

**DEPARTMENT OF MATHEMATICS  
FACULTY OF ARTS, SCIENCE AND HUMANITIES  
RESEARCH PROGRAM – M.Phil / Ph.D in Mathematics  
(2019–2020 Batch and onwards)**

Course code	Name of the course	Instruction hours / week	Credits	Maximum Marks (100)
Paper-I				
19RMAT101	Research Methodology and Pedagogy	4	4	100
Paper-II				
19RMAT201	Advanced Algebra and its Applications	4	4	100
19RMAT202	Algebra and Mathematical Analysis			
19RMAT203	Partial Differential Equations			
19RMAT204	Stochastic Processes			
Paper-III				
19RMAT301	Fuzzy Mathematics	4	4	100
19RMAT302	Advanced Topics in Fluid Dynamics			
19RMAT303	Hydrodynamic and Hydromagnetic Stability			
19RMAT304	Queueing Theory			
Program Total		12	12	300

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**19RMAT101 RESEARCH METHODOLOGY AND PEDAGOGY** **Paper-I**  
**4H – 4C**

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**Instruction Hours / week: L: 4**

**Total: 100**

**End Semester Exam: 3 Hours**

**Course Objectives:**

This course enables the students to learn

- Fundamentals of research terminology.
- The ethical principles of research, ethical challenges and approval processes.
- The quantitative, qualitative and mixed methods approaches to research.
- The components of a literature review process.
- How to critically analysed published research.
- About e-learning researches and web-based learning.

**Course Outcomes (Cos):**

After completing this course, the student will be able to:

1. Understand the basic framework of research process.
2. Understand the various research concepts of Implicit functions and extremum problems.
3. Know about the Oscillations of second order equation
4. Understand the basic concepts of LATEX.
5. Study about the Quality teaching and learning.
6. Acquiring the knowledge of e-learning researches and web-based learning.

**UNIT – I**

Research Methodology – Meaning of research, Objectives of Research, Motivation in Research – Types of Research – Research approaches – Research methods, Versus Research Methodology – Research process – Scientific method – Criteria for good research, Defining the research problem – Necessity of defining the problem – Techniques involved in defining the problem, Research Design – Meaning and need for Research Design – Features of good design – Important concepts relating to research design.

**UNIT – II**

Implicit functions and extremum problems: Introduction – Functions with non zero Jacobian determinant – Inverse function theorem – Implicit function theorem – Extrema of real valued functions of one variable and several variables. Rank Theorem – Determinants – Derivatives of Higher order-Differentiation of Integrals.

**UNIT – III**

Oscillations of second order equation-Fundamental results – Sturm comparison theorem – elementary linear oscillations – comparison theorem of Hille-Winter – Oscillations of  $x'' + a(t)x = 0$  elementary non linear oscillations – stability of linear and non linear systems – elementary critical points – system of equations with constant coefficient – the linear equations with constant coefficient – Lyapunov stability – Stability of quasi linear systems.

#### **UNIT- IV**

**LATEX:** The Basics - The Document -Bibliography - Bibliographic Databases - Table of contents, Index and Glossary - Displayed Text - Rows and Columns -Typesetting Mathematics - Typesetting - Several Kinds of Boxes - The figure environment -Cross References in LATEX - Footnotes, Marginpars, and Endnotes.

#### **UNIT-V**

Objectives and roll of higher education – Important characteristics of an effective Lecture – Quality teaching and learning – Lecture preparation – Characteristics of instructional design – Method of teaching and learning: Large group – Technique – Lecture, Seminar, Symposium, Team Teaching, Project, Small group Technique – Simulation, role playing Demonstration, Brain Storing, case discussion and assignment, Methods of evaluation – Self evaluation, Student evaluation, Diagnostic testing and remedial teaching – Question banking – Electronic media in education: e-learning researches – web based learning.

#### **SUGGESTED READINGS**

1. Kothari, C. R., (2014). Research Methodology, Method and Techniques, Second Edition, New age International publishers, New Delhi.
2. Rudin. W., 1976. Principles of Mathematical Analysis, McGraw Hill, New York.
3. Earl A. Coddington, (2002). An introduction to Ordinary Differential Equations, Prentice Hall of India Private limited, New Delhi. ( For Unit III)
4. Krishnan E., (Sep 2003). Latex Tutorials – A primer, Indian TEX users group, Trivandrum, India.
5. Panneerselvam. R, (2013). Research Methodology, Second Edition, Prentice Hall of India, New Delhi.
6. Gupta. S. P, (2011). Statistical Methods, Fourth Edition, Sultan Chand & Sons, New Delhi.
7. Vedanayagam E. G (1989). Teaching Technology for College teachers, New Delhi.
8. Kumar. K. L. (2004). Educational Technologies, New age International, New Delhi:
9. Winkler, Anthony C., and Jo Roy McCuen (1985). Writing a research paper: A Handbook, 2<sup>nd</sup> edition, Harcourt, New York.

**M.Phil / Ph.D Mathematics**

**2019-2020**

**Paper-II**

**19RMAT201 ADVANCED ALGEBRA AND ITS APPLICATIONS 4H – 4C**

**Instruction Hours / week: L: 4**

**Total: 100**

**End Semester Exam: 3 Hours**

**Course Objectives:**

This course enables the students to learn

- The concepts of finite and algebraic extensions.
- Primitive elements and Purely inseparable extensions.
- Approximation by continuous functions
- Perturbations methods and Parametric Perturbation
- Topological preliminaries and theorems.
- The concepts of diffusion equation with sources and elementary solutions of diffusion equation.

**Course Outcomes (Cos):**

After completing this course, the student will be able to:

1. Understand the Field Extensions and Normal extensions.
2. Study Riesz Representation Theorem and Topological preliminaries.
3. Understand the concepts of convex functions and inequalities.
4. Know about the Asymptotic expansion and sequential convergent versus asymptotic series.
5. An understanding of the Role of co-ordinate system.
6. Know about the diffusion equation with sources, elementary solutions of diffusion equation and separation of variables

**UNIT – I**

Field Extensions – Finite and algebraic extensions – Algebraic closure – Splitting fields and Normal extensions - Separable extensions – Finite fields – Primitive elements – Purely inseparable extensions.

**UNIT-II**

Positive Borel Measure –Riesz Representation Theorem: Topological preliminaries - Riesz Representation Theorem – Regularity properties of Borel measures – Lebesgue measure – Continuity properties of measurable functions.

**UNIT-III**

$L^p$  spaces: Convex functions and inequalities – The  $L^p$  spaces – Approximation by continuous functions.

#### **UNIT-IV**

Perturbations methods – Parametric Perturbation – Algebraic equation – The Vanderpol Oscillator – Co-ordinate Perturbation – The Bessel Equation of zeroth order-simple examples – Order Symbols and Gauge function – Asymptotic expansion and sequential convergent versus asymptotic series – Non uniform expansion – Straight forward expansion and sources of non-uniformity – Infinite domain – Duffing equation – A model for weak nonlinear instability – A small parameter multiplying the highest derivative – A second order example – Relaxation oscillation – Type change of PDE – A simple example – The presence of singularities – Shifting Singularity – Role of co-ordinate system.

#### **UNIT - V**

Elementary solutions of onedimensional wave equation-Vibrating membranes-Applications of calculus of variations-three dimensional problems – general solutions of the wave equation – Green's function for the wave equation – Non homogeneous wave equation. The use of integral transform, the use of green's function – The diffusion equation with sources - elementary solutions of diffusion equation-Separation of variables.

#### **SUGGESTED READINGS**

1. Serge Lang.,(2007). Algebra, Addison Wesley Publishing Company, Inc., Amsterdam.
2. Walter Rudin., Real and Complex Analysis, 3<sup>rd</sup> edition, McGraw Hill Book Company, New York.
3. Ross. S (2014). A first course in Probability, 9<sup>th</sup> edition, Pearson Education, Delhi
4. Ian.N.Sneedon,(1988). Elementary partial differential equations,TataMcgraw Hill Ltd.

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**2019-2020**

**19RMAT202 ALGEBRA AND MATHEMATICAL ANALYSIS**

**Paper-II  
4H – 4C**

**Instruction Hours / week: L: 4**

**Total: 100**

**End Semester Exam: 3 Hours**

**Course Objectives:**

This course enables the students to learn

- To solve systems of linear equations and application problems requiring them.
- About and work with vector spaces and subspaces.
- The basic concepts of groups and rings.
- The Structure of rings and simple and primitive rings.
- The concepts of separation theorems in the plane.
- The basic concepts of properties of the spectrum and more results on the Spectra.

**Course Outcomes (Cos):**

After completing this course, the student will be able to:

1. Understand the fundamental concepts of Commutative rings and Modules.
2. Know about the Structure of Rings.
3. Investigate symmetry using group theory
4. Know about the Cauchy's Integral formula.
5. Understand the concepts of Spectral results for Hilbert Space Operators.
6. Study more results on the spectra of self adjoint operators.

**UNIT – I**

Commutative rings and Modules : Chain Conditions – Prime and Primary Ideals – Primary Decomposition – Noetherian rings and Modules – Ring Extensions – Dedekind Domains – The Hilbert Nullstellensatz.

**UNIT – II**

The Structure of Rings: Simple and Primitive Rings – The Jacobson Radicals – Semi simple Rings – The Prime Radical; Prime and Semi prime Rings – Algebras – Divisions Algebras.

**UNIT – III**

The Fundamental Group: Homotopy of paths- The fundamental Group – Covering Spaces – The fundamental group of the circle – Retractions and fixed points – The fundamental theorem of Algebra – The Borsuk – Ulam Theorem – Deformation retracts

and Homotopy type – The fundamental Group of  $S^n$ - Fundamental groups of some surfaces.

#### **UNIT – IV**

Separation Theorems in the plane: The Jordan Separation Theorem – Invariance of Domain- The Jordan Curve Theorem – Imbedding Graphs in the plane – The winding Number of a simple closed curve – The Cauchy's Integral formula.

#### **UNIT – V**

Operators on Hilbert Spaces: Adjoint of an operator – Self Adjoint -Normal and unitary operator- Hilbert-Schmidt operator. Spectral results for Hilbert Space Operators - Some properties of the Spectrum- More results on the Spectra of Self Adjoint Operators.

#### **SUGGESTED READINGS**

1. Thomas W.Hungerford., (2011). Algebra, Springer , New York.
2. James.R. Munkers., (2002). Topology, Prentice Hall of India Pvt. Ltd., New Delhi
3. Simmons. G.F.,(2004). Introduction to Topology and Modern Analysis, Tata McGraw Hill Publishing Company, New Delhi.

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**2019-2020**

**19RMAT203 PARTIAL DIFFERENTIAL EQUATIONS**

**Paper-II**  
**4H – 4C**

**Instruction Hours / week: L: 4**

**Total: 100**

**End Semester Exam: 3 Hours**

**Course Objectives:**

This course enables the students to learn

- The fundamentals of partial differential equations.
- Laplace's equation and its properties.
- The fundamentals of wave equations.
- Numerical methods for the approximation of their solution.
- Partial derivative equation techniques to predict the behaviour of certain phenomena.
- Applications of the calculus of variations.

**Course Outcomes (Cos):**

After completing this course, the student will be able to:

1. Apply partial derivative equation techniques to predict the behaviour of certain phenomena.
2. Extract information from partial derivative models in order to interpret reality.
3. Study the solution of linear hyperbolic equations.
4. Understand the concepts Laplace's equation.
5. Know about the wave equations and its applications.
6. Study the separation of variables and use of integral transforms.

**UNIT – I**

Nonlinear partial differential equations of the first order: Cauchy's method of characteristics –Compatible systems of first order equations – Charpit's method- Special types of first order equations – Jacobi's method.

**UNIT – II**

Partial differential equations of second order: The origin of second-order equations – Linear partial differential equations with constant coefficients – Equations with variable coefficients –Characteristic curves of second-order equations- Characteristics of equations in three variables.

**UNIT – III**



The solution of linear hyperbolic equations – Separation of variables – The method of integral transforms – Nonlinear equations of the second order.

#### **UNIT – IV**

Laplace's equation : The occurrence of Laplace's equation in physics- elementary solution of Laplace's equation – Families of equipotential surfaces - boundary value problems – Separation of variables- Problems with axial symmetry.

#### **UNIT – V**

The wave equation: The occurrence of wave equation in physics – Elementary solutions of the one-dimensional wave equation – vibrating membranes: Applications of the calculus of variations – Three dimensional problems. The diffusion equations: Elementary solutions of the diffusion equation – Separation of variables- The use of integral transforms.

#### **SUGGESTED READINGS**

1. Sneddon I. N.(2006). Elements of Partial Differential Equations, McGraw-Hill Book Company, Singapore.
2. Robert C.McOwen.,(2004). Partial Differential Equations, Pearson Education, First Indian Reprint.
3. Phoolan Prasad and Renuka Ravindran., (2005). Partial Differential Equations, New Age International Pvt Ltd, New Delhi .
4. Sharma J.N., and Kehar Singh.,(2014). Partial Differential Equations for Engineers and Scientists, Narosa Publishing House, New Delhi.
5. Williams W.E.,(1980). Partial Differential Equations, Clarendon Press, Oxford.

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**2019-2020**

**19RMAT204**

**STOCHASTIC PROCESSES**

**Paper-II**  
**4H – 4C**

**Instruction Hours / week: L: 4**

**Total: 100**

**End Semester Exam: 3 Hours**

### **Course Objectives**

This course enables the students to learn

- The mathematical theory of random variables and random processes.
- How queueing theory are used as tools and mathematical models in the study of networks.
- The theoretical concepts and techniques for solving problems that arises in practice.
- The Markovian models in reliability theory.
- Laplace transforms and its properties.
- Poisson process and related distribution.

### **Course Outcomes (COs)**

On successful completion of the course, students will be able to:

1. Capable to expose the students to different types mathematical models with a view of random processes.
2. Understanding in the concept of Brownian motion.
3. Formulate some real-life problems into queueing models.
4. Study Poisson process, related distribution and birth and death process.
5. Understand the Poisson process and related distribution.
6. Know about Laplace transforms of a probability distribution a random variable.

### **UNIT-I**

Generating function – Laplace Transform – Laplace (stieltjes) transforms of a probability distribution a random variable – Classification of distributions.

### **UNIT-II**

Stochastic processes – Notation – Specification – Stationery process – Markov Chains – Definition and example and higher transition probabilities.

### **UNIT –III**

Classification of states and chains – Determination of higher transition probabilities  
– Stability of a Markov system - Limiting behavior.

#### **UNIT-IV**

Poisson process and related distribution – Generalization of Poisson process - Birth and Death process. Renewal processes - Renewal processes in continuous time – Renewal equation – Altering renewal processes.

#### **UNIT- V**

Reliability –Markovian models in reliability theory – Shock models and wear processes.

#### **SUGGESTED READINGS**

1. Medhi J., (1982). Stochastic process, New age International Private Limited publishers.
2. Samuel Karlin., (1975). First course in stochastic process, Academic press.
3. Srinivasan S., Kidambi., and K.M. Mehta., (1988). Stochastic Processes, 2<sup>nd</sup> edition, Tata McGraw Hill Publishing Company, New Delhi.
4. Saeed Ghahramani., (2015). Fundamentals of Probability with Stochastic Processes, 3<sup>rd</sup> edition, Prentice Hall.
5. Sheldon Ross., (2014). Introduction to Probability Models, 11<sup>th</sup> edition, Academic press.

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**2019-2020**

19RMAT301	FUZZY MATHEMATICS	Paper-III 4H – 4C
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**Instruction Hours / week: L: 4**

**Total: 100**

**End Semester Exam: 3 Hours**

**Course Objectives:**

This course enables the students to learn

- The basic mathematical elements of the theory of fuzzy sets.
- Differences and similarities between fuzzy sets and classical sets theories.
- The concepts of crisp set, fuzzy logic and fuzzy graphs.
- The need of fuzzy sets, arithmetic operations on fuzzy sets,
- Fuzzy relations, Fuzzy measures, Decision making in fuzzy environments.
- How to solve problems that are appropriately solved by neural networks, fuzzy logic, and genetic algorithms.

**Course Outcomes (Cos):**

After completing this course, the student will be able to:

1. Understand about the concepts of fuzzy sets and fuzzy logic.
2. Acquire the knowledge on general aggregation operations.
3. Know about the fuzzy relation equation and fuzzy graphs.
4. Describe the probability measures and fuzzy measures of fuzziness.
5. Import the knowledge on the Decision making in fuzzy environments.
6. Understand decision making in fuzzy environments.

**UNIT - I**

**Crisps sets and Fuzzy sets:** Introduction –Crisp Sets: An overview-The notion of fuzzy sets – Basics concepts of fuzzy sets –Classical logic: An overview-Fuzzy logic.

**UNIT- II**

**Operations on Fuzzy sets:** Fuzzy complement - fuzzy union – fuzzy Intersection – combinations of operation – General Aggregation operations.

**UNIT - III**

**Fuzzy relations and Fuzzy graphs:** Crisp and fuzzy relations – Binary relations- Binary relations on a single set – Equivalence and similarity relations-Compatibility or Tolerance relations – ordering- Morphisms – Fuzzy relation equations – Fuzzy graphs.

#### **UNIT- IV**

**Fuzzy Measures:** Belief and Plausibility Measures – Probability Measures – Possibility and necessity measures-Relationship among classes of Fuzzy measures of Fuzziness.

#### **UNIT- V**

**Decision making in fuzzy environments:** Fuzzy Decisions – Fuzzy Linear programming – symmetric Fuzzy LP – Fuzzy LP with crisp objective function – Fuzzy Dynamic Programming-Fuzzy Dynamic with Crisp state Transformation Function- fuzzy multi criteria Analysis– Multi objective Decision Making (MODM) – Multi Attributive Decision making (MADM).

#### **SUGGESTED READINGS**

1. George J.Klir and Tina A.Folger., (2015). Fuzzy sets – Uncertainty and information, Prentice – Hall of India Pvt. Ltd. Chapters: I, II, III & IV.
2. Zimmermann H.J.,(2007). Fuzzy set theory and its applications , Fourth Edition Springer . Chapter XIV.

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**2019-2020**

**Paper-III**

**19RMAT302**

**ADVANCED TOPICS IN FLUID DYNAMICS**

**4H – 4C**

**Instruction Hours / week: L: 4**

**Total: 100**

**End Semester Exam: 3 Hours**

**Course Objectives:**

This course enables the students to

- Understand the dynamics of fluid flows and the governing non dimensional parameters.
- Make the students to acquire the knowledge on the properties of two dimensional flow.
- Familiarize the concept of equation of motion in rotating co-ordinate system.
- Describe the main properties of the system of equations.
- Introduce the system of Magnetohydrodynamics equations and main theorems that follow from the Magnetohydrodynamics system.
- Understand the importance of fluid dynamics in diverse real life applications.

**Course Outcomes (Cos):**

After completing this course, the student will be able to:

1. Solve and Classify the fluids based on the physical properties of a fluid.
2. Compute correctly the kinematical properties of a fluid element.
3. Apply correctly the conservation principles of mass, linear momentum, and energy to fluid flow systems.
4. Extend the physics and mathematical properties of fluid flow by governing Navier-Stokes equations with proper boundary conditions and obtain solution.
5. Equip the student with the basic mathematical background and tools to model fluid motion.
6. Develop a physical understanding of the important aspects that govern fluid flows that can be observed in a variety of situations in everyday life.

**UNIT – I**

Steady unidirectional flow – Poiseuille flow – Two dimensional flow – Paint-Brush model – unsteady unidirectional flow – Flow with circular stream lines – Flow fields in which inertia forces are negligible – Lubrication theory.

## **UNIT – II**

Thermal boundary layer in laminar flow: Derivation of the energy equation – Temperature increase through adiabatic compression – Stagnation temperature – Theory of similarity in heat transfer – Exact solutions for the problem of temperature distribution in a viscous flow – Boundary layer simplifications.

## **UNIT – III**

Equation of motion in rotating co-ordinate system – Potential vorticity – vorticity equation – Ertel's theorem – Non dimensional parameters – Rossby number – Ekman number – Geostrophic flow – Taylor – Proudman theorem – Taylor column.

## **UNIT – IV**

Magnetohydrodynamics: Electrodynamics of moving media – The electromagnetic effects and the magnetic Reynolds number – Alfen's theorem – The magnetic energy – The mechanical equations – Basic equations for the incompressible MHD – Steady Laminar motion – Hartmann flow.

## **UNIT – V**

Magnetohydrodynamic waves – waves in an infinite fluid of infinite electrical conductivity – Alfen's waves – Magnetohydrodynamic waves in a compressible fluid – Magneto acoustic waves – Slow and Fast waves – Stability – Physical concepts – Linear-Pinch –Kink – Sausage and Flute types of instability – Method of small oscillations – Jeans criterion for gravitational stability.

## **SUGGESTED READINGS**

1. Batchelor. G.K.,(2002). An Introduction to Fluid Dynamics, Cambridge University Press.
2. Schlichting. H.,(2003). Boundary – Layer Theory, Springer.
3. Friedlander. S.,(1980). An Introduction to the Mathematical Theory of Geophysical Fluid Dynamics, Elsevier.
4. Ferraro .V.C.A and Plumpton. C.,(1972). An Introduction to Magneto Fluid Dynamics, Oxford University.

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**2019-2020**

**Paper-III**

**19RMAT303 HYDRODYNAMIC AND HYDROMAGNETIC STABILITY 4H – 4C**

**Instruction Hours / week: L: 4**

**Total: 100**

**End Semester Exam: 3 Hours**

**Course Objectives:**

This course enables the students to

- Learn the concept of stability of hydrodynamics systems.
- Impart the basic knowledge of hydromagnetic systems.
- Disseminate the importance of rotation of fluid in stability analysis.
- Introduce the system of Magnetohydrodynamics equations and magnetohydrodynamics system.
- Learn the Perturbation Techniques for determining the stability of superposed fluids.
- Understand the concept of important instabilities like Rayleigh-Taylor, Kelvin-Helmholtz instability.

**Course Outcomes (Cos):**

After completing this course, the student will be able to:

1. Describe the fundamental principles of the motion of ideal (inviscid) and real (viscous) fluid flows.
2. Apply analytical concepts to analyze a range of two-dimensional engineering fluid flows, with appropriate choice of simplifying assumptions and boundary conditions.
3. Provide the details of the derivation of ideal and resistive Hydrodynamic equations.
4. Demonstrate the basic properties of Hydrodynamic fluids.
5. Equip to solve the fluid flow analysis electromagnetic fields.
6. Analyze the analytical technique to characterize the hydrodynamic stability.

**UNIT – I: Introduction:**

Basic Concepts - Analysis in terms of normal modes - Non-dimensional number.



## UNIT – II: Benard Problem:

Basic hydrodynamic equations. Boussinesq approximation. Perturbation equations. Analysis into normal modes. Principle of exchange of stabilities. Equations governing the marginal state. Exact solution when instability sets in as stationary convection for two free boundaries.

## UNIT – III

**The effect of rotation:** The Perturbation equations. Analysis in terms of normal modes. Variational Principle for stationary convection. Solutions when instability sets in as stationary convection for two free boundaries. On the onset of convection as over stability; the solution for the case of two free boundaries.

## UNIT – IV

**The effect of magnetic field:** The Perturbation equations. The case when instability sets in as stationary convection; A variational principle. Solutions for stationary convection and for over stability for the case of two free boundaries. **The stability of superposed fluids.**

## UNIT – V

(i) **Rayleigh-Taylor instability:** The Perturbation equations. Inviscid case (the case of two uniform fluids of constant density separated by a horizontal boundary, the case of exponentially varying density). Effect of rotation. Effect of vertical magnetic field.

(ii) **The Kelvin-Helmholtz instability:** The perturbation equations, the case of two uniform fluids in relative horizontal motion separated by a horizontal boundary, the effect of rotation, the effect of horizontal magnetic field.

## SUGGESTED READINGS

1. Chandrasekhar. S., (1981). Hydrodynamic and Hydromagnetic Stability, Dover Publications.
2. Drazin. P.G and Reid. W.H., (2004). Hydrodynamic Stability, Cambridge University Press

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**2019-2020**

**19RMAT304**

**QUEUEING THEORY**

**Paper-III**

**4H – 4C**

**Instruction Hours / week: L: 4**

**Total: 100**

**End Semester Exam: 3 Hours**

**Course Objectives:**

This course enables the students to learn

- The fundamentals of Markov Chains.
- Classical queueing models.
- Various Markovian queueing systems.
- Multi server queueing models.
- Solve finite input source queues.
- Develop queueing models to analyze computer networks.

**Course Outcomes (Cos):**

After completing this course, the student will be able to:

1. Mastery in concepts of discrete and continuous time Markov Chains
2. Explain single server queues
3. Examine steady state solution of important queues.
4. Investigate multi server queues solution.
5. Understand input source models.
6. Model real life queueing scenarios into mathematically.

**UNIT I**

Introduction-Markov Chains- Basic ideas-Classification of states and chains- Sojourn time - Transition density matrix or infinitesimal generator - Limiting behavior: ergodicity - Transient solution -Alternative definition.

**UNIT II**

Birth-and-Death Processes: Special case: M/M/1 queue -Pure birth process-Yule-Furry process. Queueing Systems: General Concepts: Basic characteristics -The input or arrival pattern of customers -The pattern of service -The number of servers -The capacity of the system - The queue discipline. The Simple M/M/1 Queue : Steady-state solution of M/M/1 - Waiting-time distributions - The output process -Semi-Markov process analysis.

**UNIT III**

System with Limited Waiting Space: The M/M/1/K Model: Steady-state solution - Expected number in the system  $L_K$  - Equivalence of an M/M/1/K model with a two-stage cyclic model - Birth-and-Death Processes: Exponential Models - The M/M/ $\infty$  Model: Exponential Model with an Infinite Number of Servers.

#### **UNIT IV**

The Model M/M/c : Steady-state distribution - Expected number of busy and idle servers - Waiting-time distributions - The output process .The M/M/c/c System: Erlang Loss Model: Erlang loss (blocking) formula: Recursive algorithm -Relation between Erlang's B and C formulas .

#### **UNIT V**

Model with Finite Input Source : Steady-state distribution: M/M/c//m ( $m > c$ ). Engset delay model- Engset loss model M/M/c//m/( $m > c$ ) - The model M/M/c//m( $m \leq c$ ).

#### **SUGGESTED READINGS**

1. Medhi J., (2003). Stochastic models in Queueing theory, 2e, Academic press.
2. Donald Gross, John F. Shortle , James M.Thompson , Carl M., and Harris , (2008). Fundamentals of Queueing theory, Wiley.
3. Narayan Bhat U.,(2008). An introduction to Queueing theory: Modelling and Analysis in Applications, Birkhauser Basel.