (6)$

Now $y_n^1 = (y - x^2)_h = y_h - x_h^2$

$= 1.96277 - (0.8)^2$

$= 1.3230187 \rightarrow (7)$

Substituting (4), (5), (7) in (6) we get

$Y_{4,C} = 1.46820 + \frac{0.2}{3}[1.3082 + 4(1.3779) + 1.3230187]$

$= 2.0110546$

i.e. y (0.8) = 2.0110546

27. Using Taylor's series method, solve $\frac{dy}{dx} = xy + y^2$, y (0) = 1 at x = 0.1, 0.2 and 0.3 continue the solve at x = 0.4 by Milne's predictor corrector method?

Solution:

Given $y^1 = xy + y^2$, and $x_0 = 0$, $y_0 = 1$ and h = 0.1

Now $y^1 = xy + y^2$

$Y^{11} = xy^1 + y + 2yy^1$

$Y^{III} = xy^{II} + 2y^1 + 2yy^{II} + 2y^{12}$

To find y (0.1)

By Taylor series we have

$y(0.1) = y_1 + hy_0^1 + \frac{h^2}{2!} y_0^{II} + \frac{h^3}{3!} y_o^{III} + \ldots\ldots (1)$

$y_0^{II} = (xy + y^2)_0 = (x_0 y_0 + y_0^2) = 1 \ldots\ldots. (2)$

$y_0^{II} = (xy^1 + y + 2yy^1)$

$y_0^{II} = (x_0 y_0^1 + y_0 + 2y_0 y_0^1) = 3 \ldots\ldots. (3)$

$y_0^{III} = (xy_0^{II} + 2y^1 + 2yy^{II} + 2y^{12})_0 = 10 \ldots\ldots (4)$

Substituting (2). (3) and (4) in (1) we get

$Y(0.1) = 1 + 0.1 + \frac{(0.1)^2}{2} \times 3 + \frac{(0.1)^3}{6} \times 10$

$= 1 + 0.1 + 0.016 + 0.001666$

$y(0.1) = 1.11666$

To find y (0.2)

By Taylor series we have

$Y_2 = y_1 + hy_1^1 + \frac{h^2}{2!} y_1^{II} + \frac{h^3}{3!} y_1^{III} + \ldots.. (5)$

Now $y_1^1 = (xy + y^2) = x_1 y_1 + y_1^2$

$= (0.1)(1.11666) + (1.11666)^2$

$= 0.111666 + 1.2469$

$= 1.3585 \ldots\ldots (6)$

$y_1^{II} = (xy^1 + y + 2yy^1)$

$= x_1 y_1^1 + y_1 + 2y_1 y_1^1$

$= (0.1)(1.3585) + 1.11666 + 2(1.11666)(1.3585)$

= 0.13585 + 1.11666 + 3.0339

= 4.2865 …… (6)

$y_1^{III} = (xy^{II} + 2y^{I} + 2yy^{II} + 2y^{12})$

$= (x_1 y_1^{II} + 2y_1^{I} + 2y_I y_I^{II} + 2y_1^{12})$

= (0.1) (4.2865) + 2 (1.3585) + 2 (1.1167) (4.2865) + 2 $(1.3585)^2$

= 0.4287 + 2.717 + 9.5735 + 3.6916

= 16. 4102 …… (8)

Substituting (6), (7) and (8) in (5) we get

$$Y(0.2) = 1.1167 + (0.1)(1.3585) + \frac{(0.1)^2}{2}(4.2865) + \frac{(0.1)^3}{6}(16.4102)$$

Y (0.2) = 1.1167 + 0. 13585 + 0. 0214 + 0.002735

Y (0.2) = 1.27668

To find y (0.3)

By Taylor series we have

$$Y_3 = y_2 + hy_2^1 + \frac{h^2}{2!}y_2^{II} + \frac{h^3}{3!}y_2^{III} + \dots \quad (9)$$

Now $y_2^1 = (xy + y^2)_2 = (x_2 y_2 + y_2^2)$

= (0.2) (1.2767) + $(1.2767)^2$

= 0.2553 + 1.6299

= 1. 8852 …… (10)

$y_2^{II} = (xy^1 + y + 2yy^1)^2$

$= x_2 y_2^1 + y_2 + 2y_2 y_2^1$

$= (0.2)\ (1.8852) + 1.2767 + 2\ (1.2767)\ (1.8852)$

$= 0.33770 + 1.2767 + 4.8136$

$= 6.4674$ …… (11)

$y_2^{III} = (xy^{III} + 2y^1 + 2yy^{II}\ 2y^{12})_2$

$= (x_2 y_2^{II} + 2y_2^1 + 2y_2 y_2^{II} + 2y_2^{12})$

$= (0.2)\ (6.4674) + 2\ (1.8852) + 2\ (1.2767)\ (6.4774) + 2\ (1.8852)^2$

$= 1.2974 + 3.7704 + 16.5138 + 7.1079$

$= 28.6855$

Substituting (10), (11) and (12) in (9), we get

$$Y\ (0.3) = 1.2767 + (0.1)\ (1.8852) + \frac{0.1^2}{2}(6.4674) + \frac{(0.1)^3}{6}(28.6855)$$

$= 1.2767 + 0.18852 + 0.0323 + 0.004780$

$= 1.5023$

$\therefore y\ (0.3) = 1.5023$

We have the following values

$X_0 = 0 \qquad y_0 = 1$

$X_1 = 0.1 \qquad y_1 = 1.11666$

$X_2 = 0.2 \qquad y_2 = 1.27668$

$X_3 = 0.3 \qquad y_3 = 1.50233$

To find y (0.4) by Milne's predictor formula

$Y_{n+1,\,P} = y_{n+3} + \frac{4h}{3}[2y^1_{n-2} - y^1_{n-2} + 2y^1_{n}]$ ….. (1)

$Y_3^1 = (xy + y_2)_3$

$= (x_3y_3 + y_3^2)$

$= [(0.3)(1.5023) + (1.5023)^2]$

$= 0.45069 + 2.2569$

$= 2.\ 7076$

Putting n =3, we get

$Y_{4,\,P} = y_0 + \frac{4h}{3}[2y_1^1 - y_2^1 + 2y_3^1]$

$= 1 + \frac{4(0.1)}{3}[2\,(1.3585) - 1.8852 + 2\,(2.7076)]$

$= 1 + 0.1333\,[2.717 - 1.0852 + 5.4152]$

$y_{4,\,P} = 1.8329$

To find y (.04) by Milne's corrector formula

By Milne's corrector formula we have

$y_{n+1,\,C} = y_{n-1} + \frac{h}{3}[y^1_{n-1} + 4y_n^1 + y^1_{n+1}]$ …… (14)

Now $y_4^1 = (x^2 + y^2)_4 = (x_4y_4 + y_4^2)$

$= [(0.4)(1.8327) + (1.8327)^2]$

$= 0.7330 + 3.3588$

$= 4.0918$

Putting n = 3 in (14) we get

$Y_{4, C} = y_2 + \frac{h}{2}[y_2^1 + 4y_3^1 + y_4^1]$

$Y_{4, C} = 1.27668 + \frac{(0.1)}{3}[1.8852 + 4(2.7076) + 4.0918]$

= 1.27668 + 0.0333 [1.8852 + 10.8304 + 4.0918]

= 1.8369

28. Solve and get y (2) given $\frac{dy}{dx} = \frac{1}{2}(x + y)$, y (0) = 2

y (0.5) = 2.636, y (1) = 3.595; y (1.5) = 4.968 by Adam's

method?

Solution:

By Milne's method, we have $y_0^1 = \frac{1}{2}(0 + 2) = 1$

$Y_1^1 = 1.5680$, $y_2^1 = 2.2975$, $y_3^1 = 3.2340$

By Adam's predictor formula

$Y_{n+1, P} = y_n + \frac{h}{24}[55y_n^1 - 59y^1_{n-1} + 37y^1_{n-2} - 9y^1_{n-3}]$

$\therefore y_{4, P} = y_3 + \frac{h}{24}[55y^1_n - 59y_2^1 + 37y^1_1 - 9y^1_0]$ ….. (1)

$= 4.968 + \frac{0.5}{24}[55(3.2340) - 59(2.2975) + 37(1.5680) - 9(1)]$

= 68708

$y_4^1 = \frac{1}{2}(x_4 + y_4) = \frac{1}{2}(2+6.8708) = 4.4354$

By corrector, $y_{4,C} = y_3 + \frac{h}{24}[9y_n^1 + 19y_3^1 - 5y_2^1 + y_1^1]$.. (2)

$= 4.968 + \frac{0.5}{24}[9(4.4354) + 19(3.234) - (2.2975) + 1.5680]$

$= 6.8731$

29. Find y (0.1), y (0.2), y (0.3) from $\frac{dy}{dx} = xy + y^2$, y (0) = 1 by using Runge – kutta method and hence obtain y (0.4) using Adam's method?

Solution:

$f(x, y) = xy + y^2$, $x_0 = 0$, $x_1 = 0.1$, $x_2 = 0.2$,

$xy = 0.4$, $x_4 = 0.4$, $y_0 = 1$

$k_1 = hf(x_0, y_0) = (0.1) f(0, 1) = (0.1) 1 = 0.1$

$k_2 = hf(0.05, y_0 + \frac{k_1}{2}) = (0.1) f(0.05, 1.05)$

$= (0.1)[(0.05)(1.05) + (1.05)^2] = 0.1155$

$k_3 = hf(0.05, y_0 + \frac{k_2}{2}) = (0.1) f(0.05, 1.0578)$

$= (0.1)[(0.5)(1.0578) + (1.0578)^2]$

$= 0.1172$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= (0.1) f(0.1, 1.1172)$

= (0.1) [(0.10 (1.1172) + $(1.1172)^2$] = 0.13598

$y_1 = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

= 1.1169

y (0.1) = 1.1169

Again, start from y_1

k_1 = hf (x_1, y_1) = (0.1) f (0.1, 1.1169)

= 0.1359

$k_2 = hf\,(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2})$ = (0.1) f (0.15, 1.1849)

= 0.1582

$k_3 = hf\,(0.15, y_1 + \frac{k_3}{2})$ = (0.1) f (0.15, 1.196)

= 0.16098

k_4 = (0.1) f (0.2, 1.2779) = 0.1889

$y_2 = 1.1169 + \frac{1}{6}[0.1359 + 2(0.1582 + 0.16098) + 0.1889]$

y (0.2) = 1.2774

Start from (x_2, y_2) to get y_3

K_1 = hf (x_2, y_2) = (0.1) f (0.2, 1.2774) = 0.1887

$K_2 = hf\,(x_2 + \frac{h}{2}, y_2 + \frac{k_1}{2})$ = (0.1) f (0.25, 1.3718) = 0.2225

$K_3 = hf\,(x_2 + \frac{h}{2}, y_2 + \frac{k_2}{2})$

$= (0.1)\,f\,(0.25, 1.3887) = 0.2274$

$k_4 = hf\,(x_3, y_2 + \frac{k_3}{2}) = (0.1)\,f\,(0.3, 1.5048)$

$=0.2716$

$y_3 = 1.2774 + \frac{1}{6}[0.1887 + 2\,(0.2225) + 2\,(0.2274) + 0.2716] = 1.5041$

Now we use Adam's predictor formula

$Y_{4,P} = y_3 + \frac{h}{24}[55y_3^1 - 59y_2^1 + 37y_1^1 - 9y_0^1]$ …. (2)

$Y_0^1 = x_0y_0 + y_0^2 = 1$

$Y_1^1 = x_1y_1 + y_1^2 = 1.3592$

$Y_2^1 = x_2y_2 + y_2^2 = 1.8872$

$Y_3^1 = x_3y_3 + y_3^2 = 2.7135$

Using (2)

$Y_{4,P} = 1.5041 + \frac{0.1}{2}[55\,(2.7135) - 59\,(1.8872) + 37\,(1.3592) - 9\,(1)]$

$= 1.8341$

$y^1_{4,P} = x_4y_4 + y^2_4 = (0.4)\,(1.8341) + (1.8341)^2 = 4.0976$

$y_{4,P} = y_3 + \frac{h}{24}[9y_4^1 + 19y_3^1 - 5y_2^1 + y_1^1]$

$= 1.5041 + \frac{0.1}{24}[9\,(4.0976) + 19\,(2.7135) - 5\,(1.8872) + 1.3592)$

= 1.8389

y (0.4) = 1.8389

30. Solve $y^1 = \frac{y^2 - x^2}{y^2 + x^2}$; y (0) = 1 by Runge – kutta method of fourth order to find y (0.2)

Solution:

$Y^1 = f(x, y) = \frac{y^2 - x^2}{y^2 + x^2}$, $x_0 = 0$, h = 0.2, $x_1 = 0.2$

$f(x_0, y_0) = f(0, 1) = \frac{1-0}{1+0} = 1$

$k_1 = hf(x_0, y_0) = 0.2 \times 1 = 0.2$

$k_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}) = (0.2) f(0.1, 1.1)$

$= 0.2 \left[\frac{1.21 - 0.01}{1.21 + 0.01}\right] = 0.9167213$

$k_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}) = (0.2) f(0.1, 1.0983606)$

= 0.1967

$k_4 = hf(x_0 + 4, y_0 + k_3) = 0.2 f(0.2, 1.1967)$

= 0.1891

$\therefore \Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= \frac{1}{6}[0.2 + 2\,(0.19672) + 2\,(1.1967) + 0.1891]$

$= 0.19598$

$y\,(0.2) = y_1 = y_0 + \Delta y = 1.19598$

Questions

The numerical backward differentiation of y w.r.t. x once is ------------.

The second derivative of the Newton's forward differentiation is -------------.

The second derivative of the Newton's backward differentiation is -------------.

The order of error in Trapezoidal rule is -------------.

The order of error in Simpson's rule is -------------.

Numerical evaluation of a definite integral is called --------.

Simpson's ⅜ rule can be applied only if the number of sub interval is in ------.

By putting n = 2 in Newton cote's formula we get ----------- rule.

The Newton Cote's formula is also known as --------------- formula.

By putting n = 3 in Newton cote's formula we get ----------- rule.

By putting n = 1 in Newton cote's formula we get ----------- rule.

The systematic improvement of Richardon's method is called--------- method

Simpson's 1/3 rule can be applied only when the number of interval is ------.

Simpson's rule is exact for a ---------------- even though it was derived for a Quadratic.

The accuracy of the result using the Trapezoidal rule can be improved by --------

A particular case of Runge Kutta method of second order is ------------------.

Runge Kutta of first order is nothing but the --------------.

In Runge Kutta second and fourth order methods, the values of k1 and k2 are ----

The formula of Dy in fourth order Runge Kutta method is given by ---------------.

In second order Runge Kutta method the value of k2 is calculated by ------------.

__________values are calculated in Runge Kutta fourth order method.

The use of Runge kutta method gives ------------- to the solutions of the differential equation than Taylor's series method.

In Runge – kutta method the value x is taken as ----------.

In Runge – kutta method the value y is taken as ----------.

In fourth order Runge Kutta method the value of k3 is calculated by ------------.

In fourth order Runge Kutta method the value of k4 is calculated by ------------.

____________is nothing but the modified Euler method.

In all the three methods of Rungekutta methods, the values ---------- are same.

The formula of Δy in third order Runge Kutta method is given by ---------------.

The formula of Δy in second order Runge Kutta method is given by --------------.

In second order Runge Kutta method the value of k1 is calculated by ------------.

The Runge – Kutta methods are designed to give -------------- and they posses the advantage of requiring only the function values at some selected points on the sub intervals

If dy/dx is a function x alone, then fourth order Runge – Kutta method reduces to --------------.

In Runge Kutta methods, the derivatives of ----------- are not require and we require only the given function values at different points.

The use of -------------- method gives quick convergence to the solutions of the differential equation than Taylor's series method.

If dy/dx is a function x alone, then ---------- Runge – Kutta method reduces to Simpson method

If dy/dx is a function of ----------- then fourth order Runge – Kutta method reduces to Simpson method.

Opt 1	Opt 2
$f'(x) = (1/h) * (Dy_0 + (2r-1)/2 * D^2y_0 + (3r^2-6r+2)/6 * D^3y_0 + \ldots\ldots)$	$y = y_n + n\,Ñy_n + \{n(n+1) / 2!\}\, Ñ^2y_n + \{n(n+1)(n+2) / 3!\}\, Ñ^3y_n + \ldots\ldots)$
$y'' = (1/h^2) * \{D^2y_0 - D^3y_0 + (11/12)\ D^4y_0 \ldots\ldots\}$	$y'' = (1/h^2) * \{D^2y_0 + D^3y_0 + (11/12)\ D^4y_0 \ldots\ldots\}$
$y'' = (1/h^2) * \{D^2y_0 + D^3y_0 + (11/12)\ D^4y_0 \ldots\ldots\}$	$y'' = (1/h^2) * \{D^2y_0 - D^3y_0 + (11/12)\ D^4y_0 \ldots\ldots\}$
h	h^3
h	h^3
Integration	Differentiation
Equal	even
Simpson's 1/3	Simpson's ⅜
Simpson's 1/3	Simpson's 3/8
Simpson's 1/3	Simpson's ⅜
Simpson's 1/3	Simpson's ⅜
Simpson's 1/3	Simpson's ⅜
Equal	even
cubic	less than cubic
Increasing the interval h	Decreasing the interval h
Milne's method	Picard's method
modified Euler method	Euler method
same	differ
$1/6 * (k_1 + 2k_2 + 3k_3 + 4k_4)$	$1/6 * (k_1 + k_2 + k_3 + k_4)$
$h\,f(x + h/2\ , y + k_1/2)$	$h\,f(x - h/2\ , y - k_1/2)$
k_1, k_2, k_3, k_4 and Dy	k_1, k_2 and Dy
Slow convergence	quick convergence

$x = x_0 + h$	$x_0 = x + h$
$y = y_0 + h$	$y_0 = x_0 + h$
$h\ f(x - h/2\ ,\ y - k_2/2)$	$h\ f(x + h/2\ ,\ y + k_2/2)$
$h\ f(x + h/2\ ,\ y + k_1/2)$	$h\ f(x + h/2\ ,\ y + k_2/2)$
Runge kutta method of second order	Runge kutta method of third order
k_4 & k_3	k_3 & k_2
$1/6 * (k_1 + 2k_2 + 3k_3 + 4k_4)$	$1/6 * (k_1 + 4k_2 + k_3)$
k_1	k_2
$h\ f(x + h/2\ ,\ y + k_1/2)$	$h\ f(x + h/2\ ,\ y + k_2/2)$
greater accuracy	lesser accuracy
Trapezoidal rule	Taylor series
higher order	lower order
Taylor series	Euler
fourth order	third order
x alone	y alone

Opt 3	**Opt 4**	**Opt 5**
f ' (x) = (1/h)* (Dy_n + (2r+1)/2 * D^2y_n + ($3r^2$+6r+2)/6 * D^3y_n + ……….)	f ' (x) = (1/h)* ($\tilde{N}y_n$ + (2r+1)/2 * $\tilde{N}^2y_n$ + ($3r^2$+6r+2)/6 * $\tilde{N}^3y_n$ + ……….)	
y " = (1/h)* {D^2y_0 + D^3y_0 + (11/12) D^4y_0 ……….}	y " = (1/h)* {D^2y_0 – D^3y_0 + (11/12) D^4y_0 ……….}	
y " = (1/h)* {D^2y_0 + D^3y_0 + (11/12) D^4y_0 ……….}	y " = (1/h)* {D^2y_0 – D^3y_0 + (11/12) D^4y_0 ………..}	
h^2	h^4	
h^2	h^4	
Interpolation	Triangularization	
multiple of three	unequal	
Trapezoidal	Romberg	
Trapezoidal	quadrature	
Trapezoidal	Romberg	
Trapezoidal	newton's	
Trapezoidal	Romberg	
multiple of three	unequal	
linear	quadratic	
Increasing the number of iterations	altering the given function	
Modified Euler method	Runge's method	
Taylor series	none of these	
always positive	always negative	
(k_1 + $2k_2$ + $2k_3$ + k_4)	1/6 * (k_1 + $2k_2$ + $2k_3$ + k_4)	
h f(x , y)	h f(0,0)	
k_1, k_2, k_3 and Dy	none of these	
oscillation	divergence	

$h = x_0 + x$	$h = x_0 - x$
$y = y_0 – Dy$	$y = y_0 + Dy$
$h\ f(x , y)$	$h\ f(x – h/2 , y – k_1/2)$
$h\ f(x + h , y + k_3)$	$h\ f(x – h , y – k_3)$
Runge kutta method of fourth ord	Taylor series method
k_2 & k_1	k_1, k_2, k_3 & k_4
$1/6 * (4k_1 + 4k_2 + 4k_3)$	$1/6 * (k_1 + 2k_2 + 2k_3 + k_4)$
k_3	k_4
$h\ f(x , y)$	$h\ f(x – h/2 , y – k_1/2)$
average accuracy	equal
Euler method	Simpson method
middle order	zero
Runge – Kutta	Simpson method
second order	first order
both x and y	none

Opt 6 **Answers**

$f'(x) = (1/h)*(\tilde{N}y_n + (2r+1)/2 * \tilde{N}^2y_n +$
$(3r^2+6r+2)/6 * \tilde{N}^3y_n + \ldots\ldots\ldots)$
$y'' = (1/h^2)* \{D^2y_0 - D^3y_0 + (11/12)\ D^4y_0$
$\ldots\ldots\ldots\}$

$y'' = (1/h^2)* \{D^2y_0 + D^3y_0 + (11/12)\ D^4y_0$
$\ldots\ldots\ldots\}$
h^2
h^4
Integration

multiple of three
Simpson's 1/3

quadrature
Simpson's ⅜
Trapezoidal

Romberg

even

linear
Increasing the
number of iterations

Modified Euler method
Euler method

same

$1/6 * (k_1 + 2k_2 + 2k_3 + k_4)$

$h\ f(x + h/2\ ,\ y + k_1/2)$

k_1, k_2, k_3, k_4 and Dy

quick convergence

$x = x_0 + h$
$y = y_0 + Dy$

$h\ f(x + h/2\ ,\ y + k_2/2)$

$h\ f(x + h\ ,\ y + k_3)$
Runge kutta method of second order

k_2 & k_1

$1/6 * (k_1 + 4k_2 + k_3)$

k_2

$h\ f(x\ ,\ y)$

greater accuracy

Simpson method

higher order

Runge – Kutta

fourth order

x alone

Reg no-------------

(16ITU603A)

KARPAGAM ACADEMY OF HIGHER EDUCATION

Coimbatore-21

DEPARTMENT OF MATHEMATICS

Sixth Semester

I Internal Test - Dec'2018

Numerical Methods

Date: 18 -12-2018 (FN) **Time: 2 Hours**

Class: III-B.Sc IT **Maximum Marks:50**

PART-A (20×1=20 Marks)

Answer all the Questions:

1. The equation 3x – cosx – 1 = 0 is known as ------------- equation.
   a) polynomial b) transcedental
   c) algebraic d) exponential
2. If f(a) and f(b) are of opposite signs, a root of f(x) = 0 lies between -------.
   a) 0 & b b) a & b c) a & 0 d) 0 & 1
3. The convergence of bisection method is -------.
   a) linear b) quadratic c) slow d) fast
4. In Iteration method if the convergence is ---------- then the convergence is of order one.
   a) cubic b) quadratic c) linear d) zero
5. ------------- Method is also called as Bolzano method or interval having method.
   a) Bisection b) False position
   c) Newton Rapson d) Euler
6. The convergence of iteration method is ----------------.
   a) zero b) polynomial c) quadratic d) linear
7. The order of convergence of Regula falsi method may be assumed to ----
   a) 1.513 b) 1.618 c) 1.234 d)1.638
8. The Newton Rapson method fails if ---------------.
   a) $f'(x) = 0$ b) $f(x) = 0$ c) $f(x) = 1$ d) $f(x) \neq 0$
9. The order of convergence of Newton Raphson method is ----------.
   a) 4 b) 2 c) 1 d) 0
10. By Regula Falsi method, the positive root of first approximation of $x^3 - 4x + 1 = 0$ lies between ---------------.
    a) 0 & 1 b) 1 & 2 c) -1 & -2 d) 0 & –1
11. Graeffe's root squaring method is useful to find -----------------.
    a) Complex roots b) single root
    c) unequal roots d) polynomial roots
12. Gauss elimination method is a -----------------.
    a) Indirect method b) direct method
    c) iterative method d) convergent
13. The sufficient condition for convergence of iterations is -----------.
    a) $|\phi'(x)| = 1$ b) $|\phi'(x)| > 1$
    c) $|\phi'(x)| < 1$ d) $|\phi'(x)| < 0$
14. ----------------- method is also called method of tangents.
    a) Gauss Seidal b) Secant
    c) Bisection d) Newton Rapson
15. In ----------- method, first find the integral part of the equation.
    a) Iteration b) Regula Falsi
    c) Bisection d) Horner's
16. In the upper triangular coefficient matrix, all the elements above the diagonal are --------------.
    a) Zero b) non – zero c) unity d) negative.
17. The modification of Gauss – Elimination method is called ---------------------.
    a) Gauss Jordan b) Gauss Seidal
    c) Gauss Jacobbi d) Crout's
18. The bisection method is simple but ------------------.
    a) slowly convergent b) fast convergent
    c) slowly divergent d) fast divergent
19. In Newton Raphson method, the error at any stage is proportional to the ------- of the error in the previous stage.
    a)cube b) square c) square root d) equal
20. The method of false position is also known as ------------- method.
    a) Iteration b) Regula Falsi
    c) Bisection d) Newton Rapson

PART-B(3×2=6 Marks)

21. Convert the decimal number 47 into its binary equivalent.
22. Explain about Floating point Representation.
23. Write the aim of Bisection method.

PART-C (3×8=24 Marks)

24. a) Find the positive root of $x^3 - x = 1$ correct to four decimal places by bisection method.

(OR)

b) Explain about types of Error.

25. a) Solve for a positive root of $x^3 - 4x + 1 = 0$ by Regula Falsi method.

(OR)

b) Find the positive root of $f(x) = 2x^3 - 3x - 6 = 0$ by Newton Raphson method correct to five decimal places.

26. a) Solve the system by Gauss Elimination method
2x+3y-z=5; 4x+4y-3z=3; 2x-3y+2z=2.

(OR)

b) Solve the system by Gauss Jordan method
x+2y+z=3; 2x+3y+3z=10; 3x-y+2z=13.

KARPAGAM ACADEMY OF HIGHER EDUCATION

Department of Mathematics

I Internal Answerkey

Subject : Numerical Methods

Class : III B.Sc IT

Subject code : 16ITU603A

PART - A

1. transcendental
2. a & b
3. slow
4. linear
5. Bisection
6. quadratic
7. 1.618
8. $f'(x) = 0$
9. 2
10. 0 & 1
11. complex roots
12. direct method
13. $|\phi'(x)| < 1$
14. ~~Secant~~ Newton Raphson
15. Horner's
16. Zero
17. Gauss Jacobbi
18. slowly convergent
19. Square root
20. Regula Falsi

PART - C

24

a) Given :

$x^3 - x = 1$, $f(x) = x^3 - x - 1 = 0$

To Find :

The positive root by bisection method

Soln :

finding $f(0)$, $f(1)$, $f(2)$

the root lies b/w 1 & 2.

$x_0 = \frac{2-1}{2} = 1.5$

Continuing this Process until getting approximation root.

→ 5 marks.

Ans :

→ 1 mark.

b) Types of Error :

Local Truncation error → 2 marks

Global Truncation error. → 2 marks.

round off error → 2 marks

25

a) <u>Soln :</u>

$f(x) = x^3 - 4x + 1$

$f'(x) = 3x^2 - 4$

~~$x_1 = x_0 + \frac{f(x)}{f'(x)}$~~

Continue this Process till getting approximate root

→ 5 marks

Ans :

→ 1 mark

b) <u>Soln</u>

$f(x) = 2x^3 - 3x - 6$

$f'(x) = 6x^2 - 3$

~~$x_0 = \frac{f(b) - f(a)}{b - a}$~~ $x_1 = x_0 + \frac{f(x)}{f'(x)}$

Continue this Process till getting approximation. → 5 marks

Ans : 1.78376.

→ 1 mark.

26.

a) Gauss Elimination method

$$(A, B) = \left[\begin{array}{ccc|c} 2 & 3 & -1 & 5 \\ 4 & 4 & -3 & 3 \\ 2 & -3 & 2 & 2 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 2 & 3 & -1 & 5 \\ 0 & -2 & -1 & -7 \\ 0 & 0 & 6 & 18 \end{array}\right]$$

→ 4 marks

$z = 3$

$y = 2$

$x = 1$ ⟶ 2 marks.

b) Gauss Jordan Method :

$$(A, B) = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 2 & 3 & 3 & 10 \\ 3 & -1 & 2 & 13 \end{array}\right]$$

Reducing as diagonal Matrix

⟶ 5 marks

Finding x, y, z

⟶ 1 mark.

Reg no-------------

(16ITU603A)

KARPAGAM ACADEMY OF HIGHER EDUCATION

Coimbatore-21

DEPARTMENT OF INFORMATION TECHNOLOGY

Second Semester

II Internal Test - Feb'2019

Numerical Methods

Date: 05-02-2019 (FN) **Time: 2 Hours**

Class: III-B.Sc IT **Maximum Marks:50**

PART-A(20×1=20 Marks)

ANSWER ALL THE QUESTIONS

1. Gauss Seidal method always --------- for a special type of systems.
   a) converges b) diverges
   c) oscillates d) zero
2. Condition for convergence of Gauss Seidal method is ----
   a) coefficient matrix is not diagonally dominant
   b) pivot element is Zero
   c) coefficient matrix is diagonally dominant
   d) diagonal zero
3. The process of computing the value of the function inside the given range is called ________
   a) Interpolation b) extrapolation
   c) reduction d) expansion
4. If the point lies inside the domain $[x_0, x_n]$, then the estimation of f(y) is called _________
   a) Interpolation b) extrapolation
   c) reduction d) expansion

   Modified form of Gauss Jacobi method is ------------------- method.
   a) Gauss Jordan b) Gauss Seidal
   c) Gauss Jacobi d) Gauss elimanation
5. In Gauss elimination method by means of elementary row operations, from which the unknowns are found by ----------------- method.
   a) random b) back substitution
   c) Forward substitution d) null
6. Δ is called __________ difference operator.
   a) forward b) backward c) central d) none
7. In the forward difference table y_0 is called ___________ element.
   a) leading b) ending c) middle d) positive
8. Gauss Seidal method always converges for ----------- of systems
   a) Only the special type b) all types
   c) quadratic types d) first type
9. ∇ is called __________ difference operator
   a) forward b) backward c)central d) none
10. The elimination of the unknowns is done not only in the equations below,but also in the equations above the leading diagonal is called ------------
    a) Gauss elimination b) Gauss Jordan c) Gauss Jacobi d) Gauss siedal
11. The difference of first forward difference is called ________
    a) divided difference b) 2^{nd} forward difference
    c) 3^{rd} forward difference d) 4^{th} forward difference
12. In the forward difference table $\Delta y_0, \Delta^2 y_0, \ldots$ are called ___________ difference.
    a) leading b) ending c) middle d) positive
13. Gregory – Newton forward interpolation formula is also called as Gregory – Newton forward _________formula.
    a) Elimination b) iteration c) difference d) distance

14. In Gregory – Newton forward interpolation formula 1st three terms of this series give the result for the _________ interpolation.

a) ordinary linear b) ordinary differential
c) parabolic d) central

15. Gauss – Jacobi method is ________ Method.

a)direct b)indirect
c)elimination d) interpolation

16. In Gregory – Newton forward interpolation formula 1st two terms of this series give the result for the ________ interpolation.

a) Ordinary linear b) ordinary differential
c) parabolic d) central

17. Gregory – Newton forward interpolation formula is mainly used for interpolating the values of y near the _____________ of the set of tabular values.

a) beginning b) end
c) centre d) side

18. The value of E is

a) delta -1 b) 1-delta
c) delta+1 d)delta+2

19. The difference value $\nabla y1 - \nabla y0$ in a Newton backward difference table is denoted by ________.

a) $\nabla^2 y_0$ b) $\nabla^2 y_1$ c) ∇y_0 d) ∇y_1

20. The (n+1) th and higher differences of a polynomial of the nth degree are --------

a)zero b)one c)two d)three

PART-B (3×2=6 Marks)
ANSWER ALL THE QUESTIONS

21. Write the formula for Newton forward and backward difference.
22. Define iterative method.
23. Write the difference between Gauss Jacobi and seidel method.

PART-C (3×8=24 Marks)
ANSWER ALL THE QUESTIONS

24. a) Solve the following system of equations by Gauss-Jacobi method

$$10x - 5y - 2z = 3$$
$$4x - 10y + 3z = -3$$
$$x + 6y + 10z = -3$$

(OR)

b) Solve the following system of equations by Gauss-Seidal method.

$$28x + 4y - z = 32$$
$$x + 3y + 10z = 24$$
$$2x + 17y + 4z = 35$$

25. a) Find the values of y at x = 21 and x = 28 from the following data.

x:	20	23	26	29
y:	0.3420	0.3907	0.4384	0.4848

(OR)

b) From the following data find the value of θ at x = 43 and x= 84

x:	40	50	60	70	80	90
y:	184	204	226	250	276	304

26. a) Solve the following system by Gauss Jacobi method**.**

$$8x + y + z = 8$$
$$2x + 4y + z = 4$$
$$x + 3y + 3z = 5$$

(OR)

b) From the following table, find the value of tan 45°15′

x°	45	46	47	48	49	50
$\tan x^\circ$	1.0000	1.0355	1.0723	1.1106	1.1503	1.1917

KARPAGAM ACADEMY OF HIGHER EDUCATION

II Internal Answer key

Subject : Numerical Methods

Class : III B.Sc IT

Subject code : 16ITU603A

PART-A

1. converges
2. coefficient matrix is diagonally dominant
3. Interpolation
4. Interpolation
5. Gauss Seidal.
6. Forward
7. leading
8. Only the Special type
9. backward
10. Gauss jordan
11. 2^{nd} forward difference
12. leading
13. difference
14. Parabolic
15. indirect
16. Ordinary linear
17. beginning
18. delta +1
19. $\nabla^2 y_1$
20. three.

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PART – B

21 Forward difference:

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{2p-1}{2!}\Delta^2 y_0 + \cdots\right]$$

$$p = \frac{x - x_0}{h}$$

→ 1 mark

Backward difference:

$$y_p = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \cdots$$

$$p = \frac{x - x_n}{h}$$

→ 1 mark

22 Iterative method:

Defn → 1 mark

example → 1 mark.

23 Difference b/w Gauss Jacobi & Seidel method.

i) Slowdown Process. Fast Process

ii) Indirect method Indirect method

→ 2 marks

PART – C

24)

a) finding

$$x = \frac{1}{10}[5y + 2z + 3]$$

$$y = \frac{1}{10}[-4x - 3z - 3]$$

$$z = \frac{1}{10}[-x - 6y - 3]$$

finding solution by iterations.

→ 5 marks

Ans :

$x = 0.341$

$y = 0.285$

$z = -0.505$ → 1 mark.

b) finding

~~$x = \frac{1}{28}[-4y + z + 32]$~~

~~$y = \frac{1}{3}[24 - x - 10]$~~

$$x = \frac{1}{28}[-4y + z + 32]$$

$$y = \frac{1}{17}[35 - 2x - 4z]$$

$$z = \frac{1}{10}[24 - x - 3y]$$

finding solution by iterations

→ 5 marks

Ans :

$x = 0.9936$

$y = 1.5069$

$z = 1.8481$ → 1 mark.

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25

a) For difference table

→ 2 marks

Forward difference formula

finding: → 1 mark

$y(2) = 0.3583$ → 1 mark

backward difference formula

→ 1 mark

Finding

$y(28) = 0.4695$ → 1 mark.

b). difference table

→ 2 marks.

Formula (Forward) → 1 mark

$y(43) = 189.79$ → 1 mark

Formula → 1 mark.

$y(84) = 286.96$ → 1 mark

26

a) Finding

$$x = \frac{1}{8}[8 - y - z]$$

$$y = \frac{1}{4}[2x - z + 4]$$

$$z = \frac{1}{3}[-x - 3z + 5]$$

finding root of x, y, z → 5 marks

Ans → 1 mark.

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Reg no-------------
(16ITU603A)

KARPAGAM ACADEMY OF HIGHER EDUCATION
Coimbatore-21
DEPARTMENT OF INFORMATION TECHNOLOGY
Sixth Semester
III Internal Test - Mar'2019
Numerical Methods

Date: 12 -03-2019 (FN) **Time: 2 Hours**
Class: III-B.Sc IT **Maximum Marks:50**

PART-A(20×1=20 Marks)
ANSWER ALL THE QUESTIONS

1. The Euler Method Predicator is -------------
   a) $Y_{n+1} = y_n + y_n'$ b) $\mathbf{Y_{n+1} = y_n + h y_n'}$
   c) $x_{n+1} = X_n + h X_n'$ d) $Y_{n+1} = y_n - h y_n'$
2. The differences of constant functions are -----
   a)Not equal to zero b) **zero**
   c) one d) two
3. The Euler Method of second category are called ------------
   a)one step method b) two step method
   c) **step by step method** d) Multi step method
4. In R - k method derivatives of higher order are ------------
   a)required b) **not required**
   c) may be required d) must required
5. If y(x) is linear then use ____________.
   a) simphson's 3/8 b) simphson's 1/3
   c) trapezoidal d) taylor series
6. The numerical integral of a single integral is known as
   a) boundary quadrature
   b) mechanical quadrature
   c) initial quadrature
   d) classical quadrature
7. The number of interval is multiple of three the use _____________
   a) simpson's 1/3 b) trapezoidal
   c) simpson's 3/8 d) taylor series
8. The x values of Interpolating polynomial of newton -Gregory has _______________
   a) even space **b) equal space**
   c) odd space d) unequal
9. In simpson's 3/8 rule, we calculate the polynomial of degree
   a) degree n b) linear
   c) second degree d) **third degree**
10. Differentiation of sin x is
    a) cosx b) tanx
    c) sinx d) logx
11. In divided difference the value of any difference is ----- of the order of their argument
    a) **independent** b) dependent
    c) inverse d) none of these
12. 18. If g is continuous on interval (a,b) and g (x) Ɛ (a,b) for all x Ɛ (a,b) then --------
    a) **g has fixed point in [a, b]**
    b) g has not fixed point in [a, b]
    c) g has fixed point in (a, b)
    d) none of these
13. In Newton cote formula if f(x) is interpolate at equally spaced nodes by a polynomial of degree three then it represents
    a)**Trapezoidal rule** b) Simpson's rule
    c) 3/8 Simpson's rule d) Booles rule
14. In Euler's Method averaged the ------------
    a) points b) **slopes**
    c) slopes and points d) chords

15. In Euler's Method solution of the differential equation denoted by -----------

a) **continuous line graph** b) graph

c) line graph **d)** diagram

16. In Modified Euler's Method averaged the ------------

a) **points** b) slopes

c) slopes and points d) chords

17. The Euler Method and Modified Euler's Method are ------

a) convergent b) **slow convergent**

c) fast Convergent d) divergent

18. The order of error in Trapezoidal rule is -------------

a) h b) h^3 **c)** h^2 d) h^4

19. The accuracy of the result using the Trapezoidal rule can be improved by --------

a) increasing the interval h

b) **decreasing the interval h**

c) increasing the number of iterations

d) altering the given function

20. The augment matrix is the combination of ------------------.

a) coefficient matrix and constant matrix

b) unknown matrix and constant matrix

c) coefficient matrix and Unknown matrix

d) **coefficient unknown and constant matrix**

PART-B(3×2=6 Marks)
ANSWER ALL THE QUESTIONS

21. Write the difference between Euler and modified Euler Method.
22. Write the Simpson's 3/8th rule formula.
23. Define R-K method with formula.

PART-C(3×8=24 Marks)
ANSWER ALL THE QUESTIONS

24. a) Evaluate $\int_{-3}^{3} x^4 dx$ by using (i) Trapezoidal rule (ii) Simpson's rule. Verify your results by actual integration.

(OR)

b) Evaluate $\int_{0}^{6} \frac{dx}{1+x^2}$ by (i) Trapezoidal rule (ii) Simpson's rule.

Also check up the result by actual integration

25. a) Compute y at x=0.25 by modified Euler method. Given $y'=2xy, y(0)=1$

(OR)

b) Apply fourth order Runge-Kutta method to find y(0.2) given that $y' = x + y, y(0) = 1$.

26. a) Obtain the values of y at x=0.1,0.2 using R-K method of

(i) second order

(ii) third order

(iii) fourth order

For the differential equation $y' = -y$ given $y(0) = 1$.

(OR)

b) Using Euler method, find y(0.2),y(0.1) given $dy/dx=x^2+y^2, y(0)=1$

KARPAGAM ACADEMY OF HIGHER EDUCATION

Department of Information Technology

III Internal Answer key.

Subject : Numerical Methods

Class : III B.Sc IT

Subject code : 16ITU603A

PART- A

1. $y_{n+1} = y_n + h y_n'$
2. Zero
3. Step by Step method
4. not required
5. trapezoidal
6. mechanical quadrature
7. Simpson's 1/3
8. equal Space
9. third
10. cos x
11. independent
12. g has fixed Point in [a,b]
13. Trapezoidal rule
14. Slopes.
15. Continuous line graph
16. Points
17. Slow convergent
18. h^2
19. decreasing the interval h.
20. Coefficient unknown and constant Matrix.

PART - B

21.

Euler	Modified Euler
i) $y_{n+1} = y_n + h f(x_n, y_n)$	$y_{n+1} = y_n + h f(x_n + 1/2 h, y_n + 1/2 h f(x_n, y_n))$
ii) $f(x)$ is not reduced	it reduced to half

22. $3/8^{th}$ rule:

$$\int f(x)dx = \frac{3h}{8}\left[(y_0 + y_n) + 2(y_3 + y_6 + \dots) + 3(y_1 + y_2 + y_4 \dots\right.$$

→ 2 marks

23 R K Method formula.

$k_1 = f(x, h)$

$k_2 = f(x + 1/2 h, y + 1/2 k_1)$

$k_3 = f(x + 1/2 h, y + 1/2 k_2)$

$\Delta y = (k_1 + 2k_2 + 2k_3)$

PART - C

24

a) Trapezoidal rule

$\int_{-3}^{3} x^4 dx = 115$ → 2 marks

Simpsons rule : $\int_{-3}^{3} x^4 dx = 98$ → 2 marks

Actual integration $\int_{-3}^{3} x^4 dx = 97.2$ → 2 marks

25

a) Modified Euler method

Formula

→ 2 marks

$y(0.25)$ for $y' = 2xy$ is 1.0625

→ 4 marks

b). $y' = x+y$, $y(0) = 1$

Fourth order R-K method formula

→ 2 marks

$y(0.2) = 1.110342$ → 2 marks

$\Delta = 1.2428$ → 2 marks

26

a) Second order

$y(0.1) = 0.905$

$y(0.2) = 0.819$

Third order → 2 marks

$y(0.1) = 0.91$

$y(0.2) = 0.823$

→ 2 marks

Fourth order

$y(0.1) = 0.0904$

$y(0.8) = 0.9048$

$\Delta = 0.81873$

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b) $\frac{dy}{dx} = x^2+y^2$ (Eulermethod)

$y(0) = 1$

$y(0.2) = 1.1105$ → 3 marks

$y(0.1) = 1.2502$.

→ 3 marks.